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Small Area Estimation with State-Space Common Factor Models for Rotating Panels

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Summary

Macro-economic indicators about the labour force, published by national statistical institutes, are predominantly based on rotating panels. Sample sizes of most Labour Force Surveys in combination with the design-based or model-assisted mode of inference, usually applied in survey sampling, obstructs the publication of such indicators on a monthly frequency. It is shown how multivariate structural time series models can be used to obtain more precise model-based estimates. These models take advantage of sample information observed in preceding periods, in other domains, or strongly correlated auxiliary series. In addition they account for the rotation group bias and the serial correlation induced by the rotating panel design. The models are applied to real life data obtained with the Dutch LFS. A relatively parsimonious version of these models is currently used by Statistics Netherlands to produce official monthly figures about the labour force.

Keywords: Survey sampling, Structural time series modelling, cointegration, labour force survey, Kalman filter

1. Introduction

The purpose of national statistical institutes is to produce reliable macro-economic indicators. Some of the most important indicators are monthly, quarterly and annual figures about the labour force, which are generally obtained with the Labour Force Survey (LFS). National statistical institutes often apply rotating panel designs for their LFS. Design-based or model-assisted estimation techniques, known from classical sampling theory, like the general regression (GREG) estimator (Särndal et al., 1992) are widely applied by national statistical institutes to produce official statistics about the employed and unemployed labour force. A major drawback of these estimators is that they have unacceptably large standard errors in the case of small sample sizes. Sample sizes of LFS's are often too small to produce sufficiently reliable monthly figures with these design-based estimators. Since there is a large demand at labour force indicators at a monthly frequency, many national statistical institutes publish each month a rolling quarterly figure as a pragmatic solution. Another problem often encountered with rotating panels are the systematic differences between population parameter estimates that are based on the observations obtained in the subsequent waves of a panel. This phenomenon is known in the literature as rotation group bias (RGB), Bailar (1975).

Previous research proposed a multivariate structural time series model to solve problems with small sample sizes and RGB simultaneously, Pfeffermann (1991), Van den Brakel and Krieg (2009). With this time series model sample information observed in preceding periods is used to improve the precision of the current monthly estimate, which is a form of small area estimation. Simultaneously modelling the series observed in the different waves enables exclusion of the RGB from the time series estimates of labour force parameters. This model is used by Statistics Netherlands since June 2010 to produce official monthly statistics about the labour force at national level and a breakdown in six domains that is based on a cross classification of gender and age in three categories.

Design-based and model-assisted estimators only use the sample units observed in a particular domain, subpopulation or reference period to estimate population parameters for such domains or periods. Small area estimation refers to the problem where the available sample is too small to apply design-based or model-assisted estimators to produce sufficiently precise sample estimates for domains of interest. In such situations model-based estimation procedures are considered, where an explicit statistical model is used to improve the effective sample size to estimate parameters for the different domains or reference periods with sample information that is observed in preceding periods or other domains, Rao (2003), Pfeffermann (2013).

In a design-based inference frame work, the population parameters of interest are regarded as fixed but unknown values that are estimated with observation obtained in the sample. Applying this concept in repeated surveys implies that only the sample information in each particular reference period is used to estimate the population parameters of interest for this period. Generally this is not a very efficient approach since in reality the population parameters for subsequent periods are strongly correlated with each other. The unemployed labour force in month t e.g. will be closely related to the values in the preceding months. One possibility to take advantage of the sample information that is observed in preceding periods, is to consider the values of the finite population parameters of interest as the outcome of a stochastic process that can be modelled with a time series model. The idea to apply time series models to improve time series obtained with repeated surveys, dates back to Scott and Smith (1974).

Using time series models to improve the precision of direct estimates with sample information observed in preceding periods is one possibility of small area estimation. If time series of different domains are correlated, parameter estimates for the individual domains can be further improved by modelling the observed time series simultaneously in a multivariate model. Pfeffermann and Burck (1990), Pfeffermann and Bleuer (1993) modelled the correlation between the trends of domain series in a multivariate structural time series model. This improves the precision of direct domain estimates by taking advantage of sample information observed in preceding periods as well as other domains. Another approach to improve the precision of domain estimates is to benchmark the time series estimates of domains to more precise outcomes obtained at a higher aggregation level, as proposed by Pfeffermann and Tiller (2006). Krieg and Van den Brakel (2012) modelled domain series in a multivariate time series model and applied the concept of cointegration to construct more parsimonious common trend models. Another well-known approach to use sample information from preceding periods and other domains are time-series multilevel model versions of the Fay-Herriot model (1979), see e.g. Rao and Yu (1994) and Datta et al. (1999). Harvey and Chung (2000) considered a bivariate state-space model for the estimated unemployment series of the LFS and the series of claimant counts in the UK to obtain stable and more precise estimates of change in unemployment. Durbin and Quenneville (1997) used state-space models to benchmark survey time series data to strongly related auxiliary series to estimate the absolute bias in the survey estimates.

The purpose of this paper is to illustrate how effective econometric modelling of time series can be used to improve the precision of time series of macro-economic indicators produced by national statistical institutes using traditional design-based inference procedures. Modelling time series observed with repeated surveys requires a component that models the stochastic structure of the underlying population parameter as well as components that account for the data collection of the underlying survey. In this paper the starting point is a multivariate structural time series model for the series of the different waves for each domain separately. The time series component for the population parameter of interest increases the effective sample size for domain estimates with sample information observed in preceding periods within each domain. Two other components are used to model the data collection process and accounts for RGB, serial correlation induced by the rotating panel and for heteroscedasticity in the variance due gradual changes in the sample size over time. This model is extended in two different directions.

The first extension considered in this paper is a multivariate model that combines the series of the waves for all publication domains. Earlier papers, e.g. Pfeffermann and Burck (1990), Pfeffermann and Bleuer (1993) and Krieg and Van den Brakel (2012), modelled the correlation between trends of cross-sectional domain series in a multivariate structural time series model. This paper combines the wave series of the rotating panel of all domains in a multivariate model to take advantage of the correlation between the trend component, seasonal component and RGB of the domains. The concept of cointegration is applied to construct common factors for each of these components. It is investigated to what extent the precision of the domain estimates can be improved further with these common factor models.

The second extension considered in this paper is the inclusion of the series of an auxiliary variable that is highly correlated with the target variable. For the monthly unemployed labour force, a series of the number of people that receive unemployment benefits can be added to the model. The correlation between trend component of the auxiliary series and the series of the target variable can be modelled to improve further the precision of time series model estimates for the target parameter. An alternative approach is to model the auxiliary series as an additional wave and account for bias in the level and seasonal component. Both approaches will be applied to monthly unemployed labour force and their advantages and disadvantages will be discussed.

The models considered in this paper are not limited to LFS data, but are also applicable to other time series that are observed by rotating panel designs. Models for time series observed with cross-sectional repeated survey follow directly as a special case.

The paper starts in Section 2 with a brief review of design-based inference that is typically used in survey sampling and official statistics. The survey design of the Dutch Labour Force Survey (LFS) is summarised in Section 3. In Section 4, the time series model for the five waves of a domain population parameter is described. In Section 5, this model is generalised to models that allow for common trend, common seasonals or common RGB components. In Section 6 it is explained how the model from Section 4 can be extended with a related auxiliary series. Section 7 briefly summarises how these multivariate state-space models are analysed. Results for the common factor models from Section 5 are presented in Section 8. Results for the models with auxiliary series are presented in Section 9. The paper concludes with a discussion in Section 10.

2. Design-based inference

Estimation techniques employed by national statistical institutes are largely design based. This implies that statistical inference is predominantly based on the stochastic structure of the sampling design, while statistical models only play a minor role. The GREG estimator (Särndal et al., 1992) is an example of this class of estimators. This estimator expands or weights the observations obtained in the sample with the so-called survey weights such that the sum over the weighted observations is approximately design unbiased for the unknown population total. The survey weights are initially derived from the sampling design, by taking the weights equal to the inverse of the inclusion probabilities of the sampling units. In a second step these design-weights are calibrated, such that the sum over the weighted auxiliary variables in the sample equate to the known population totals. Under the model-assisted approach, the GREG estimator is derived from a linear regression model that specifies the relationship between the values of a certain target parameter and a set of auxiliary variables.

This class of estimators has nice properties that make them very attractive to be used in a production process to compile timely official statistics. GREG estimators are, in the absence of non-response, asymptotically design unbiased and consistent, see Isaki and Fuller (1982), and Robinson and Särndal (1983). This provides a form of robustness in the case of large sample sizes. If the underlying linear model of the GREG estimator explains the variation of the target parameter in the finite population reasonably well, then the use of auxiliary information results in a reduction of the design variance and also decreases the bias due to selective non-response. Model misspecification might result in an increase of the design variance but the property that the GREG estimator is approximately design unbiased remains. From this point of view, the GREG estimator is robust against model misspecification. Additionally these estimators only require one set of weights to estimate all possible target variables, which is an attractive practical advantage in multipurpose surveys.

Major drawbacks of GREG estimators are the relatively large design variances in the case of small sample sizes, and the fact that they do not handle measurement errors effectively. Most national statistical institutes apply a rotating panel design for their LFS to produce official statistics about the labour force. In most countries the monthly sample size of the LFS is too small to rely on the GREG estimator to produce reliable official statistics about the monthly employed and unemployed labour force. Therefore many national statistical institutes publish each month rolling quarterly figures about the labour force. Rolling quarterly figures have the obvious disadvantages that monthly seasonal patterns are smoothed out and that they are less timely since the monthly publications refer to the latest rolling quarter instead of the latest month.

The second problem is RGB. In the case of the Dutch LFS, the estimated unemployed labour force in the first wave is systematically larger than the subsequent waves, Van den Brakel and Krieg (2009). This is the result of systematic differences in measurement errors in the different waves, due to e.g. differences in questionnaires and data collection modes between the waves, panel attrition and panel effects.

In such situations, model-based procedures can be used to produce more reliable estimates. Pfeffermann (1991) proposed a multivariate structural time series model to model the series observed in the separate waves of a rotating panel. This model improves the precision of the GREG estimates for the labour force by using sample information from preceding periods, account for the RGB by explicitly modelling the differences between series observed in the subsequent waves and account for the autocorrelation in the survey errors due to the rotating panel design. Van den Brakel and Krieg (2013) applied this model to the Dutch LFS and illustrate how this model-based approach enables Statistics Netherlands to produce reliable indicators about the labour force on a monthly frequency. They also extended the model to account for systematic differences in the series induced by a redesign of the underlying survey process.

3. Dutch Labour Force Survey

The objective of the Dutch LFS is to provide reliable information about the Dutch labour force. The target population of the LFS consists of the non-institutionalised population aged 15 years and over, residing in the Netherlands. The sampling frame is a list of all known occupied addresses in the Netherlands, which is derived from the municipal basic registration of population data. Each month a stratified two-stage cluster design of addresses is selected. Strata are formed by geographical regions. Municipalities are considered as primary sampling units and addresses as secondary sampling units. All households residing at an address, up to a maximum of three, are included in the sample and can be regarded as the ultimate sampling units. Most target parameters of the LFS concern people aged 15 through 64 years. Different subpopulations are oversampled to improve the accuracy of the official releases, for example addresses with persons registered at the employment office and subpopulations with low response rates.

Since October 1999, the LFS is conducted as a rotating panel design. Data in the first wave are collected by means of computer assisted personal interviewing (CAPI). The respondents aged 15 through 64 years are re-interviewed four times at quarterly intervals by means of computer assisted telephone interviewing (CATI). During these re-interviews a condensed questionnaire is applied to establish changes in the labour market position of the respondents. Participation of households with the Dutch LFS is on a voluntary basis. The monthly gross sample size for the first wave averaged about 8000 addresses commencing the moment that the LFS changed to a rotating panel design and gradually declined to about 6500 addresses at the end of the period of the series considered in this paper, i.e. June 2010. The response rate is about 55% in the first wave and in the subsequent waves about 90% with respect to the responding households from the preceding wave.

In July 2010 the data collection in the first wave changed from CAPI to a mix of CAPI and CATI. This survey redesign resulted in discontinuities that required an extension of the time series model, see Van den Brakel and Krieg (2013). In this paper the focus is on the extension to multivariate models that use sample information from other domains or from auxiliary series. Therefore the data are used, which are obtained with the panel until July 2010 to avoid unnecessarily complex model formulations. Extensions to time series models that account also for discontinuities are relatively straightforward, as explained in Van den Brakel and Krieg (2013).

Key parameters of the LFS are the employed, unemployed and total labour force, which are defined as population totals. Another important parameter is the unemployment rate, which is defined as the ratio of the unemployed labour force over the total labour force. Monthly estimates for these parameters are produced at the national level as well as a breakdown in six domains that is based on the cross classification of gender and three age classes. Monthly estimates are obtained with the following estimation procedure. Each month data are collected in five independent waves. The GREG estimator is applied to produce five independent estimates for a target parameter. Inclusion probabilities reflect the sampling design described above as well as the different response rates between geographic regions. The weighting scheme is based on a combination of different socio-demographic categorical variables. This results in five series of monthly GREG estimates for each target parameter, which are the input for a multivariate structural time series model described in Section 4. With this model reliable estimates for the population parameters are obtained by taking advantage of the sample information observed in preceding periods. The model also accounts for RGB and autocorrelation induced by the rotating panel design. Since 2010 this approach is applied to produce official monthly figures about the labour force, Van den Brakel and Krieg (2013).

