Towards the implementation of a multivariate Benchmark method on Dutch National Accounts

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Explanation of symbols

.	= data not available
*	= provisional figure
x	= publication prohibited (confidential figure)
–	= nil or less than half of unit concerned
–	= (between two figures) inclusive
0 (0,0)	= less than half of unit concerned
blank	= not applicable
2005-2006	= 2005 to 2006 inclusive
2005/2006	= average of 2005 up to and including 2006
2005/'06	= crop year, financial year, school year etc. beginning in 2005 and ending in 2006
2003/'04–2005/'06	= crop year, financial year, etc. 2003/'04 to 2005/'06 inclusive

Due to rounding, some totals may not correspond with the sum of the separate figures.

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Towards the implementation of a multivariate Benchmark method on Dutch National Accounts

Summary: DMK participates in a project, with the aim of implementing a new, efficient method for the benchmarking of National Accounts. Benchmarking is the process to achieve mathematical consistency between low-frequency (e.g. annual) and high-frequency (e.g. quarterly) accounts.

In 2008 an extended Multivariate Denton method for benchmarking has been developed and implemented in proto-type software. Furthermore, an experiment was carried out on the Dutch supply and use tables. In this paper we briefly describe the method, the experiment and we review potential research topics for 2009. The aim of this paper is to ask the methodology advisory council's advise and opinion on these topics.

Keywords: Benchmarking, Data reconciliation, Denton method, National accounts, HEcS+

1. Introduction

Currently, many methods that are applied to the Dutch national accounts are being redesigned, under the scope of the HEcS+ project (meaning: redesign economic statistics). This project has several aims, one of them is to anticipate future requirements, the current methods were designed long ago and may no longer be optimal (in the future), another aim is to improve efficiency. The processes that are topics of the HEcS+ project take place at two divisions of Statistics Netherlands: Division of Macro-economic Statistics and Dissemination, where the National Accounts are constructed, and the division of Business Statistics.

One of the subprojects of HEcS+ is the benchmarking of national accounts. That is, achieving (mathematical) consistency between high-frequency (e.g. quarterly) and low-frequency (e.g. annual) accounts. For instance: the sum of four quarterly values has to be equal to an annual value. The current benchmark method will be replaced by a more efficient, automated method.

DMK participates in this subproject of HEcS+. In this paper we will explain the activities performed by DMK in 2008, discuss potential research topics for 2009 and ask the methodology advisory council for its opinion on these topics.

DMK also participates in other project that deal with the application of data reconciliation techniques, see for instance Boonstra et al. (2008) for an application to trade and transport statistics.

The paper is organised as follows. In Section 2 we explain the benchmarking method, currently used by Dutch national accounts. In Section 3 the intended process for future use will be described shortly. Sections 4, 5 and 6 deal with: the development of a theoretically model, the development of proto-type software and the conduction of a practical experiment on the Dutch supply and use tables. In Section 7 potential research activities for 2009 are listed.

2. The current cycle of compiling national accounts

The compilation of annual supply and use tables occurs according to a pre-specified plan. For each year under review, three systems of annual national accounts are compiled. For some year t the preliminary accounts are published in the following year ($t + 1$), the semi-final accounts are compiled two years later ($t + 2$) and finally in the year $t + 3$ the final version of the national accounts is published.

In other words: each year t, three systems of annual national accounts are compiled:

1) The preliminary system of $t - 1$;
2) The semi-final system of $t - 2$;
3) The final system of $t - 3$.

The difference between these three versions is that more information is used for the latest publications. The preliminary and semi-final annual accounts are based on a very limited number of sources. Only after three years all sources are available.

The compilation of annual accounts at Statistics Netherlands is (mainly) a manual process. It involves a balancing process: correcting the data so that certain account-ting constraints are satisfied, for instance production = intermediate use + value added. These constraints should hold true for each time period.

National accounts are not only made on an annual basis, but also on a quarterly basis. Since quarterly accounts adopt the same framework as annual accounts they have to be consistent over time with them. This implies, in the case of flow variables, that the sum of the quarterly data has to be equal to the annual figures for each year. Therefore, after each new compilation of annual accounts (three times a year), a new set of quarterly accounts has to be compiled, that has to be consistent with the annual accounts. This process is called *benchmarking*.

The results of the current benchmarking process may suffer from the *step problem*, i.e. spurious gaps between the fourth quarter of one year and the first quarter of the next year, see Figure 2.1 for an example. These jumps arise due to the fact that the benchmarking is done for each year independently.

