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Explanation of symbols

.	= data not available
*	= provisional figure
x	= publication prohibited (confidentional figure)
–	= nil
–	= (between two figures) inclusive
0 (0,0)	= less than half of unit employed
a blanc	= category not applicable
2003–2004	= 2003 to 2004 inclusive
2003/2004	= average for the years 2003 up to and including 2004
2003/'04	= cropyear, financial year, school year etc., beginning in 2003 and terminating in 2004
2001/'02–2003/'04	= book year etc., 1993/'94 up to and including 2003/'04

Detailed items in tables do not necessarily add to totals because of rounding.

Revised figures are not marked as such.

Calculating weekly patterns with X-12-ARIMA

M. Jansen [1)]

Introduction

The regression model of the seasonal adjustment program X-12-ARIMA is suitable for deriving weekly patterns (average percentage contribution per weekday) from monthly series. Benchmark analyses have demonstrated reliable results. [2)] The method will be explained in this note. A detailed example is annexed.

Method

Many statistical series show a weekly pattern: retail sales, for example, are usually higher on Fridays and Saturdays than on Mondays and Tuesdays. Because of these weekly patterns, monthly figures are not fully comparable with each other. Except for February, months are longer than exactly four weeks which means that some months have an additional Friday and Saturday, while others have an additional Sunday and Monday.
Adjustments for these weekday patterns can be performed by X-12-ARIMA, a program actually developed for seasonal adjustment. In order to correct for seasonal patterns, series are split into different components. This decomposition normally takes place with a multiplicative or additive method.
X-12-ARIMA supports adjustment for calendar effects because it renders the seasonal decomposition of series more accurate. Calendar effects can be removed from series by regression techniques. The regression model for weekday adjustments (also called trading day model) is the basis for our method to derive weekly patterns from series.
The method works as follows. We apply the regression model with seven weekday variables to a series for which we want to determine a weekday pattern. This results in a parameter estimate β_i for each weekday. In the case of a *multiplicative decomposition* we apply this formula:

$$X_{it} = \left(\frac{12}{365} + \beta_i\right) Y_t$$

where

i = 1,..,7 the weekday concerned
β_i = the parameter estimate of the trading day model for weekday i
X_{it} = the calculated value of weekday i in month t if the month has five days i
Y_t = observed value in month t

In the case of an *additive decomposition* we apply a slightly different formula:

$$X_{it} = \left(\frac{12}{365}\right)\overline{Y} + \beta_i$$

where

$$\overline{Y} = \frac{\sum_{t=1}^{n} Y_t}{n}$$

and

n = number of observations.

The next step is the calculation of the percentage contribution of day i in the total week value (α_i). For both additive and multiplicative decompositions this is calculated as follows:

$$\alpha_i = \frac{X_{it}}{\sum_{i=1}^{7} X_{it}}$$

Because Y_t and $\overline{Y}$ respectively fall away in the calculation, α_i is independent of t.

We can conclude from the formulas that if for all i's $\beta_i = 0$ calendar effects do not exist. X_{it} then equals the average value per day for all i's, e.g. (12/365) Y_t and (12/365) $\overline{Y}$ respectively. If $\beta_i \neq 0$, calendar effects are present and the contribution of the weekday concerned deviates from the average. The distribution of observed values over the weekdays can be determined with α_i.

We recommend that the regression model be extended with outlier detection and leap year and holiday regression. The results will then be more accurate. Of course the implemented regression variables in the model must be statistically significant. The model span can considerably influence the weekday pattern and should therefore be carefully chosen. Also the type of decomposition can influence the calculation of weekday patterns.

Annex

In this Annex we apply the method to the turnover figures of the Dutch food retail for the period 1994–2000.

We calculate a (multiplicative) regression model with X-12-ARIMA. On the basis of the P-value (not shown here) we conclude that the weekly pattern is significant on the whole. The t-values of the separate days give information about the reliability of the effects. In the model below, only Tuesday is not significant (|t|<1,96), but this is caused by the small parameter estimate. Notice that the trading day model has been extended with leap year regression.

Regression Model

Variabel	Parameter estimate	Standard error	T-value
Trading day			
Mon	–0.0205	0.00272	–7.55
Tue	–0.0007	0.00264	–0.28
Wed	–0.0088	0.00263	–3.36
Thu	0.0153	0.00267	5.75
Fri	0.0237	0.00273	8.68
Sat	0.0166	0.00274	6.04
*Sun (derived)	–0.0255	0.00266	–9.59
Leap year	0.0243	0.00806	3.01

In the next step we determine the weekly pattern with the help of the trading day parameters. While Y_t plays no role in the calculation of α_i, we define Y_t =1 arbitrarily. This results in the following calculations:

Day	X_{it} for Y_t =1	α_i
Monday	(12/365) – 0.0205 = 0.012377	5.38%
Tuesday	(12/365) – 0.0007 = 0.032177	13.98%
Wednesday	(12/365) – 0.0088 = 0.024077	10.46%
Thursday	(12/365) + 0.0153 = 0.048177	20.92%
Friday	(12/365) + 0.0237 = 0.056577	24.57%
Saturday	(12/365) + 0.0166 = 0.049477	21.49%
Sunday	(12/365) – 0.0255 = 0.007377	3.20%
Total	(84/365) = 0.230	100%

Notes

1) The author wishes to thank colleague G. van Leeuwen for the methodological assistance in writing this note.

2) Benchmark analyses were performed by Statistics Netherlands on the basis of confidential information provided by retailers. For this reason we cannot publish the results of our benchmark analyses.