4. Structural time series model for separate domains

With a structural time series model, a time series is decomposed in a trend component, a seasonal component, other cyclic components, a regression component and an irregular term. The different components are stochastic, which allows the different components to be time dependent. See Harvey (1989) and Durbin and Koopman (2001) for an introduction in structural time series models and state-space models.

Let $\hat{y}^{j}_{t,d}$ denote the GREG estimator for the unknown population parameter $\theta_{t,d}$ for domain *d* at time *t* based on the *j*-th wave observed at time *t*, *j*= 1, …, 5. Due to the applied rotation pattern, each month a vector $\hat{\mathbf{y}}_{t,d} = (\hat{y}^{1}_{t,d}\ \hat{y}^{2}_{t,d}\ \hat{y}^{3}_{t,d}\ \hat{y}^{4}_{t,d}\ \hat{y}^{5}_{t,d})^{T}$ is observed. As a result, a five dimensional time series with GREG estimates for the monthly employed and unemployed labour force is obtained. According to Pfeffermann (1991), Van den Brakel and Krieg (2009), this vector can be modelled as

$$\hat{\mathbf{y}}_{t,d} = \mathbf{1}_{5}\theta_{t,d} + \boldsymbol{\lambda}_{t,d} + \mathbf{e}_{t,d} , \qquad (1)$$

with $\mathbf{1}_{5}$ a five dimensional vector with each element equal to one, $\boldsymbol{\lambda}_{t,d} = (\lambda^{1}_{t,d}\ \lambda^{2}_{t,d}\ \lambda^{3}_{t,d}\ \lambda^{4}_{t,d}\ \lambda^{5}_{t,d})^{T}$ a vector with time dependent components that account for the RGB, and $\mathbf{e}_{t,d} = (e^{1}_{t,d}\ e^{2}_{t,d}\ e^{3}_{t,d}\ e^{4}_{t,d}\ e^{5}_{t,d})^{T}$ the corresponding survey errors for each wave estimate.

The population parameter $\theta_{t,d}$ in (1) can be decomposed in a trend component, a seasonal component, and an irregular component, i.e.

$$\theta_{t,d} = L_{t,d} + S_{t,d} + \varepsilon_{t,d} . \qquad (2)$$

Here $L_{t,d}$ denotes a stochastic trend component. In this paper the so-called smooth trend model is used, i.e.

$$\begin{aligned} &L_{t,d} = L_{t-1,d} + R_{t-1,d} , \\ &R_{t,d} = R_{t-1,d} + \eta_{R,t,d} , \\ &E(\eta_{R,t,d}) = 0, \quad Cov(\eta_{R,t,d}, \eta_{R,t',d}) = \begin{cases} \sigma^{2}_{R,d} & \text{if} \quad t = t' \\ 0 & \text{if} \quad t \neq t'. \end{cases} \end{aligned} \qquad (3)$$

This component models the low frequency variation, including economic cycles. A slightly more general trend model is the local linear trend model. This model also has a disturbance term for the level parameter $L_{t,d}$ in (3). Previous research shows that applying the local linear trend model does not improve the fit to the data (Van den Brakel and Krieg, 2009).

Furthermore, $S_{t,d}$ denotes a trigonometric stochastic seasonal component,

$$S_{t,d} = \sum_{l=1}^{6} S_{l,t,d} \ , \tag{4}$$

where

$$\begin{aligned}
&S_{l,t,d} = S_{l,t-1,d}\cos(h_l) + S^*_{l,t-1,d}\sin(h_l) + \eta_{\omega,l,t,d}\\
&S^*_{l,t,d} = S^*_{l,t-1,d}\cos(h_l) - S_{l,t-1,d}\sin(h_l) + \eta_{\varpi,l,t,d}\,,\ h_l = \frac{\pi\, l}{6},\ \ l = 1,\ldots,6,\\
&E(\eta_{\omega,l,t,d}) = E(\eta_{\varpi,l,t,d}) = 0,\\
&Cov\,(\eta_{\omega,l,t,d},\eta_{\omega,l',t',d}) = Cov\,(\eta_{\varpi,l,t,d},\eta_{\varpi,l',t',d}) = \begin{cases}\sigma^2_{\omega,d} & \text{if} \quad l = l' \ \text{and} \ t = t'\\ 0 & \text{if} \quad l \neq l' \ \text{or} \ t \neq t'\end{cases},\\
&Cov\,(\eta_{\omega,l,t,d},\eta_{\varpi,l,t,d}) = 0,\ \forall\, l,\ \forall\, t.
\end{aligned} \tag{5}$$

Finally, $\varepsilon_{t,d}$ denotes the irregular component, which contains the unexplained variation and is modelled as a white noise process:

$$E(\varepsilon_{t,d}) = 0, \quad Cov\,(\varepsilon_{t,d},\varepsilon_{t',d}) = \begin{cases}\sigma^2_{\varepsilon,d} & \text{if} \quad t = t'\\ 0 & \text{if} \quad t \neq t'.\end{cases} \tag{6}$$

Through component (2) values for $\theta_{t,d}$ are related with the population values from preceding periods. This component shows how sample information observed in preceding periods is used to improve the precision of the estimates for $\theta_{t,d}$ in a particular month.

The systematic differences, i.e. the RGB, between the subsequent waves are modelled in (1) with $\boldsymbol{\lambda}_{t,d}$. The absolute bias in the monthly labour force figures cannot be estimated from the sample data only. In this application the model is identified by taking the first component of $\boldsymbol{\lambda}_{t,d}$ equal to zero. This implies that it is assumed that an unbiased estimate for $\theta_{t,d}$ is obtained with the first wave, i.e. $\hat{Y}^1_{t,d}$. The other elements of $\boldsymbol{\lambda}_{t,d}$ measure the time dependent differences with respect to the first wave. To this end $\lambda^j_{t,d}$ are modelled as random walks for j = 2,3,4, and 5. As a result it follows that

$$\lambda^1_{t,d} = 0, \quad \lambda^j_{t,d} = \lambda^j_{t-1,d} + \eta_{\lambda,j,t,d}\,, \qquad j = 2, 3, 4, 5, \tag{7}$$

$$E(\eta_{\lambda,j,t,d}) = 0, \quad Cov\,(\eta_{\lambda,j,t,d},\eta_{\lambda,j',t',d}) = \begin{cases}\sigma^2_{\lambda,d} & \text{if} \quad t = t' \quad \text{and} \quad j = j'\\ 0 & \text{if} \quad t \neq t' \quad \text{or} \quad j \neq j'.\end{cases}$$

Finally a time series model for the survey errors $\mathbf{e}_{t,d}$ in (1) is developed. The direct estimates for the design variances of the survey errors are available from the micro data and are incorporated in the time series model using the survey error model $e^j_{t,d} = k^j_{t,d}\tilde{e}^j_{t,d}$ where $k^j_{t,d} = \sqrt{V\hat{a}r(\hat{y}^j_{t,d})}$, proposed by Binder and Dick (1990). Here $V\hat{a}r(\hat{y}^j_{t,d})$ denotes the estimated variance of the GREG estimate for the d-th domain. Choosing the survey errors proportional to the standard error of the GREG estimates allows for non-homogeneous variance in the survey errors, that arise e.g. due to the gradually decreasing sample size over the last decade.

The sample of the first wave is observed for the first time at time *t*, so survey errors are not correlated with survey errors in the past. It is, therefore, assumed that $\tilde{e}^1_{t,d}$ is white noise with $E(\tilde{e}^1_{t,d}) = 0$ and $Var(\tilde{e}^1_{t,d}) = \sigma^2_{e1,d}$. As a result, the variance of the survey error equals $Var(e^1_{t,d}) = (k^1_{t,d})^2 \sigma^2_{e1,d}$, which is approximately equal to the direct estimate of the variance of the GREG estimate for the first wave if the maximum likelihood (ML) estimate for $\sigma^2_{e1,d}$ is close to one. The survey errors of the second, third, fourth and fifth wave are correlated with survey errors of preceding periods and are modelled with an AR(1) model; $\tilde{e}^j_{t,d} = \rho_d \tilde{e}^{j-1}_{t-3,d} + \nu^j_{t,d}$, with ρ_d the first order autocorrelation coefficient of domain *d* that is estimated from the survey data using the approach proposed by Pfeffermann et al. (1998). Furthermore $E(\nu^j_{t,d}) = 0$, and $Var(\nu^j_{t,d}) = \sigma^2_{ej,d}$ for $j = 2,3,4,5$. Since $Var(e^j_{t,d}) = \sigma^2_{ej,d}(k^j_{t,d})^2/(1-\rho^2_d)$, $Var(e^j_{t,d})$ is approximately equal to $V\hat{a}r(\hat{y}^j_{t,d})$ provided that the ML estimate for $\sigma^2_{ej,d}$ is close to $(1-\rho^2_d)$.

With the time series model estimates for the population parameter are obtained, which is defined as the trend (3) plus the seasonal component (4). In this paper, this is further referred to as the *signal*. Another important indicator is the trend (3) of the population parameter. Both are currently published as official monthly indicators.

5. Multivariate time series models for D domains

The model in Section 4 is applied to each domain separately and therefore only uses sample information observed in preceding periods within each domain separately. Simultaneously modelling the D domains gives rise to a model that also uses sample information from different domains. A model that combines the D multivariate state-space models defined by (1) for the domains is

$$\begin{pmatrix} \hat{\mathbf{y}}_{t,1} \\ \vdots \\ \hat{\mathbf{y}}_{t,D} \end{pmatrix} = \begin{pmatrix} \mathbf{1}_5 \theta_{t,1} \\ \vdots \\ \mathbf{1}_5 \theta_{t,D} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\lambda}_{t,1} \\ \vdots \\ \boldsymbol{\lambda}_{t,D} \end{pmatrix} + \begin{pmatrix} \mathbf{e}_{t,1} \\ \vdots \\ \mathbf{e}_{t,D} \end{pmatrix} \equiv \hat{\mathbf{Y}}_t = \boldsymbol{\Theta}_t + \boldsymbol{\Lambda}_t + \mathbf{E}_t . \tag{8}$$

This model allows for correlation between disturbances of the different components of the structural time series model. For the population parameters it makes sense to allow for correlation between the slope disturbances of the trend components. This implies that the covariance structure for the slope disturbances in (3) is replaced by

$$Cov(\eta_{R,t,d}, \eta_{R,t',d'}) = \begin{cases} \sigma^2_{R,d} & \text{if} \quad t = t' \text{ and } d = d' \\ \varsigma_{R,dd'} & \text{if} \quad t = t' \text{ and } d \neq d' \\ 0 & \text{if} \quad t \neq t' . \end{cases} \tag{9}$$

If the model detects a strong correlation between the slope disturbances, then the trends of the domains will develop into the same direction more or less simultaneously. In this way sample information observed in other domains is used to improve the precision of the separate domain estimates, which is another form of small area estimation. In a similar way it makes sense to allow for correlation between the disturbances of the seasonal components of the domains, resulting in the following extension of the covariance structure defined by (5)

$$Cov(\eta_{\omega,l,t,d}, \eta_{\omega,l',t',d'}) = Cov(\eta_{\varpi,l,t,d}, \eta_{\varpi,l',t',d'}) = \begin{cases} \sigma^2_{\omega,d} & \text{if} \quad l = l' \text{ and } t = t' \text{ and } d = d' \\ \varsigma_{\omega,dd'} & \text{if} \quad l = l' \text{ and } t = t' \text{ and } d \neq d' \\ 0 & \text{if} \quad l \neq l' \text{ or } t \neq t' . \end{cases} \tag{10}$$

Also the RGB observed in the domains might develop simultaneously into the same direction, and can be modelled by extending the covariance structure in (7) to:

$$Cov(\eta_{\lambda,j,t,d}, \eta_{\lambda,j',t',d'}) = \begin{cases} \sigma^2_{\lambda,d} & \text{if} \quad t = t' \text{ and } j = j' \text{ and } d = d' \\ \varsigma_{\lambda,dd'} & \text{if} \quad t = t' \text{ and } j = j' \text{ and } d \neq d' \\ 0 & \text{if} \quad t \neq t' . \end{cases} \tag{11}$$

Strong correlation between the disturbances of the seasonals or the RGB implies that these components develop in the same direction. Taking advantages of this correlation might result in more precise estimates for these components as well as the population parameters of interest. For the white noise of the population parameters as well as the survey errors between domains, non-diagonal covariance structures are not considered.

6. Time series models for separate domains with auxiliary series

Another possibility to improve the precision of the monthly labour force figures is to extend model (1) with an auxiliary series that is strongly correlated with the population parameter measured with the LFS. In the case of estimating the unemployed labour force, the number of people that receive employment benefits is a potential auxiliary variable to be included in the model. Let $x_{t,d}$ denote the auxiliary series for domain *d*, *t=1,…,T*. There are different ways to include this information in the time series model. The first approach is to extend model (1) as

$$\begin{pmatrix} \hat{\mathbf{y}}_{t,d} \\ x_{t,d} \end{pmatrix} = \begin{pmatrix} \mathbf{1}_5 \theta^y_{t,d} \\ \theta^x_{t,d} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\lambda}_{t,d} \\ 0 \end{pmatrix} + \begin{pmatrix} \mathbf{e}_{t,d} \\ 0 \end{pmatrix}. \tag{12}$$

The series of the LFS and the auxiliary series from the register both have their own population parameter that can be modelled with two separate time series models defined similarly to (2), i.e. $\theta^z_{t,d} = L^z_{t,d} + S^z_{t,d} + \varepsilon^z_{t,d}$, where $z = y$ or $z = x$. The trend, seasonal and white noise components are defined by (3) through (6). Since the auxiliary series is based on a registration, this series does not have a RGB or a survey error component. Therefore the last element of the vector for the RGB and the vector of the survey errors in (12) are equal to zero.