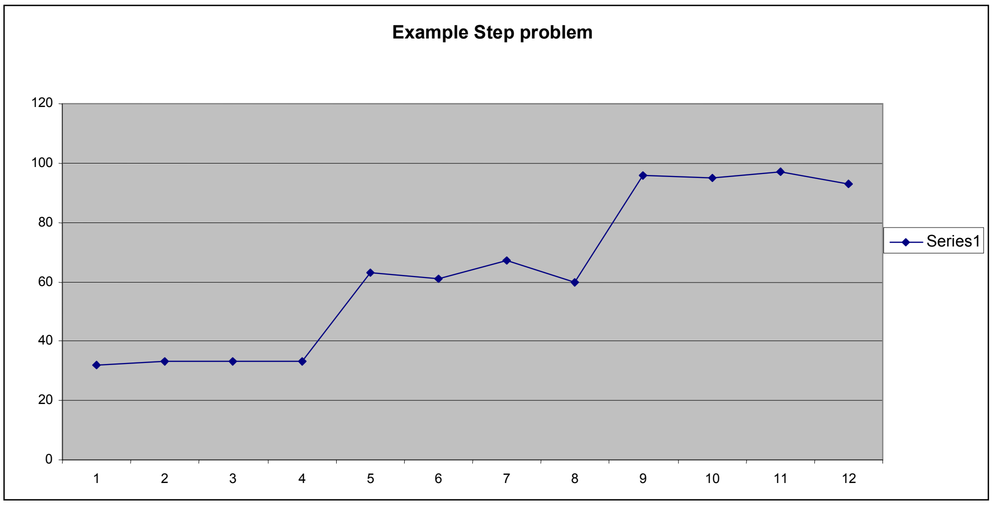

Figure 2.1 An example of the step problem from Dutch supply and use tables of the years 2004-2006.

3. The future cycle of making National accounts

The planned future cycle of some year t starts with a compiled system of final annual national accounts for $t - 3$ and a set of compiled systems of quarterly accounts for the year $t - 1$, $t - 2$ and $t - 3$. At the starting point of the process there is no connection between the quarterly and annual accounts.

The information that comes from the limited number of sources, that are currently used in the compilation process of the preliminary and semi-final annual accounts (of $t - 1$ and $t - 2$), will also be used in the planned, future process. However, these two systems of annual accounts will no longer be manually compiled, i.e. the manual balancing process, that is needed to satisfy the accounting rules within each year, will be skipped. This will improve the efficiency of the process. Thus, at the start of the future benchmarking process the annual data of $t - 1$ and $t - 2$ will not satisfy the accounting constraints (in contrast with the current process).

A formal, mathematical model will be used to perform the benchmarking. The data of three years will be benchmarked simultaneously. A key feature of this model is that it preserves (as much as possible) all quarter-to-quarter changes. Thus, the step problem is avoided (at least within the period of three years). In the literature this phenomenon is called the *movement preservation principle*.

Furthermore, all feature economical accounting constraints are included in the model, for each time period (quarter and year). Moreover, for each time series the restriction is included that the sum of the first four quarterly values (i.e. year $t - 3$) is equal to the final, annual value. Further, the outcomes of the model will be influenced by the few figures from the sources of the preliminary and semi-final annual accounts, to a less or greater degree. And finally, expert knowledge may be elaborated into the model, for instance: the knowledge that the value of a ratio of two variables should not differ much from some target value. These kind of constraints are necessary to model the continuity of prices in the national accounts, amongst others.

A system of annual accounts for the preliminary and semi-final year ($t-1$ and $t-2$) are obtained by adding up the outcomes of the four underlying quarters.

Figure 3.1 illustrates the main principle of the intended process for the univariate case. The horizontal line indicates the average value of the first year, averaged over the four quarters. As this value is higher than the average value of the original, first four quarters, the whole time series is shifted upward, without changing the quarter-to-quarter changes.