The model allows for correlation between the disturbances of the slope from the trend model of the LFS and the auxiliary series by assuming the following covariance structure for the trend:

$$Cov(\eta^z_{R,t,d}, \eta^{z'}_{R,t',d'}) = \begin{cases} \sigma^2_{R,d,z} & \text{if} \quad t = t' \text{ and } d = d' \text{ and } z = z' \\ \varsigma_{R,d,zz'} & \text{if} \quad t = t' \text{ and } d = d' \text{ and } z \neq z' \\ 0 & \text{if} \quad t \neq t' \text{ or } d \neq d'. \end{cases} \tag{13}$$

The correlation between both series is determined by the model. If the model detects a strong correlation, then the trends of both series will develop into the same direction more or less simultaneously. In the case of strong correlations, the auxiliary information will result in an increased precision of the model estimates for the monthly unemployment figures. Correlations between the disturbances of the seasonal components and white noise are not considered.

Model (12) is a variation of the model proposed by Harvey and Chung (2000). They proposed a bivariate structural time series model for the estimated series of unemployed labour force obtained with the LFS and a series of claimant counts in the UK and use the correlation between both series to improve the precision the estimated change in unemployment. Their model accounts for the serial correlation of the survey errors but ignores the RGB of the rotating panel applied in the LFS of the UK.

Model (12) assumes that the correlation between the slope disturbances $\varsigma_{R,d,zz'}$ of the LFS series and the auxiliary series is fixed over the entire period of the series. A similar assumption is made in the bivariate model of Harvey and Chung (2000). This is a strong assumption that can be violated easily. Amendments of the law with respect to unemployment benefits, changes in the period that persons receive unemployment benefits, introduction of new inspection procedures concerning the detection of illegally receiving benefits or changes in the mode of operation of the employment office might result in sudden or gradual differences of the number of people that receive unemployment benefits. As a result the correlation between the slope disturbances of the LFS series and the auxiliary series becomes time dependent. This is not reflected by the time independent covariance structure in (13) and might give rise to model misspecification. It is of course possible to define separate covariances for different time intervals. The problem remains that a gradual change in correlation cannot be detected immediately, resulting in a period where the model is misspecified.

The second approach to include auxiliary information in model (1) is to define one population parameter and add an additional component to model the difference between the trends of the LFS and the auxiliary series similar to the RGB of the second through the fifth wave. Furthermore a term for the seasonal component is added to account for discrepancies between the seasonal pattern of the LFS and the auxiliary series. As a result the following model is obtained:

$$\begin{pmatrix} \hat{\mathbf{y}}_{t,d} \\ x_{t,d} \end{pmatrix} = \mathbf{1}_6 \theta_{t,d} + \begin{pmatrix} \boldsymbol{\lambda}_{t,d} \\ \lambda^x_{t,d} \end{pmatrix} + \begin{pmatrix} 0 \\ S^x_{t,d} \end{pmatrix} + \begin{pmatrix} \mathbf{e}_{t,d} \\ 0 \end{pmatrix}. \tag{14}$$

In this model $\lambda^x_{t,d}$, stands for the bias in the trend of the auxiliary variable with respect to the series of GREG estimates observed in the first wave of the LFS panel. This can be modelled with a random walk in a similar way as the RGB of the other waves using formula (7). It turns out that it is more appropriate to use the smooth trend model defined by (3) to capture the systematic difference between the trend of the auxiliary variable and the trend in the LFS parameter. The component $S^x_{t,d}$ is a stochastic trigonometric seasonal component, as defined by (4) and (5), that models the systematic difference between the seasonal pattern in the auxiliary variable and the LFS parameter. The reason to consider (14) as an alternative for (12) is that differences between the auxiliary series and the LFS parameter in the trend and the seasonal component are modelled with time dependent components. As a result (14) might, compared to (12), (13), be more flexible to capture gradual changes in the relation between the auxiliary series and the LFS series.

An alternative way to incorporate auxiliary information in the model is to extend the time series model (2) for the population parameter of the LFS with a regression component for the auxiliary series, i.e. $\theta_{t,d} = L_{t,d} + S_{t,d} + b_d x_{t,d} + \varepsilon_{t,d}$, where b_d denotes the regression coefficient expressing the relation between the target series *Y* and the auxiliary series *X*. The major drawback of this approach is that the auxiliary series will partially explain the trend and seasonal effect in $\theta_{t,d}$, leaving only a residual trend and seasonal effect for $L_{t,d}$ and $S_{t,d}$. This hampers the estimation of a trend for the target variable. This approach is not considered in this paper.

7. Estimation

The structural time series models discussed in the preceding sections can be expressed in the so-called state-space representation. A state-space model contains a measurement equation and a transition equation. First, let $\boldsymbol{\alpha}_t$ denote a vector containing the state variables, i.e. level and slope parameter of the trend in (3), the seasonal components (5), the RGB parameters (7) and the sampling errors $\tilde{e}_{t,d}$. The measurement equation, $\hat{\mathbf{Y}}_t = \mathbf{Z}_t\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t$, specifies how the observed series depend on the vector of unknown state variables $\boldsymbol{\alpha}_t$ through a known and time dependent design matrix $\mathbf{Z}_t$ and a vector with measurement errors $\boldsymbol{\varepsilon}_t$. The transition equation $\boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \boldsymbol{\eta}_t$ specifies how the state variables evolves gradually over time where $\mathbf{T}$ is a known design matrix and $\boldsymbol{\eta}_t$ a vector with the disturbances of the stochastic processes for the trend, seasonal component, RGB and the sampling error. It is assumed that the vectors with disturbances are normally and independently distributed, i.e. $\boldsymbol{\varepsilon}_t \cong N(\mathbf{0},\mathbf{H})$, with $\mathbf{H} = \text{cov}(\boldsymbol{\varepsilon}_t)$ and, $\boldsymbol{\eta}_t \cong N(\mathbf{0},\mathbf{Q})$, with $\mathbf{Q} = \text{cov}(\boldsymbol{\eta}_t)$. Furthermore it is assumed that $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\eta}_t$ are independently distributed. The diagonal elements of $\mathbf{Q}$ contain the variances of the disturbance terms of the stochastic processes for the trend $\sigma^2_{R,d}$ in (3), the seasonal components $\sigma^2_{\omega,d}$ in (5), the RGB $\sigma^2_{\lambda,d}$ in (7) and $\sigma^2_{ej,d}$ for the survey errors. The off-diagonal elements of $\mathbf{Q}$ contain the covariances between the disturbance terms of the trend $\varsigma_{R,dd'}$ in (9), the seasonals $\varsigma_{\omega,dd'}$ in (10), the RGB $\varsigma_{\lambda,dd'}$ in (11) or the slope disturbances between the LFS and the auxiliary series in (13).

If the model is expressed in state-space form, the Kalman filter can be applied to obtain optimal estimates for the state variables, see e.g. Durbin and Koopman (2001). The Kalman filter assumes that the variance and covariance terms in $\mathbf{H}$ and $\mathbf{Q}$ are known in advance and are often referred to as hyperparameters. In practise these hyperparameters are not known and are therefore substituted with their ML estimates. Estimates for state variables for period t based on the information available up to and including period t are referred to as the *filtered estimates*. They are obtained with the Kalman filter where the ML estimates for the hyperparameters are based on the complete time series. The filtered estimates of past state vectors can be updated, if new data become available. This procedure is referred to as smoothing and results in *smoothed estimates* that are based on the complete time series.

In this application interest is mainly focussed on the estimation results obtained with a structural time series model based on the complete set of information that would be available in the regular production process to produce a model-based estimate for the monthly unemployment figures for month t. This can be approximated with the filtered estimates. Filtered estimates obtained with the Kalman filter, however, use the ML estimates for the hyperparameters that are based on the complete time series. To mimic the estimates as obtained in a production process, the so-called *concurrent estimates* are defined. They are defined as the estimates for state variables for period t based on the information available up to and including period *t* using ML estimates for the hyperparameters that are also based on the information available up to and including period *t*.

The covariance matrix of the transition equation is non-diagonal and therefore implemented as a Cholesky decomposition, i.e. $\mathbf{Q} = \mathbf{ADA}^t$ with **D** a diagonal matrix and **A** a lower triangular matrix with ones on the diagonal. The off-diagonal elements of **A** are zero, except the lower triangular elements that correspond to non-zero covariances. For the multivariate model (8) for D domains, non-zero covariances are defined by $\varsigma_{R,dd'}$ for the slopes in (9), $\varsigma_{\omega,dd'}$ in for the seasonals in (10) or $\varsigma_{\lambda,dd'}$ for the RGB in (11). For the model with auxiliary variables in (12), non-zero covariances are defined by $\varsigma_{R,d,zz'}$ in (13). This decomposition has several advantages. First it ensures that the ML estimates for **Q** are always positive-semidefinite. Second it allows for the detection of common factors. The diagonal elements of **D** are estimated on the log-scale to avoid negative variance estimates.

If in (8) the disturbances for the *D* domains of a component, i.e. slope, seasonal or RGB, are strongly correlated, then this component might be driven by $p<D$ common factors. This implies that **Q** is not of full rank. In the context of structural time series models this is often referred to as cointegration. If *D-p* ML estimates of the diagonal elements of **D** for a component tend to zero, then the variances of the disturbance terms of this component are obtained as a linear combination of *p* common factors. Also the state variables of the different components for the *D* domains can be expressed as a linear combination of *p* common factors, up to a simplified correction term. In a similar way the disturbances of the slope of the LFS population parameter and the auxiliary series in (12) can be driven by one instead of two factors. Expressing the model in terms of a common factor model results in more parsimonious models and improves the efficiency of the estimation procedure. See Harvey (1989), Section 8.5 or Koopman et al. (2007), Section 9.1 for more details concerning cointegration and common factor state-space models.

The analysis is conducted with software developed in OxMetrics in combination with the subroutines of SsfPack 3.0, see Doornik (2009) and Koopman e.a. (1999, 2008). All state variables are non-stationary with the exception of the survey errors. The non-stationary variables are initialised with a diffuse prior, i.e. the expectation of the initial states are equal to zero and the initial covariance matrix of the states is diagonal with elements diverging to infinity. The survey errors are stationary and therefore initialised with a proper prior. The initial values for the survey errors are equal to zero and the covariance matrix is available from the aforementioned model for the survey errors. The exact initial solution for the Kalman filter with diffuse initial conditions, proposed by Koopman (1997) is used. ML estimates for the hyperparameters are obtained using the numerical optimization procedure maxBFGS in OxMetrics.

8. Results common factor models for six domains

In this section we compare the following four versions of model (8):

Model 1: Equation (8) with a diagonal covariance structure for the trend, seasonal and RGB, i.e. $\varsigma_{\eta,dd'}$ in (9), $\varsigma_{\omega,dd'}$ in (10), and $\varsigma_{\lambda,dd'}$ in (11), are equal to zero. This is similar to applying model (1) to each domain separately.

Model 2: Equation (8) with a non-diagonal covariance structure (9) for the trend and a diagonal covariance structure for the seasonal and RGB components, i.e. $\varsigma_{\omega,dd'}$ in (10), and $\varsigma_{\lambda,dd'}$ in (11), are equal to zero. This implies a common factor structure for the trend only.

Model 3: Equation (8) with a non-diagonal covariance structure for the trend (9) and the seasonal component (10), and a diagonal covariance structure for the RGB, i.e. $\varsigma_{\lambda,dd'}$ in (11), are equal to zero. This implies a common factor structure for the trend and the seasonal component.

Model 4: Equation (8) with a non-diagonal covariance structure for the trend (9) and the RGB (11), and a diagonal covariance structure for the seasonal component, i.e. $\varsigma_{\omega,dd'}$ in (10), are equal to zero. This implies a common factor structure for the trend and the RGB.

In the appendix, the ML estimates for the standard deviations of the trend, seasonal, and RGB under the four different models are specified. Model M1 is nested in M2. In a similar way, model M2 is nested in M3 and M2 is also nested in M4. This enables to test the improvement of the model by allowing for non-zero covariances for the trend, the seasonal component and the RGB with the likelihood-ratio test. Results are specified in Table 1. Each model contains 186 state variables. The number of hyperparameters are different and specified in Table 1. The likelihood-ratio tests where the number of degrees of freedom are based on the total number of off-diagonal elements, i.e. 15, are conservative since no advantage is taken from the more parsimonious model formulation that can be obtained with the common factor structures. From the ML estimates for the $\mathbf{ADA}^t$ of the trend it follows that three common factors are required to model the covariance structure of the slope disturbances. A full covariance structure results in a significant increase of the likelihood component, even if the parsimonious common factor structure is disregarded. For the seasonal component two common factors are required. The increase in the likelihood is much smaller compared to the trend, but still significant at a 5% significance level due to the small number of common factors. Modelling a full covariance structure for the RGB does not result in a significant model improvement. The increase of the likelihood is small, while three diagonal elements of the $\mathbf{ADA}^t$ decomposition tends to zero, implying that three common factors are required to model the covariance structure of the disturbance terms.

The correlation matrix for the slope disturbances of the trend is specified in Table 2. The results in Table 2 are obtained with model M2. The correlation matrices for the slope disturbances in M3 and M4 are the same. In general there is a strong correlation between the domains, except between men/15-24 and women/45-64.

The correlation matrix for the disturbances of the seasonal component is specified in Table 3. In this case there is a clear clustering into two groups. The first group contains the domains men/15-24, women/15-24 and men/25-44, who show strong positive or negative correlations. The second group contains the domains of women/25-44, men/45-64 and women/46-64 which all show a strong positive correlation.

The correlation matrix of the disturbances of the RGB are specified in Table 4. The correlations are clearly weaker compared to the patterns observed for the trend and the seasonals.

Table 1: likelihood-ratio tests

Model	Log likelihood	Number of hyper-parameters	hypothesis	LR test statistic	Full covariance matrix		Common factor model		
					df	p-value	cf	df	p-value
M1	24821.2	54							
M2	24855.6	69	M1=M2	68.8	15	0.0000	3	9	0.0000
M3	24862.6	84	M2=M3	14.0	15	0.5255	2	5	0.0156
M4	24862.0	84	M2=M4	12.8	15	0.6177	3	9	0.1719

Cf: number of common factors; df: degrees of freedom.

Table 2: correlation matrix slope disturbances based on model M2

	M/15-24	W/15-24	M/25-44	W/25-44	M/45-64
W/15-24	0.76				
M/25-44	0.93	0.94			
W/25-44	0.65	0.99	0.88		
M/45-64	0.47	0.93	0.75	0.98	
W/45-64	0.07	0.70	0.42	0.80	0.91

Table 3: correlation matrix disturbances seasonal component based on model M3

	M/15-24	W/15-24	M/25-44	W/25-44	M/45-64
W/15-24	-1.00				
M/25-44	-0.99	0.99			
W/25-44	-0.13	0.13	0.00		
M/45-64	-0.05	0.05	-0.07	1.00	
W/45-64	-0.29	0.29	0.17	0.99	0.97

Table 4: correlation matrix disturbances RGB based on model M4

	M/15-24	W/15-24	M/25-44	W/25-44	M/45-64
W/15-24	-0.59				
M/25-44	0.34	0.41			
W/25-44	-0.10	0.67	0.24		
M/45-64	-0.77	0.94	0.29	0.39	
W/45-64	0.31	0.14	0.86	-0.30	0.18

Comparing the standard errors of M1 with the GREG estimator illustrates the increased precision obtained by using sample information observed in preceding periods within each domain separately. In small area estimation this is often referred to as borrowing strength over time. Comparing the standard errors of model M1 with respectively M2, M3, and M4 illustrates the additional increase in precision by using sample information from other domains by modelling the correlation between different time series components of the domain series. In small area estimation literature, this generally referred to as borrowing strength over space. Based on the results of likelihood-ratio tests in Table 1, an additional increase might be expected from model M2 and M3 but not from M4.

Monthly GREG estimates for the unemployed labour force are required to analyse the variance reduction obtained with the different state-space models. These estimates are obtained by weighting the response of the five waves observed in each month. A correction for RGB is based on a crude correction where the GREG estimate is multiplied with a ratio. The numerator of this ratio contains a GREG estimate for the same target variable that is based on the observation in the first wave over the last three years. The denominator contains the GREG estimate for the same target variable using the data observed in the waves over the last three years. As a result the monthly GREG estimate is adjusted to the level of the first wave. This approach was used in the past to correct for RGB in the rolling quarterly figures that were published until June 2010, see Van den Brakel and Krieg (2009) for details.

In Figure 1, the filtered signal obtained with model M1 for women/25-44 is compared with the GREG estimates for the monthly unemployed labour force. This figure shows that the series of the GREG estimates is more erratic, mainly due to large sampling errors in the monthly GREG estimates. Figure 2 and 3 compares the filtered signals and the filtered trends for this domain with model M1, M2 and M3. Modelling the correlation between the slope disturbances result in less volatile trends and in small adjustments of the level of the filtered series (compare M1 with M2). If in addition the correlation between the disturbances of the seasonals are modelled, then the filtered trend or signal are hardly affected (compare M2 with M3). Modelling the correlation between the RGB disturbances (model M4) does not affect the filtered signals and trends compared to model M2. Results are therefore not presented.

Figure 1: Monthly unemployed labour force Women/25-44; filtered signal model M1 versus GREG estimator

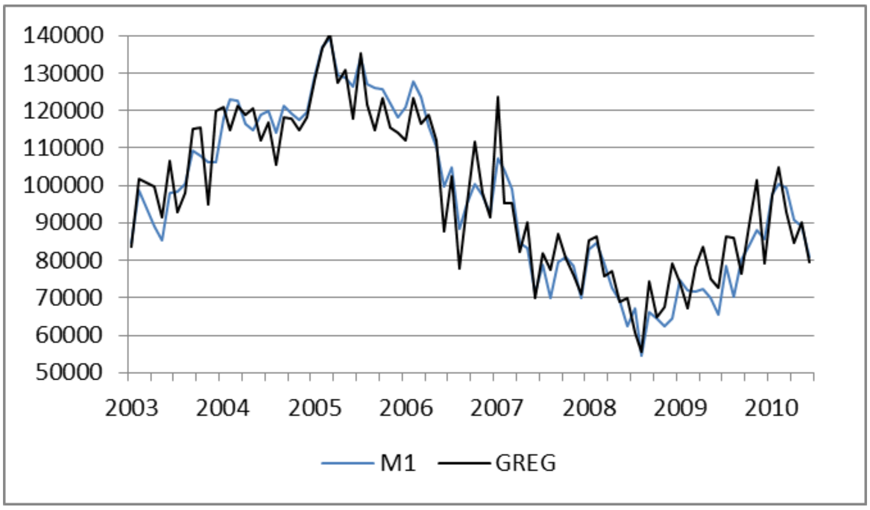


Figure 2: Monthly unemployed labour force Women/25-44; filtered signal model M1, M2 and M3

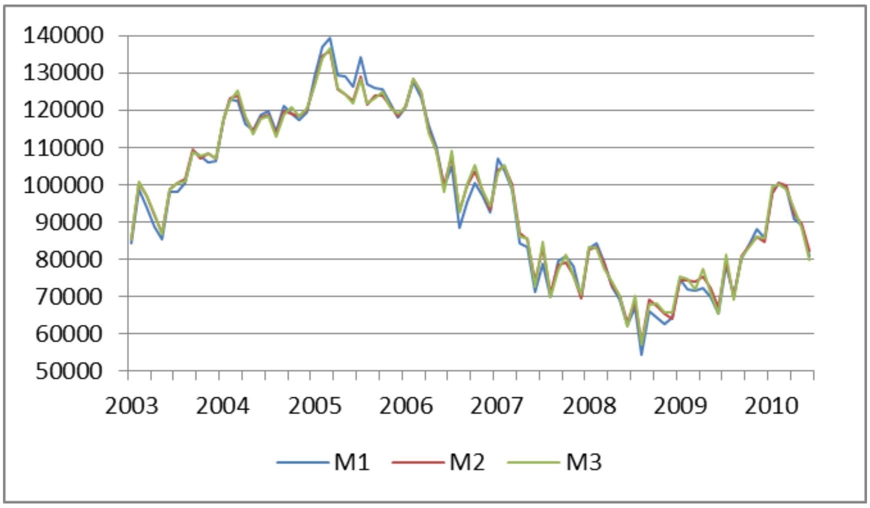
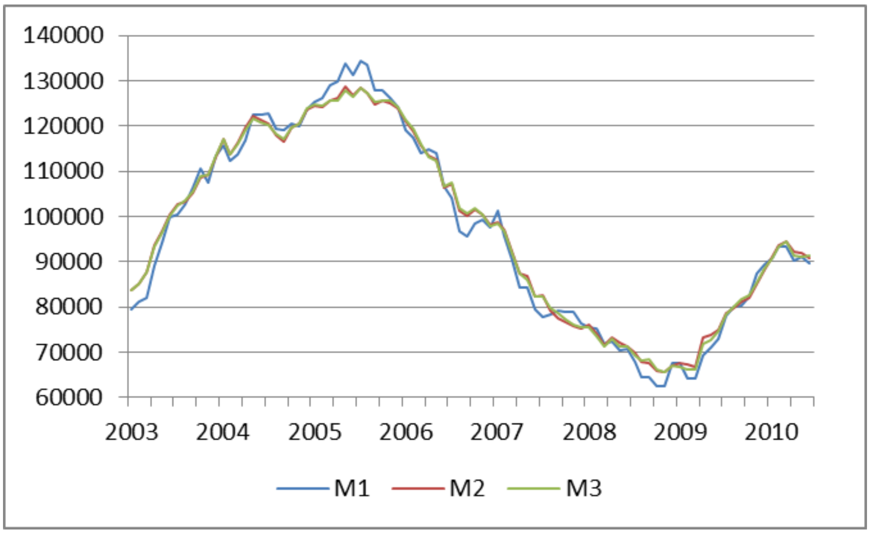
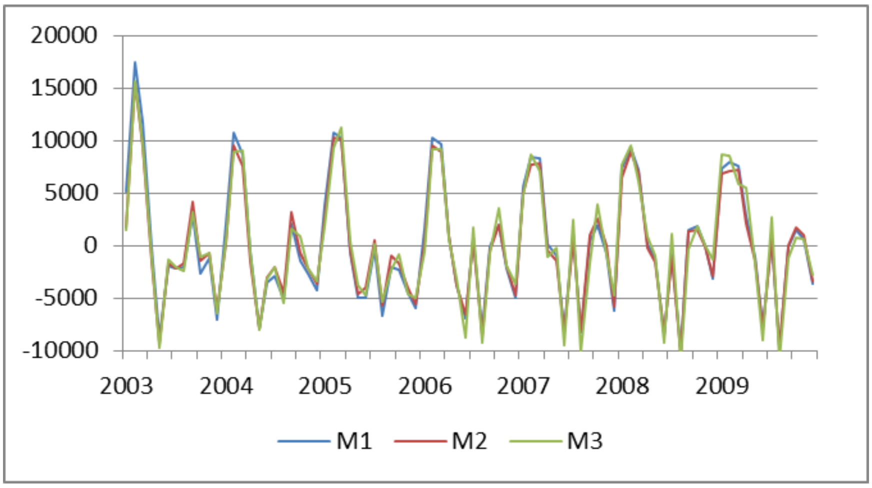


Figure 3: Monthly unemployed labour force Women/25-44; filtered trend model M1, M2 and M3



In Figure 4, the filtered seasonal effects under model M1, M2 and M3 are presented for the domain women/25-44. Modelling correlation between the slope disturbances does not affect the seasonal component as might be expected (compare M1 with M2). Modelling the correlation between the disturbances of the seasonals does not change the seasonal patterns either (compare M2 with M3 or M1 with M3). Model M4 result in a similar seasonal component (result not presented).

Figure 4: Monthly unemployed labour force Women/25-44; filtered seasonals model M1, M2 and M3



To summarise differences between point estimates obtained under two different models or estimation procedures the mean absolute relative difference are used, which is defined as

$$\text{MARD}\ (A,B)=\frac{1}{T}\sum_{t=1}^{T}\frac{\left|y_t^A-y_t^B\right|}{y_t^A}\times 100\ , \tag{15}$$

where y_t^x is the point estimate for a population parameter for period t obtained under estimation procedure x. This abbreviation stands for a GREG estimate or a Kalman filter estimate obtained under model M1 through M4.

In Table 5 the point estimates obtained with the GREG estimator and model M1, M2 and M3 for the six domains over the last 24 months are compared with each other using MARD defined in (15). Comparing MARD(GREG,M1), MARD(GREG,M2) and MARD(GREG,M3), illustrates that modelling the correlation between the trend or the seasonals does not further increase the differences between the filtered and GREG estimates. Modelling the covariance between trends has some effect on the filtered signal and trend as follows from MARD(M1,M2). Further extension to a model that allows for the correlation between seasonals has a very small effect on the filtered trend and signals as follows from MARD(M2,M3). Filtered trends and signals under model M4 are almost exactly equal to M2. Results are therefore omitted.

Table 5: MARD for GREG and filtered estimates under model M1, M2, and M3 for the last 24 months of the series.

Domain	GREG versus filtered signal			Filtered signal			Filtered trend		
	(GREG,M1)	(GREG,M2)	(GREG,M3)	(M1,M2)	(M1,M3)	(M2,M3)	(M1,M2)	(M1,M3)	(M2,M3)
M/15-24	7.37	7.95	7.83	2.72	2.85	1.05	3.16	3.17	0.38
W/15-24	10.47	10.43	9.63	3.01	3.28	1.94	3.34	3.12	0.52
M/25-44	8.00	7.43	7.45	2.44	2.79	0.71	3.39	3.08	0.88
W/25-44	7.45	7.28	7.61	2.12	2.30	1.39	2.46	2.16	0.67
M/45-64	10.74	8.42	8.37	5.50	5.40	0.54	6.03	5.79	0.66
W/45-64	8.76	9.61	9.55	1.46	2.07	1.48	1.62	1.88	0.50

Figure 5 compares the standard errors of the GREG estimator and the filtered signals under model M1, M2 and M3 for the domain women/25-44. Figure 6 compares the standard errors of the filtered trend for this domain under models M1, M2 and M3. The difference between GREG and M1 in Figure 5 illustrates the increase in precision for the monthly unemployment estimates by borrowing strength over time. Modelling the correlation between the slope disturbances results in another substantial increase of the precision, which follows from the difference between M1 and M2 in Figures 5 and 6. The difference between M1 and M2 is the additional precision obtained by borrowing strength over space. Modelling the correlation between the disturbances of the seasonals does not further improve the precision of the filtered estimates for the unemployed labour force. Modelling the correlation between the RGB disturbances slightly increases the standard errors compared to M2 (results not shown).