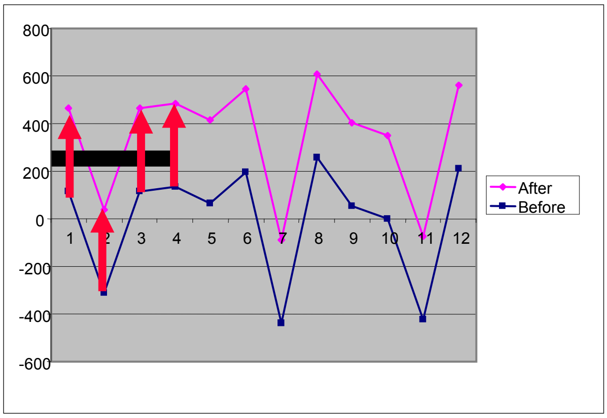

Figure 3.1 The intended, future method applied to an univariate time-series

There are several ways to set up the model. One of the biggest challenges of the project was to choose the way to incorporate the annual sources of the preliminary and semi-final accounts into the model. Several model set ups have been tested, and one of them has been selected for future use. In the 'best' set up other, simulated 'annual' figures have been added to the model, to ensure that the (few) sources of the preliminary and semi-final annual accounts influence the outcomes of the model in the correct way.

Finally, it should be mentioned that the purpose of the model is not the correction of the errors in the source information. It is intended in the future to correct errors before the benchmarking. However, in the current process error correction and data balancing are conducted simultaneously.

4. The mathematical model

4.1 Literature

The benchmarking methods in the literature can be classified into purely numerical methods and model-based methods, see for instance Bloem et al. (2001, Chapter VI) and Dagum and Cholette (2006). A classical reference to a method in the numerical field is Denton (1971). The Denton method is a quadratic programming method and is proposed for univariate data. Di Fonzo and Marini (2003) have extended the

Denton method to multivariate data. Subsequently, Bikker and Buijtenhek (2006) have introduced reliability weights to the multivariate Denton method. These weights can be used to describe differences in reliability of time-series' sources.

However, to satisfy all needs imposed by Statistics Netherlands the multivariate Denton has been further extended. The extensions involve: soft constraints, ratio constraints and inequality constraints.

4.2 The univariate Denton model

The aim of this method is to find a benchmarked time-series $\hat{x}_t$, t = 1,…,T, that preserves as much as possible all quarter-to-quarter changes of the original series x_t, subject to the annual benchmarks.

Denton proposed several measures to define the quarter-to-quarter changes. We consider the proportional first-order function and the additive first-order function. The additive function keeps additive corrections ($\hat{x}_t - x_t$) as constant as possible over all periods. The proportional function is designed to preserve the (proportional) growth rates of x_t and therefore keeps the relative corrections ($\hat{x}_t / x_t$) as constant as possible over all periods.

In mathematical terms the additive Denton model is:

$$\min_{\hat{x}} \sum_{t=2}^{T} \left((\hat{x}_t - \hat{x}_{t-1}) - (x_t - x_{t-1}) \right)^2 \tag{4.1}$$

$$\textit{such that} \sum_{t=(j-1)+1}^{t=(j-1)+4} \hat{x}_t = \hat{y}_j \qquad j = 1,\ldots, T/4 \tag{4.2}$$

and the proportional Denton model is given by

$$\min_{\hat{x}} \sum_{t=2}^{T} \left((\hat{x}_t / x_t) - (\hat{x}_{t-1} / x_{t-1}) \right)^2 \tag{4.3}$$

$$\textit{such that} \sum_{t=(j-1)+1}^{t=(j-1)+4} \hat{x}_t = \hat{y}_j \qquad j = 1, \ldots, T/4 \tag{4.4}$$

where j is an index of the year and $\hat{y}_j$ is an annual value. The set of restrictions expresses the alignment of four quarters to annual totals.

4.3 The extension to the multivariate case

The extension of the univariate Denton model to the multivariate case is straightforward. The multivariate additive model is given by:

$$\min_{\hat{x}} \sum_{i=1}^{N} \sum_{t=2}^{T} \frac{1}{\left(w_i^P\right)^2} \left((\hat{x}_{it} - \hat{x}_{it-1}) - (x_{it} - x_{it-1}) \right)^2 \tag{4.5}$$

$$such\ that\ \ C^H \hat{x} = b^H , \tag{4.6}$$

where i is the index for the time-series, N denotes the number of time-series, and w_i^P is a reliability weight that is attached to the i'th time-series. The set of restrictions may involve both contemporary (i.e. over different time-series, one period) and intertemporal (over different periods, and one time-series) restrictions.

The extension for the proportional model is similar. In our application the proportional and the additive models are combined. The user has to specify for each time-series whether the model is proportional or the additive.