To summarise differences between the standard errors of the point estimates obtained under two different models or estimation procedures the mean relative difference of the standard errors is used, which is defined by

$$\text{MRDSE}\ (A, B) = \frac{1}{T}\sum_{t=1}^{T}\frac{SE\,(y_t^A) - SE\,(y_t^B)}{SE\,(y_t^A)} \times 100\ , \tag{16}$$

where $SE\,(y_t^x)$ denotes the standard error of y_t^x.

Figure 5: Monthly unemployed labour force Women/25-44; standard error filtered signal model M1, M2 and M3 and GREG estimator

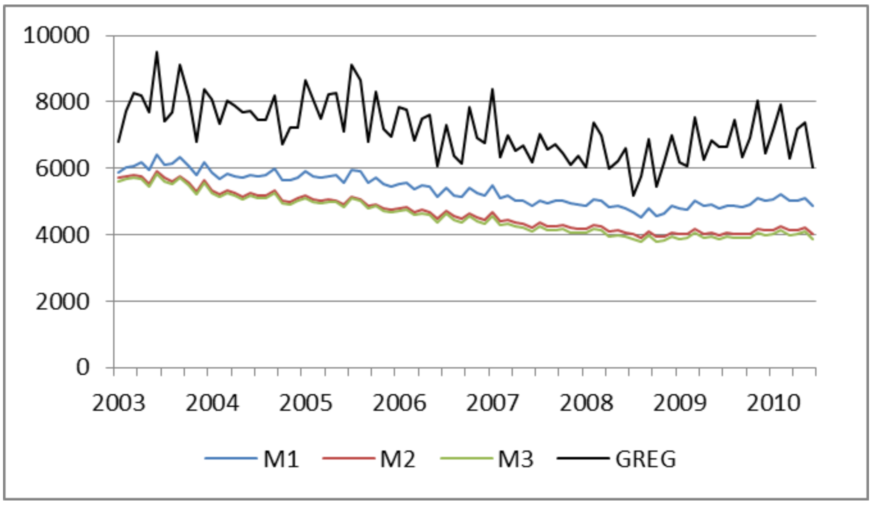
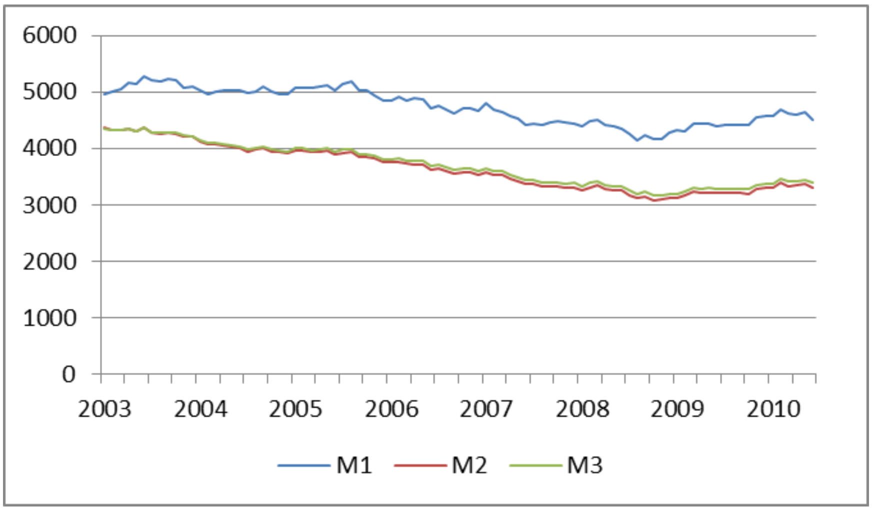


Figure 6: Monthly unemployed labour force Women/25-44; standard error filtered trend model M1, M2 and M3



In Table 6 the standard errors of the GREG estimates and the filtered estimates under M1, M2 and M3 are compared using MRDSE defined by (16) over the last 24 months of the series. The results for MRDSE(GREG,M1) show that borrowing strength over time results in the strongest decrease of the standard errors. Comparing MRDSE(GREG,M1) with MRDSE(GREG,M2) shows that a smaller but worthwhile decrease of the standard errors is obtained with M2 where sample information from other domains is used by modelling the correlation between the slope disturbances. This conclusion also follows from the results of MRDSE(M1,M2) for the filtered trend and the filtered signal. The results for MRDSE(GREG,M3) and the MRDSE(M2,M3) for the filtered trend and the filtered signal illustrate that modelling the correlation between the disturbances of the seasonal does not further decrease and sometimes even increase the standard errors of the filtered trend or signal although the likelihood-ratio test indicate a significant improvement if the model is extended with a common seasonal component. The same conclusion holds for M4 where the correlation between the RGB disturbances is modelled (results not shown).

Table 6: MRDSE for GREG and filtered estimates under model M1, M2, and M3 for the last 24 months of the series.

Domain	GREG versus filtered signal			Filtered signal			Filtered trend		
	(GREG,M1)	(GREG,M2)	(GREG,M3)	(M1,M2)	(M1,M3)	(M2,M3)	(M1,M2)	(M1,M3)	(M2,M3)
M/15-24	37.19	45.91	46.03	13.94	14.13	0.22	26.06	26.47	0.56
W/15-24	27.66	44.59	43.86	23.51	22.46	-1.38	39.35	39.68	0.55
M/25-44	29.49	32.56	32.92	4.45	5.02	0.62	12.14	10.11	-2.30
W/25-44	26.04	38.30	40.18	16.65	19.17	3.02	27.00	25.27	-2.37
M/45-64	30.84	46.53	46.33	22.78	22.49	-0.37	30.72	29.63	-1.58
W/45-64	39.46	45.31	44.77	9.69	8.80	-0.99	14.17	13.35	-0.96

The foregoing analyses indicate that borrowing strength over time through M1 result in a significant increase of the precision of the monthly GREG estimates. The likelihood-ratio tests indicate that a substantial improvement can be obtained by modelling the correlation between the slope disturbances of the trend component with M2. The MRDSE also show that M2 results in a substantial increase of the precision of the filtered trends and signals compared to M1. Modelling the correlation between the disturbances of the seasonals or the RGB does not further improve the model fit and also does not further improve the precision of the filtered trends and signals. As a result it can be concluded that M2 is the most promising model to produce monthly unemployment figures.

So far filtered estimates are analysed to compare different time series models and GREG estimates with each other. As explained in Section 7, concurrent estimates mimic exactly the results obtained if the time series models are used in production since they also recompute the ML estimates for the hyperparameters for each time period. For the time series models considered in the section, this is extremely time consuming. The numerical optimization procedure to obtain ML estimates takes about 25 to 30 hours for model M2, M3 and M4. Therefore we decided to use the filtered estimates as an approximation. The discrepancies between the filtered and concurrent estimates are only analysed for the model that is finally selected, i.e. M2.

In Figure 7, the filtered and concurrent trend for the domain Women/25-44 for the last 24 months of the time series are compared. The standard errors of both estimates are plotted in Figure 8. Table 7 summarises the MARD and MRDSE for the filtered and concurrent trends and signals under M2 over the last 24 months of the series for the six domains. Also the GREG estimates and the concurrent signals under M2 are compared. It can be concluded that differences between filtered and concurrent estimates are small.

Figure 7: Monthly unemployed labour force Women/25-44; filtered and concurrent trend model M2

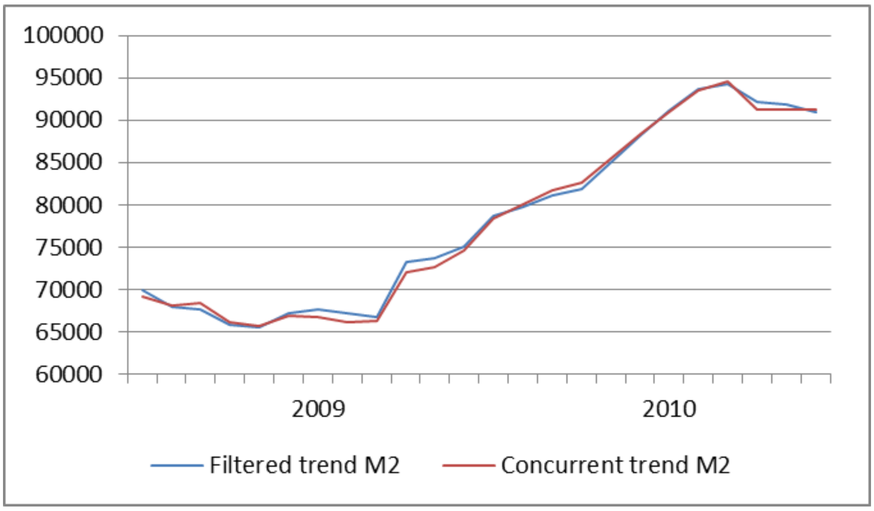


Figure 8: Monthly unemployed labour force Women/25-44; standard error filtered and concurrent trend model M2

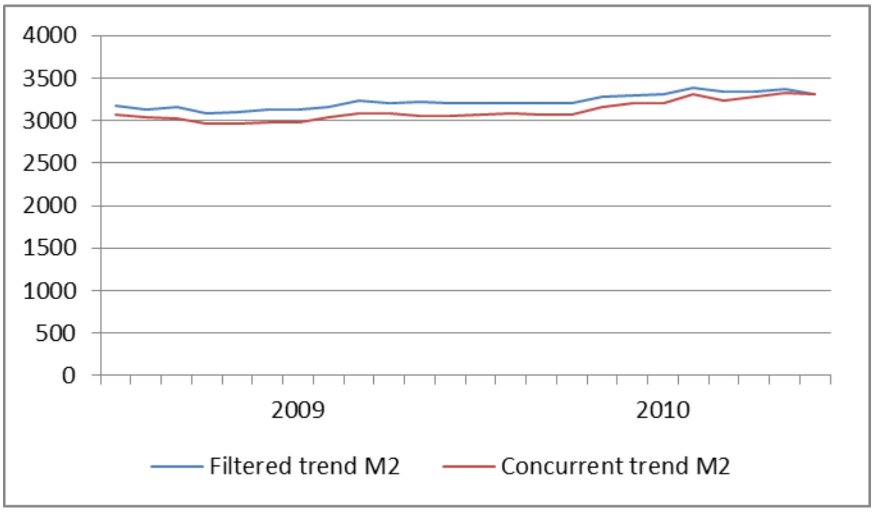


Table 7: Comparison GREG, filtered and concurrent estimates for M2 for the last 24 months of the series

Domain	signal		trend		GREG versus signal	
	MARD(filt,con)	MRDSE(filt,con)	MARD(filt,con)	MRDSE(filt,con)	MARD(GREG,con)	MRDSE(GREG,con)
M/15-24	1.19	8.33	0.77	4.03	8.03	50.41
W/15-24	1.17	2.12	1.15	-0.81	10.01	45.76
M/25-44	2.34	0.44	2.00	0.69	8.89	32.54
W/25-44	1.17	-3.53	0.93	3.51	6.79	36.08
M/45-64	1.56	-0.04	1.57	0.38	8.15	46.58
W/45-64	0.88	0.13	0.76	3.89	9.64	45.36

9. Results time series models with auxiliary variables

In this section we compare the following models for the six domains:

Model 1: the same model as in Section 8, i.e. a model where only the sample information of the domain itself is used as input.

Model 5: equation (12), i.e. a model where the auxiliary information is modelled as a separate series with a non-zero correlation between auxiliary series and the series of the LFS parameter.

Model 6: equation (14), i.e. a model where the auxiliary information is modelled as the sixth wave of the panel.

In Section 9.1 it is investigated to what extent the model estimates and their standard errors are affected by using the auxiliary series in the model. Currently the auxiliary information for month *t* is not available at the moment that the official monthly figures about the labour force are produced. The use of auxiliary information in the estimation procedure implies a delay in the timeliness of these official releases. This can be avoided if missings are accepted for the auxiliary series. Therefore the concurrent estimates under model 5 and 6 are analysed in a situation where there are no missing observations in the auxiliary series and in a situation where the last two months in the auxiliary series are missing. These models are denoted by M5M0, M6M0, M5M2, and M6M2 respectively.

As explained in Section 6 there are various reasons that can result in an evolution of the auxiliary series that is not related to the real development of the unemployed labour force. This raises the question how robust model 5 and 6 are to these kind of disturbances in the auxiliary series. In Section 9.2 it is simulated how such changes in the auxiliary series can affect the estimates of the unemployment figures.

The computation time of the models in this section allows the computation of concurrent estimates in all cases.

9.1 Results with original auxiliary series

In the appendix the ML estimates for the hyperparameters for M5 and M6 are specified. Likelihood ratio tests indicate that the null hypothesis that the covariance terms between the slope correlations are zero, i.e. $\varsigma_{R,d,zz'}$ in (12) and (13), is not rejected for all the domains under M5M0 and M5M2 (largest p-value is 0.02 for Men/15-24 for M5M0). Table 8 shows the mean and the standard deviation of the ML estimates for the correlation between the slope disturbances of the LFS and the auxiliary series under M5 over the period July 2008 – June

2010. The correlations are quite high, for Men/15-24 and for Women/45-64 they are slightly smaller. However, the ML estimates are not constant over the entire period, especially for Men/15-24. This follows from the standard deviations of the ML estimates in Table 8 and is also illustrated in Figure 9 for the domain Men/15-24. Here the ML estimates for the correlation increases with the length of the observed time series. Possible reasons are that the ML estimates become more reliable if the length of the series increases or that the relation between the auxiliary series and the series of the LFS parameter is not constant in time. For the other domains, the variation in the estimated correlation over time are smaller, Women/25-44 is shown as an example in Figure 10. The improvement of the precision of the LFS parameters by using auxiliary information will increase as the ML estimates for the correlations increase. Table 8 shows that especially for the domains Women/15-24, Men/25-44 and Women/25-44 a large improvement can be expected since the trends tend to be cointegrated. Since the estimates of the ML estimates for the correlation fluctuate over time the use of concurrent estimates instead of filtered estimates is important. Particularly for Men/15-24 the ML estimate for the correlation is the highest at the end of the series. Using filtered instead of concurrent estimates might underestimate the standard errors for the preceding periods.

Table 8: mean and standard deviation of the ML estimates for the correlation between the slope disturbances of the LFS and the auxiliary series over the period July 2008 – June 2010

	Mean		Standard deviation	
	M5M0	M5M2	M5M0	M5M2
Men/15-24	0.83	0.85	0.143	0.115
Women/15-24	0.99	0.99	0.006	0.005
Men/25-44	0.99	0.98	0.006	0.006
Women/25-44	0.99	0.99	0.004	0.005
Men/45-64	0.93	0.93	0.024	0.023
Women/45-64	0.87	0.86	0.029	0.035

Figure 11 and 12 show the evolution of the ML estimates of the hyperparameter for the bias in the trend of the auxiliary series (under model 6) for Men/15-24 and for Women/25-44. The ML estimates for this hyperparameter for Men/15-24 increase (with some instabilities) similarly as the estimated correlation increases in Figure 9. Note that this is a contradictory result, since a smaller hyperparameter for the bias of the auxiliary series, implies more similarity between the trend of the auxiliary series and the LFS series. The hyperparameter estimate for Women/25-44 in Figure 12 is after the first years more or less stable.

Figure 9: ML estimates slope disturbance correlations Men/15-24

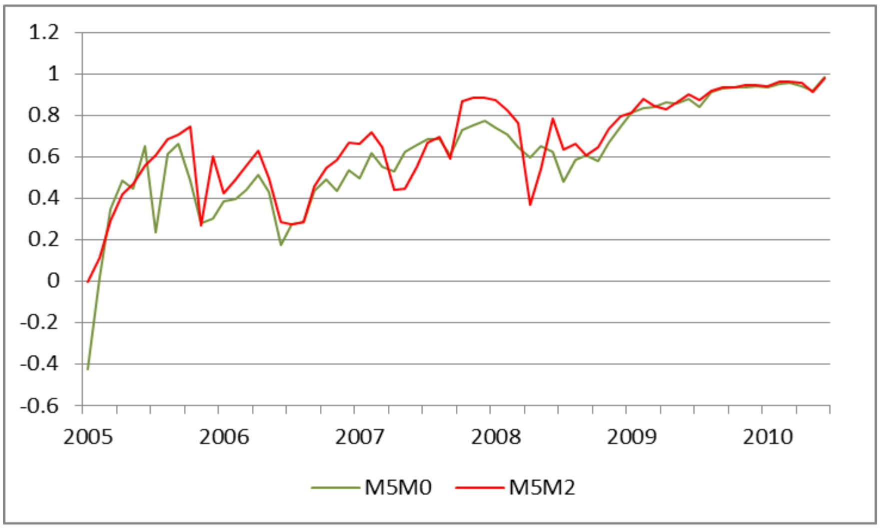


Figure 10: ML estimates slope disturbance correlations Women/25-44

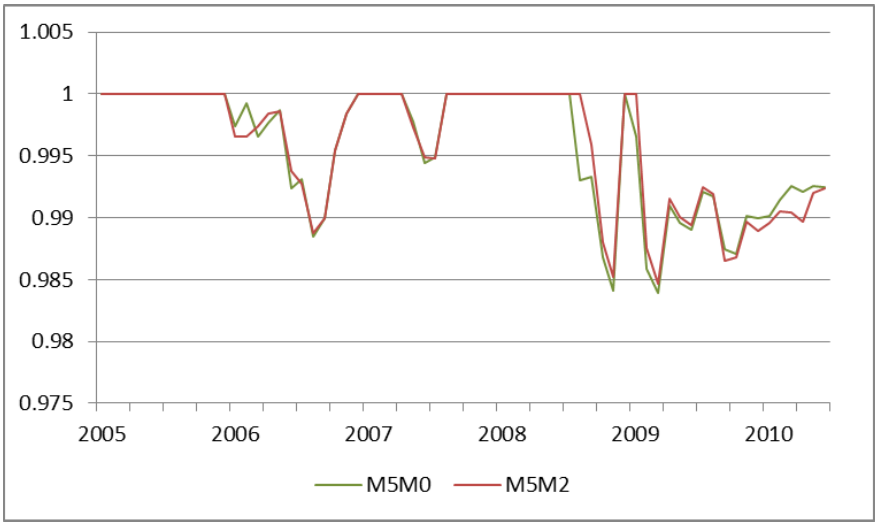


Figure 11: ML estimate slope hyperparameter bias auxiliary trend Men/15-24

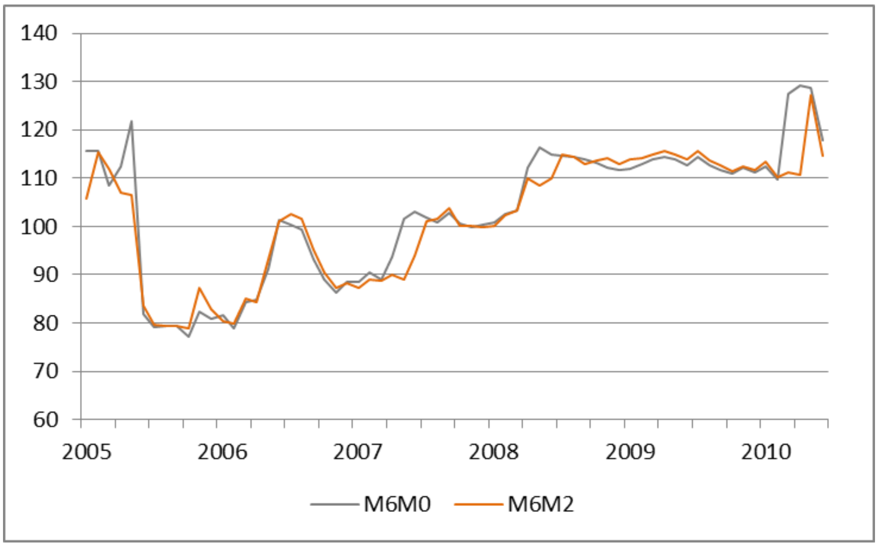
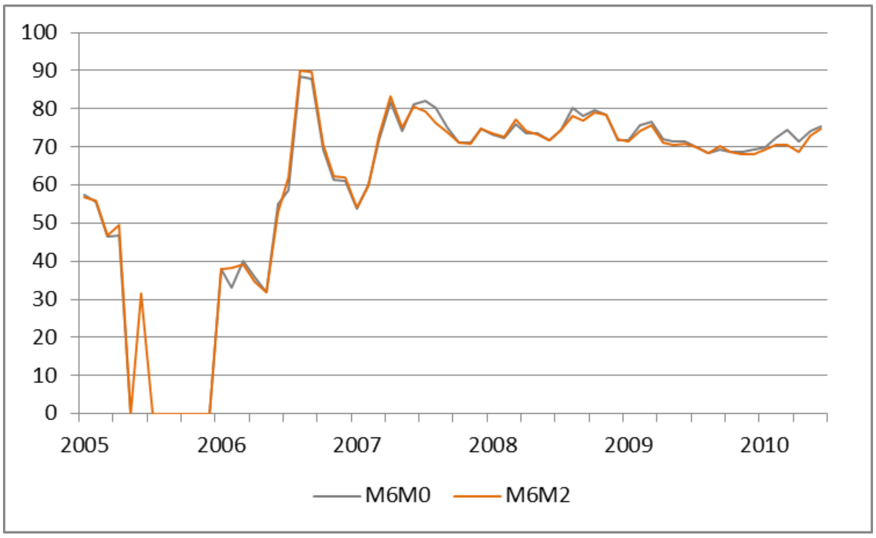


Figure 12: ML estimate slope hyperparameter bias auxiliary trend Women/25-44



As mentioned in Section 6, a random walk model for the bias of the auxiliary series is less appropriate. In Figure 13 the concurrent estimates for the trend of domain Women/25-44 under different models are compared including M6 with a random walk for the bias in the trend of the auxiliary series. The trend under M1 is used as a benchmark since the level of this trend is not influenced by information from other domains or auxiliary series. The estimated trend under M6 with a random walk for the bias of the auxiliary series shows the strongest deviation from the trend under M1. With this model the auxiliary series has the strongest influence on the level as well as the volatility of the series of the target parameter. Plots of the standardized innovations show that the innovation are positive or negative for long periods. Sample correlograms indicate strong positive autocorrelation, also for large lags, in the standardized innovations. For the other domains, similar results are found. Therefore M6 with a random walk for the bias of the auxiliary trend is no longer considered.

Figure 13 shows that the evolution of the concurrent trends under M1, M5 and M6 are more or less similar. M5 and M6, however, result in considerably smoother concurrent trends than M1. The evolution of the auxiliary series is indeed much smoother compared to the survey estimates of the LFS. Both models M5 and M6 borrow information from these auxiliary series, resulting in a smoother trend for the LFS parameter. Comparing the concurrent trends for the domains of Men/25-44, Men/45-64 and Women/45-64 under M1, M5 and M6 show a similar picture as obtained for Women/25-44 in Figure 13.

In Figure 14 and 15 the concurrent trends under M1, M5 and M6 are compared for Men/15-24 and Women/15-24 respectively. For these domains the differences between M6 and M1 are considerable, in particular for Women/15-24. For the last domain the standardized innovation under M6 show clear patterns of autocorrelation. For Men/15-24, the estimates under model M1 and model M5 are very similar since the ML estimates for the correlation are small (Figure 11). For Women/15-24, there are also some substantial differences between M1 and M5, especially in 2005 and in 2010.

The differences between the concurrent trends under a model with the last two observations in the auxiliary series missing and without missings are negligible. Therefore the estimates under the models with two missings are not shown in the figures.

Figure 13: Concurrent trend Women/25-44

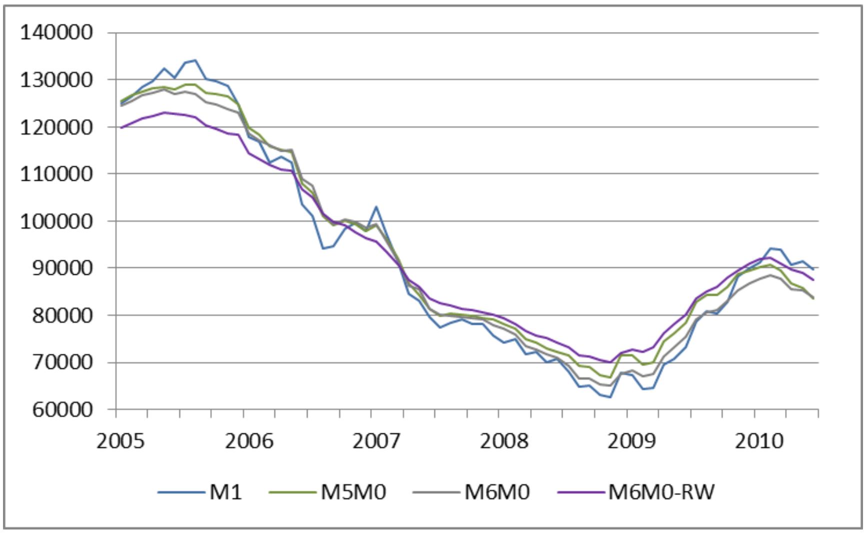


Figure 14: Concurrent trend Men/15-24

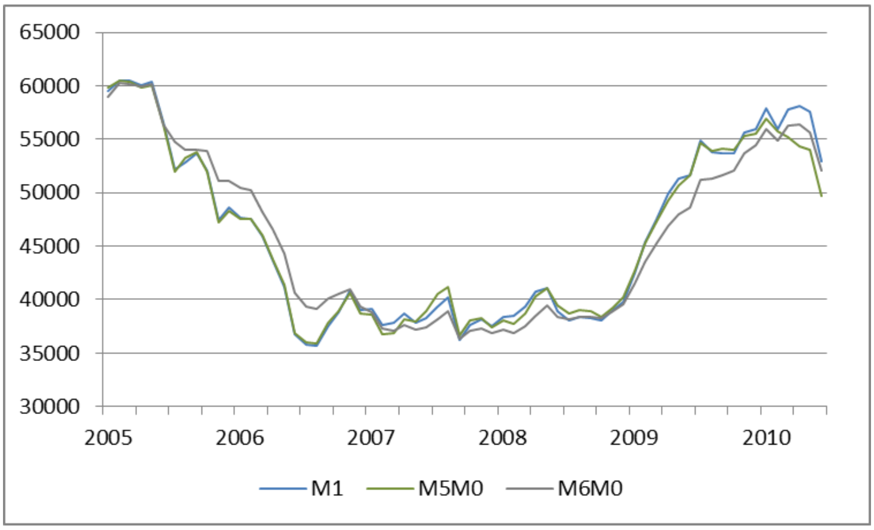
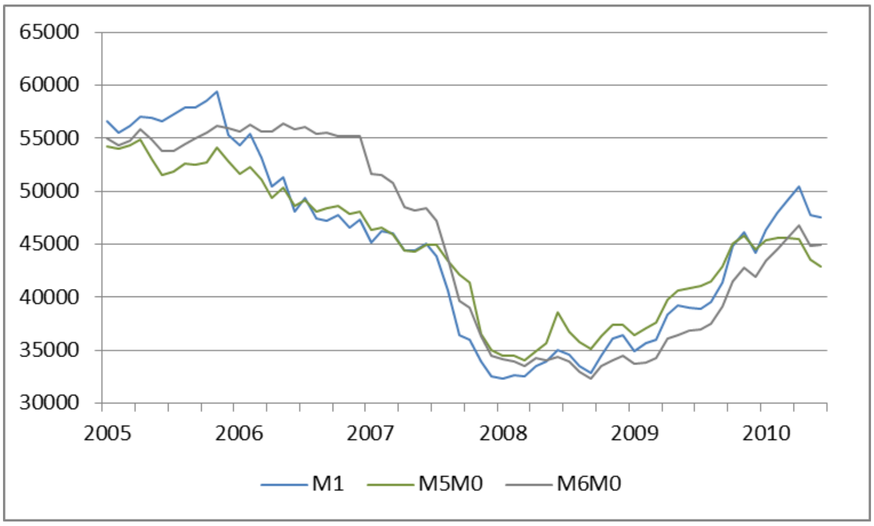