4.4 The extension for soft constraints, ratio constraints, and inequalities

A set of soft constraints is given by

$$C^S \hat{x} \approx b^S . \tag{4.7}$$

The superscript S denotes that the constraints are soft (where the superscript H in (4.6) indicates that a constraint is hard). These constraints are included in the model by adding the following penalization terms to the objective function (4.5):

$$+\sum_{r=1}^{R^S} \frac{1}{\left(w_r^S\right)^2} \left(b_r^S - \sum_{t=1}^{T} \sum_{i=1}^{N} c_{rit}^S \hat{x}_{it} \right)^2 , \tag{4.8}$$

where r is the index for the soft constraints, R^S denotes the total number of constraints and w_r^S is the weight for constraint r.

Hard and soft ratio constraints are respectively given by

$$\hat{x}_{n.} / \hat{x}_{d.} = v \ \text{ and } \ \hat{x}_{n.} / \hat{x}_{d.} \approx v, \tag{4.9}$$

where x_n denotes the numerator time-series, x_d denotes the denominator time series. The sign . indicates the quarter, which is irrelevant here, since these restrictions will be the same for all time periods and v is some predetermined value. Ratio constraints can be linearised, in the following way

$$\hat{x}_{n.} - v\hat{x}_{d.} = 0 \text{ and } \hat{x}_{n.} - v\hat{x}_{d.} \approx 0 \tag{4.10}$$

These linearised ratios are included in the model, as they can be added to (4.6) and (4.7).

Note that, essentially, there is no difference between hard linear constraints and hard linearised ratio's, but there is a difference in the soft case. For soft linearised ratios, the value of v, at the left-hand side of the ≈ sign of (4.10), is uncertain. But in the case of a linear constraint the uncertainty is in the values of b^S, at the right hand side of the ≈ sign, see for instance Magnus et al. (2000). Due to this difference the weights of soft ratio's are computed in a different way than the weights of soft, linear constraints (see also Section 4.5).

The constraints (4.6) are equalities. However, in the standard formulation of a quadratic optimization (QP) problem, inequalities are allowed as well. Therefore a set of inequalities, given by

$$A\ \hat{x} \leq z, \tag{4.11}$$

can be easily included to the model.

The problem, defined by (4.5), (4.6), (4.8) and (4.11), is a standard (convex) quadratic programming (QP) problem. The problem is well known in the literature, and a lot of efficient solving techniques are available.

4.5 Methodological challenges

One of the biggest challenges in extending the model is to tune the weights of the different components of the objective function (i.e. additive changes, proportional changes, soft ratio's and soft, linear constraints). The weights have to be determined, so that the model meets certain desired properties, see Öhlén (2006). We focussed on two properties:

- Invariance: If all input data are multiplied by the same nonnegative scalar, the outcomes must also be changed by this factor;
- Symmetry: It does not matter for the results if a soft ratio in the benchmarking model would be replaced by its reciproke, i.e. if the restriction $x / y \approx r$ is replaced by $y / x \approx 1 / r$.

But most of all it should be kept in mind that expressions for weights should be easy to use, even for users without a mathematical background. We proposed expressions for all kind of weights in the aforementioned model. These expressions will not be mentioned in this paper, due to their technical character. One feature of our expressions is that it is possible to change all weights that are related to one time-series (i.e. both the weights of the quarter-to-quarter changes of that time-series and the weights of constraints that involve the time-series) by adjusting only the value of one parameter.

5. Software implementation

In order to be useful for practical implementation at Statistics Netherlands, the benchmarking software has to be able to handle very large data sets. To our knowledge large-scale applications of the multivariate case of the Denton method do not exist at NSIs (National Statistical Institutes). The lack of adequate technology used to be an obstacle for the implementation of automated benchmarking methods.

However, the current powerful optimization solvers, like CPLEX and XPRESS, are able to handle a large amount of data. Statistics Netherlands has developed a prototype of benchmarking software, using CPLEX as a solver.

A benchmarking problem with 13.790 time series, 3 years, 12 quarters has been translated into a quadratic optimization problem with approximately 160,000 variables and 75,000 constraints. By using CPLEX the optimal solution was found in 45 seconds. We expect that even larger optimization problems will have to be solved in the future, for example about 500,000 variables and 120,000 constraints. On the basis of some tests with simulated data, we strongly suspect that these large sizes are no problem.

Statistics Netherlands has also been using MATLAB to solve benchmarking problems. However, the method in MATLAB does not include inequality constraints. In MATLAB the solution of a benchmarking problem is found by solving a linear system, which implies inverting matrices. From our test we saw that CPLEX and XPRES can handle more variables than MATLAB.

Currently Statistics Netherlands is making a production version of benchmarking software. This software has to be user-friendly, directed for users without a mathematical background.