Figure 15: Concurrent trend Women/15-24



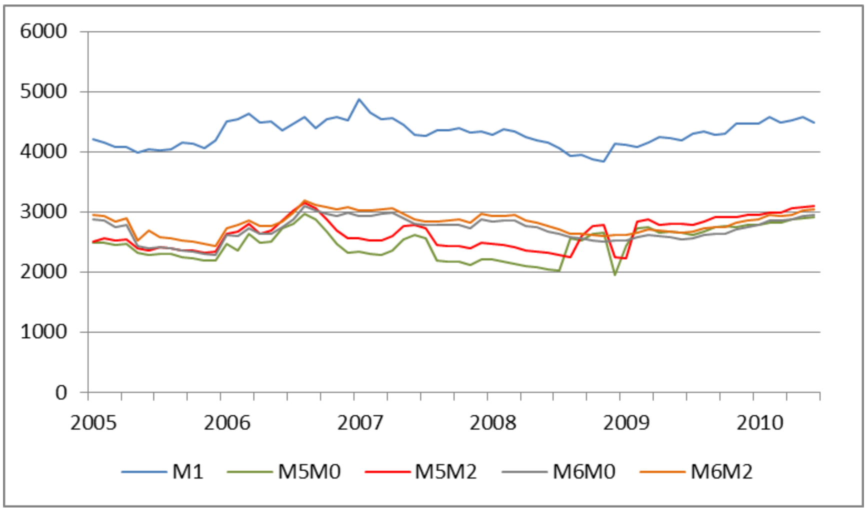
In Table 9, the differences between the concurrent estimates for the signals under M1, M5M0, M5M2, M6M0, M6M2 and the GREG estimator over the last 24 months of the series are summarized using MARD (15). MARD(M5M0,M5M2) and MARD(M6M0,M6M2) show that for all domains, the point estimates are very similar if the last two values of the auxiliary information are available or missing. Comparing MARD(GREG,M1), MARD(GREG,M5M0) and MARD(GREG,M6M0) shows that the differences between the model estimates and the GREG estimates are more or less equal. There are nevertheless some differences between the concurrent signals under the different models.

Table 9: *MARD for concurrent estimates under model M1, M5, and M6 and under GREG for the last 24 months of the series.(G stands for GREG)*

	Trend					Signal		
	(M1,M5M0)	(M1,M6M0)	(M5M0,M5M2)	(M6M0,M6M2)	(M5M0,M6M0)	(G,M1)	(G,M5M0)	(G,M6M0)
M/15-24	1.68	2.96	0.83	0.42	3.13	7.58	7.73	9.78
W/15-24	4.78	5.25	1.11	0.26	7.00	10.36	10.65	13.75
M/25-44	5.69	4.11	2.45	1.70	4.43	8.54	8.89	10.19
W/25-44	5.28	3.25	1.27	1.11	3.19	7.16	6.24	7.34
M/45-64	4.70	4.45	1.15	1.07	0.89	10.61	8.95	9.15
W/45-64	2.25	2.17	0.52	0.56	0.81	8.53	8.64	9.37

The standard errors of the concurrent trends under M1, M5M0, M5M2, M6M0 and M6M2 are compared in Figure 16 for the domain Women/25-44. The use of auxiliary information results in an increased precision of the estimates. The standard errors under M5 and M6 are more or less equal. Under M5, there are some sudden jumps in the standard errors, which can be explained by the changes in the ML estimates for the correlation between the slope disturbances. The standard error increases only slightly when the last two values of the auxiliary information are missing for both models.

Figure 16: standard error concurrent trend Women/25-44



In Table 10, the standard errors of the concurrent trend and signal estimates under the different models for the last 24 months of the series are compared using MRDSE (16) for all domains. Compared to M1, the use of auxiliary information in M5 and M6 results in a substantial improvement of the precision. The reduction of the standard error under M5 depends on the ML estimate for the correlation. From Table 8 it follows that the trends for the domains Women/15-24, Men/25-44 and Women/25-44 tend to be cointegrated. These domains indeed have the strongest decrease in their standard errors in Table 10. For M6, more information from the auxiliary series is used when the bias of the auxiliary series is less volatile, i.e. if the slope hyperparameter of this bias is small compared to the slope hyperparameter of the trend of the population parameter. Compare MRDSE(M1,M6) in Table 10 with the ratio of the ML estimates of the slope hyperparameter of the LFS over the ML estimate of the slope hyperparameter of the bias in the auxiliary series under M6 in the Appendix. Comparing M5 and M6 with M2 shows that modelling the correlation between the slope disturbances of the domains result in four domains in smaller standard errors and in two domains in larger standard errors compared to a model that includes an auxiliary series.

Table 10: MRDSE concurrent trend and signal

Domain	(M1,M5M0)		(M1,M6M0)		(M5M0,M5M2)		(M6M0,M6M2)		(M5M0,M6M0)	
	trend	signal	trend	signal	trend	signal	trend	signal	trend	signal
M/15-24	9.44	2.65	14.18	12.90	-2.15	0.43	-0.37	-0.24	5.12	10.52
W/15-24	23.38	13.71	21.14	19.43	-3.95	-1.92	-0.31	-0.18	-3.11	6.57
M/25-44	37.29	21.07	14.38	14.84	-20.02	-7.30	-3.31	-2.51	-40.66	-9.02
W/25-44	37.54	18.95	37.28	33.01	-4.87	-1.50	-3.38	-1.74	-1.06	17.19
M/45-64	19.56	14.25	14.28	13.47	-1.81	-1.14	-1.18	-0.83	-6.79	-1.07
W/45-64	9.72	5.68	11.35	11.76	-1.04	-0.53	-0.79	-0.52	1.80	6.44
	(M2,M5M0)		(M2,M5M2)		(M2,M6M0)		(M2,M6M2)			
	trend	signal	trend	signal	trend	signal	trend	signal		
M/15-24	-22.24	-15.41	-24.84	-14.88	-15.84	-3.25	-16.27	-3.50		
W/15-24	-25.70	-14.88	-30.65	-17.08	-29.52	-7.33	-29.91	-7.52		
M/25-44	27.03	15.85	13.61	9.92	0.50	9.21	-2.76	6.94		
W/25-44	14.70	7.23	10.67	5.85	14.36	23.24	11.46	21.90		
M/45-64	-14.65	-9.95	-16.70	-11.19	-22.19	-11.01	-23.62	-11.93		
W/45-64	-7.95	-4.08	-9.07	-4.63	-6.00	2.61	-6.84	2.10		

In conclusion, it follows that the use of auxiliary information results in a substantial variance reduction. In some domains there is, however, also a substantial adjustments of the level of the concurrent trends and signals over longer periods. The question raises whether strong adjustments on the point estimates can be considered as improvements since the auxiliary series does not measure unemployment according to the definition of the LFS. In the most extreme cases, the level adjustments of the trend result in an increased autocorrelation in the standardized innovations, which is an indication that the auxiliary information increases the bias.

9.2 Changes in the auxiliary series

In this section the robustness of the model estimates to changes in the auxiliary series not related with real economic developments is tested in a simulation. For the domain Women/25-44, the auxiliary series are modified in different ways, and the effect on the model estimates is analysed. It is assumed that there are no missing observations in the auxiliary series. The effect of the following modifications in the auxiliary series are simulated:

1. A gradual increase added to the auxiliary series. In September 2007, 1000 is added to the original series, in October, 2000 is added. This number increases stepwise to 6000 in February 2008. This last number is added in all months from February 2008.
2. A gradual decrease of the auxiliary series. In September 2007, 500 is subtracted from the original series. The amount that is subtracted in each month increases stepwise to 3000 in February 2008.
3. A gradual increase added to the auxiliary series starting in September 2007. The monthly increase is half as large as under simulation 1. In September 2007, 500 is added to the original series. This amount increases each month increases stepwise to 3000 in February 2008.
4. A sudden increase or shock of 3000 that is added to the auxiliary series for all months from September 2007.

The original and the four modified auxiliary series are shown in Figure 17. The four changes are added to the auxiliary series in a period where the ML estimate for the correlation between slope disturbances is exactly 1, see Figure 10. These changes are moderate, and might occur in the register due to reasons that are not related to real economic developments as mentioned in Section 6. Sudden shocks as simulated under 4 are, compared to the gradual changes, easier recognised and can be modelled with a dummy intervention. In a production process it remains, however, difficult to detect an outlier or shock in the last observation of the series. The gradual changes remain harder to detect, also after longer periods. Therefore the simulation is interesting to detect the robustness of this estimation procedure to evolutions in the auxiliary series that are unrelated to developments in the labour force.

In the ideal situation, modifications in the auxiliary series are picked up in M5 by a decreasing ML estimate for the correlation between the slope disturbances. In M6 the bias for the auxiliary trend should pick up the simulated adjustments in the auxiliary series. This might require a larger ML estimate for the slope hyperparameter used to model the bias in the auxiliary trend.

Figure 17: auxiliary series for Women/25-44, original series and four simulated modifications, period 2007-2008

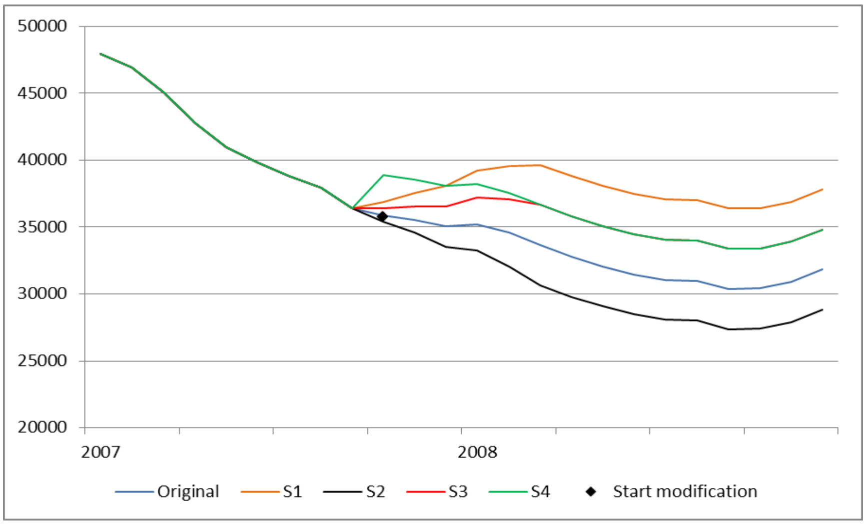


Figure 18 show the ML estimates for the correlation between the slope disturbances under the different simulations under M5. Figure 19 compares the concurrent trends obtained with M5. Under the first simulation, the ML estimate for the correlation drops in March 2008. During the period September 2007 until March 2008, the strong ML estimates for the correlation result in an overestimation of the concurrent trend. The concurrent trend incorrectly suggest a turning point in the last quarter of 2007 and again in the first quarter of 2008. The third simulation shows a similar but less extreme pattern, as expected. Under the second simulation, the ML estimate for the correlation suggests cointegration for a much longer period. This results in an underestimation of the concurrent trend for a period of about 12 months. It takes about 7 months before the simulated shock in the fourth simulation results in an adjustment of the ML estimate for the correlation. This translates into an overestimation of the concurrent trend for the same period including two spurious turning points. It is noted that this kind of shock effects are detected more easily and can be modelled with an appropriate intervention variable for the auxiliary series. The effect of the simulated changes in the auxiliary series are only partially visible in the standardized innovations but not in the sample correlograms.