6. A first simulation exercise

6.1 The organization of the experiment

A simulation exercise has been conducted on the Dutch supply and use tables to test whether the proposed benchmarking method works in practise. The key question is whether it is possible to obtain results, that satisfy all requirements brought up by experts of the national accounts department. We only wanted to know whether it is possible to satisfy all needs, there was no intention to obtain results that indeed satisfy all requirements. Thus, we did not model all relations between the variables in the supply and use table. For some classes of similar restrictions we only worked out examples.

The (2008) experiment was organised as follows:

1) A first set up of the model was made, together with subject matter experts. The set up involves: data, constraints and tuning parameters like weights;
2) The automated benchmarking method was carried out;
3) Subject-matter specialists inspected the results of the model;
4) The comments of the specialists were translated to changes of the input of the model. For instance: different weights, other or more restrictions;
5) Go back to 2).

About four iterations have been carried out. Consultation sessions with subject matter specialists have been organised to improve the set up of the model. To save time the improvement proposals were applied to a (small) subset of the data only.

6.2 Results

The results of this experiment are excellent. We were able to incorporate all (non-technical) comments of the subject-matter specialists into our model. Although it was not the aim of the experiment, we obtained outcomes that closely resemble the published figures of the Dutch supply and use tables, especially for the main aggregates, like GDP (Gross domestic product). However, at a lower aggregation level differences were larger.

These results can be explained by the fact that the initial discrepancies of the constraints were small. As the experiment started with compiled systems of national accounts, there were no violations of the constraints within the quarters and years. The only discrepancies were those between quarterly and annual figures, but these were mainly small.

Another important result is a setup of a benchmarking model consisting of a combination of quarterly and annual data, constraints, weights and tuning parameters. This setup will be used for the final test in 2009.

6.3 Challenges in the model building

One aspect which makes it difficult to build the model is that conflicting source information is used in a benchmarking process. For instance: one of the assumptions of the supply and use table is that the ratio between production and intermediate use should be kept at its original value as much as possible, for each quarter. However the quarterly data have to align to annual values. The annual data also imply values for the ratio of production and intermediate use, which may be different from the target values implied by the quarterly data. This kind of inconsistencies may lead to unexpected model results (for instance negative values). For this kind of problems specific solutions should be found.

Another difficulty when modelling the process is that including some classes of constraints is nearly impossible. For instance, an expert argued that the change of (some of the) trade margins should be about the same as the change of consumption. This constraint can be written as:

$$A / B \approx C / D,$$

(note that a change means that one value is divided by another value). However, the Denton method we proposed is not able to cope with this kind of constraint. This problem has been solved by including different constraints, that have the same effect on the results. In the aforementioned example, a ratio constraint is introduced that keeps the ratio of the margins and consumptions as much as possible at its original value for each quarter. However, this approximation only works, because the constraint that the changes of trade margins are about the same as the changes of consumption, is already satisfied in the source data.

6.4 Challenges in the interpretation of the results

It is not always easy to find the reason for unexpected results. Since all time-series are connected by constraints, errors in the source data of one time series may lead to erratic results for an other time-series, that describes a totally different subject from a conceptual point of view. There are no algorithms to track down the cause of a problem.

In the experiment the task of finding the cause of problems has been performed by methodologists. However, in the future this task should be taken over by employees of the national accounts division.

7. Benchmarking in 2009

In 2009 a final test will be carried out. The aim of that test is to prove that the intended benchmark process will lead to results that are acceptable for subject-matter specialists. The model is planned to be adopted into the production process if the results of the test are positive. For the definitive implementation it is very important that the knowledge about using the model is passed on to the employees of the national accounts department.

There are also plans to start up a project with the aim of automating the reconciliation process of the quarterly and annual supply and use tables. This is more challenging, as the initial discrepancies are bigger and there may be constraints, that are different for each period. Furthermore, Statistics Netherlands wants to integrate the benchmarking process of several parts of the national accounts, i.e. the supply and use tables, the Sectoral accounts and the labour accounts, although this project has not been planned yet.

Further, the multivariate Denton method is planned to be presented at the NTTS 2009 congress.

Several methodological questions about the current Denton model are open for further research. These are:

- The definitions of the weights. The definition may be evaluated and improved. This could lead to a benchmarking model with more appealing properties than the current one;
- Rounding: the results of the benchmarking model have to be rounded to integer values. Currently, the rounding process is conducted after the benchmarking process. Possibly, both processes could be integrated in one;
- Non-linear constraints: there may be better methods for modelling difficult non-linear constraints (see Section 6.3).