Figure 18: Concurrent ML estimates correlation between slope disturbances with different simulated auxiliary series under M5

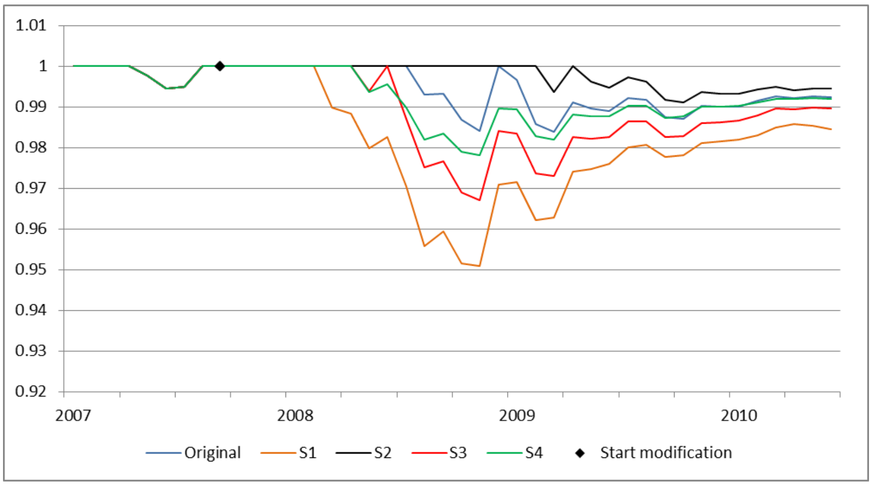


Figure 19: Concurrent trends with different simulated auxiliary series under model M5

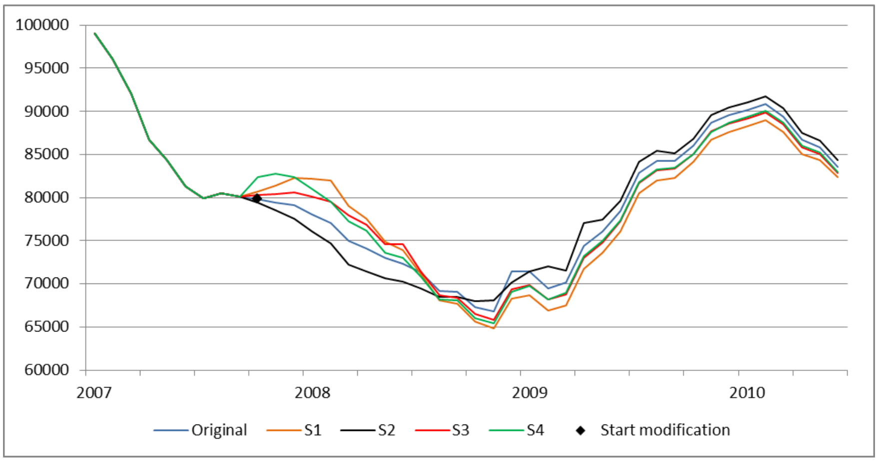


Figure 20 compares the concurrent estimates for the slope hyperparameter of the bias for the auxiliary trend of M6 under the different simulations. Figure 21 compares the concurrent estimates for the bias in the trend of the auxiliary series. Figure 22 compares the concurrent trends obtained with M6. Contrary to the ML estimates for the correlation in M5, the ML estimates for the slope hyperparameter of the bias in the auxiliary trend is directly influenced by the simulated modifications. After a period of about a year, the concurrent estimates for this bias completely accounts for the simulated differences that are added to the auxiliary series is all four simulations. This results in a period of a year where the trends are incorrectly adjusted, see Figure 22. The adjustments of the concurrent trends are considerably smaller compared to M5, compare Figure 19 with 22. Under the first and third simulation there is still a positive bias in the concurrent trend, but there are no clear spurious turning points as in the case of M5. The shock in the fourth simulation is still visible in the concurrent trend, but less extreme. Again, an extension of the model with an intervention variable would be an appropriate solution. In the second simulation, the ML estimate for the slope hyperparameter of the bias in the auxiliary trend becomes smaller. This implies that this bias is less flexible and suggest that the trend of the auxiliary series and the LFS parameter evolve more similar. This is equivalent to the overestimation of the correlation of M5 under simulation 2 and results in an incorrect downwards adjustment of the concurrent trend under M6.

It can be concluded that both models are sensitive for changes in the auxiliary series that are unrelated to the real evolution of the labour force. The concurrent target parameter estimates under M6, however, are more robust compared to M5.

Figure 20: ML estimates slope hyperparameter bias auxiliary trend with different simulated auxiliary series under model M6

Figure 21: Concurrent bias auxiliary trend with different simulated auxiliary series under model M6

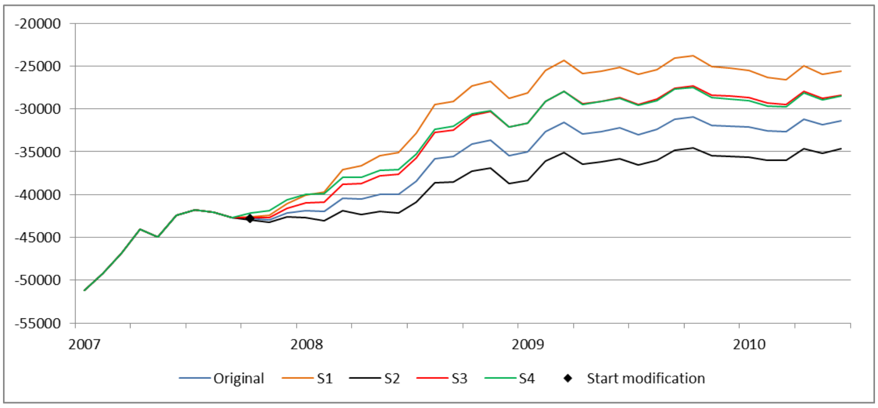
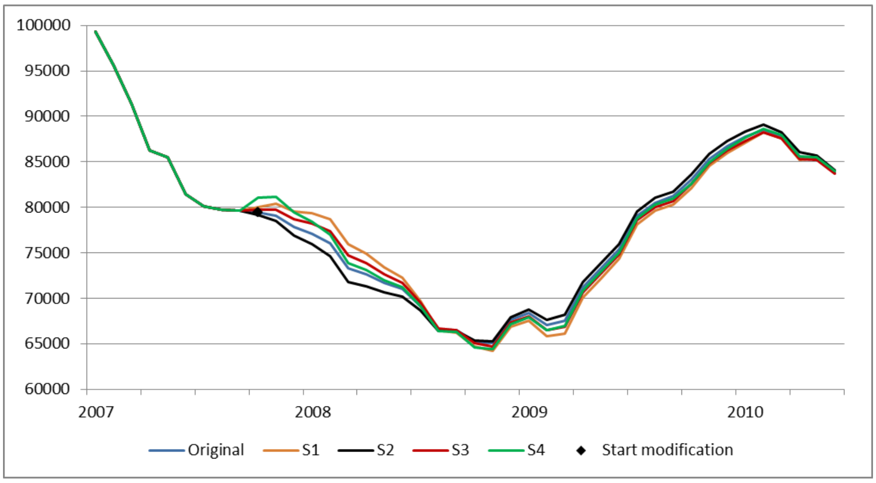


Figure 22: Concurrent trends with different simulated auxiliary series under model M6



10. Discussion

This paper illustrates how multivariate structural time series models can improve the accuracy of survey sample statistics obtained with a rotating panel. A multivariate model for the waves for separate domains reduces the sampling error by borrowing strength over time, accounts for the systematic differences between the waves as well as the serial correlation in the survey errors. The model enables Statistics Netherlands to publish a reliable macro-economic indicator about the labour force on a monthly frequency which is not possible with the standard design-based approach, known from classical sampling theory.

An empirical result is that a multivariate time series model for the wave series of all domains of interest further increases the precision of the survey estimates by modelling the covariance between the slope disturbances. This improvement is, however, not as large as the improvement obtained with borrowing strength over time within a domain. Modelling the correlation between the disturbances of the seasonal components does not further improve the precision of the parameter estimates although the seasonal components are also cointegrated. The correlation between the disturbances of the RGB is the weakest and does not result in more precise estimates.

Extending the time series model for the wave series of a separate domain with an auxiliary series is another way to further improve the precision of the sample estimates. One possibility is to model the covariance between the slope disturbances of the LFS parameter and the auxiliary series. Another possibility is to consider the auxiliary series as an additional wave and model the systematic differences between the level and the seasonals as time dependent bias components. The main advantage of the latter approach is that it is more flexible to account for gradual changes in the relation between the LFS parameter and the auxiliary series.

Another empirical finding is that the use of people that receive unemployment benefits as an auxiliary series results in a substantial improvement of the precision estimated unemployed labour force, compared to a model that only borrows strength over time. The concurrent trends and signals for the unemployed labour force are less volatile but the adjustment of the level can be undesirable large. A simulation reveals that adding modifications to the auxiliary series result in spurious adjustments of the concurrent trends and signals. Since it can be expected that auxiliary series contain evolution that are partially induced by factors unrelated to real developments of the unemployed labour force, the use of auxiliary series in these models is not recommended. These results are in line with the results reported by Harvey and Chung (2000). They found that the trends of the LFS series and claimant counts in a bivariate state-space model are cointegrated, resulting in a substantial decrease of the standard errors of the estimated change in unemployment. They did not consider the robustness of the model to possible evolutions in the claimant counts that are not related to real developments of the unemployed labour force.

In conclusion, the multivariate time series model for all domains that accounts for non-zero correlation between the slope disturbances is the most appropriate from a statistical point of view. The only drawback is the required computation time.

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Appendix: Maximum likelihood estimates standard deviations

Hyperparameter	domain	M1	M2	M3	M4	M5M0	M5M2	M6M0	M6M0
Slope	M/15-24	259	258	263	261	411	386	179	178
	W/15-24	179	130	132	140	278	280	94	95
	M/25-44	704	22	19	21	758	773	654	664
	W/25-44	468	0	0	0	425	423	357	356
	M/45-64	434	0	0	0	335	338	354	357
	W/45-64	229	0	0	0	225	225	207	207
Seasonal	M/15-24	141	152	180	149	159	150	0	0
	W/15-24	57	67	0	50	82	82	2	2
	M/25-44	336	342	286	336	226	224	30	32
	W/25-44	134	131	0	128	110	110	0	2
	M/45-64	0	0	0	0	0	0	0	0
	W/45-64	86	88	0	88	79	79	0	0
RGB	M/15-24	918	847	828	863	957	952	1114	1117
	W/15-24	465	415	398	610	402	402	766	761
	M/25-44	198	266	309	193	342	341	2006	1967
	W/25-44	694	675	673	0	652	652	582	581
	M/45-64	0	0	0	0	1	2	924	923
	W/45-64	415	428	441	0	390	390	387	387
bias slope auxiliary	M/15-24	NA	NA	NA	NA	NA	NA	118	115
	W/15-24	NA	NA	NA	NA	NA	NA	69	70
	M/25-44	NA	NA	NA	NA	NA	NA	211	223
	W/25-44	NA	NA	NA	NA	NA	NA	76	75
	M/45-64	NA	NA	NA	NA	NA	NA	135	135
	W/45-64	NA	NA	NA	NA	NA	NA	98	98
Correlation Slope disturbances	M/15-24	NA	NA	NA	NA	0.98	0.98	NA	NA
	W/15-24	NA	NA	NA	NA	0.99	0.99	NA	NA
	M/25-44	NA	NA	NA	NA	0.99	0.99	NA	NA
	W/25-44	NA	NA	NA	NA	0.99	0.99	NA	NA
	M/45-64	NA	NA	NA	NA	0.96	0.96	NA	NA
	W/45-64	NA	NA	NA	NA	0.89	0.89	NA	NA
Bias seasonal auxiliary series	M/15-24	NA	NA	NA	NA	6	6	6	6
	W/15-24	NA	NA	NA	NA	2	2	0	0
	M/25-44	NA	NA	NA	NA	29	31	0	0
	W/25-44	NA	NA	NA	NA	0	1	0	1
	M/45-64	NA	NA	NA	NA	23	23	23	24
	W/45-64	NA	NA	NA	NA	0	0	0	0

NA: not applicable

ML estimates of hyperparameters white noise, and survey errors for M1. Results for other models are similar and therefore not presented

Standard deviation	*M/15-24*	*W/15-24*	*M/25-44*	*W/25-44*	*M/45-64*	*W/45-64*
White noise	1520	3510	4290	4288	3329	201
Survey error panel 1	0.98	1.11	1.08	0.90	0.99	1.10
Survey error panel 2	0.96	1.03	1.04	0.94	1.12	1.02
Survey error panel 3	1.04	1.10	1.03	0.95	1.06	0.93
Survey error panel 4	1.08	1.19	0.99	0.96	0.99	0.92
Survey error panel 5	1.02	1.01	0.99	0.98	1.08	0.89
$\hat{\rho}$	0.13	0.12	0.39	0.22	0.44	0.38

Explanation of symbols

.	Data not available
*	Provisional figure
**	Revised provisional figure (but not definite)
x	Publication prohibited (confidential figure)
–	Nil
–	(Between two figures) inclusive
0 (0.0)	Less than half of unit concerned
empty cell	Not applicable
2013–2014	2013 to 2014 inclusive
2013/2014	Average for 2013 to 2014 inclusive
2013/’14	Crop year, financial year, school year, etc., beginning in 2013 and ending in 2014
2011/’12–2013/’14	Crop year, financial year, etc., 2011/’12 to 2013/’14 inclusive

Due to rounding, some totals may not correspond to the sum of the separate figures.

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