# Probabilistic population and household forecasts for the Netherlands 

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#### Abstract

Population forecasters have a long tradition of explicitly stating the uncertainty of their forecasts by means of specifying alternative variants. In addition to a medium variant, usually high and low variants are published. One problem in using high and low variants is that it is unknown how likely it is that the interval between these variants will cover the actual population size. Probabilistic population forecasts provide information about the probability that a specific forecast interval will cover the true future value. The development of high speed computers has made it possible to make probabilistic forecasts by means of simulations. The probability distribution of future population is assessed on the basis of assumptions about the degree of uncertainty of future fertility, mortality, and migration. In assessing the uncertainty of household forecasts one major question is how low and high variants should be specified. It is not evident whether one should combine the high population variant with high or low average household size. Since the high population variant is based on high fertility, it seems logical to combine the high population variant with large household size and similarly the low population variant with small family size. However, this would imply that the difference between the numbers of households in the low and high variants would be rather small, which may lead to underestimating the degree of uncertainty of household forecasts. On the other hand combining high population growth with low household size may lead to overestimating uncertainty. One benefit of making probabilistic


household forecasts is that the forecaster does not need to choose one combination of population and household variants.
This paper discusses concisely the methodology of making probabilistic population and household forecasts. The paper focuses on the way the underlying assumptions are specified. The discussion is based on an analysis of Dutch data.

## 1. Introduction

Population forecasts are based on assumptions about the future development of fertility, mortality, and migration. In most industrialised countries the official population forecasts include one medium, main, central or baseline variant. This variant usually aims to project either a continuation of observed trends in fertility, mortality, and migration, or the future developments that are considered most probable by the forecasters. Of course, these two criteria may coincide: forecasters may assume that a continuation of trends is the most probable future.
Uncertainty of population forecasts depends on the uncertainty of the validity of the assumptions about future fertility, mortality, and migration. Uncertainty on future fertility, mortality, and migration can be taken into account by means of calculating the effect of alternative values of these three components on the future size and age structure of the population. Each combination of values for the three components results in one set of outcomes of population size and age structure. Usually high and low variants are based on combining high and low values of fertility, life expectancy, and net migration.
In assessing the width between low and high variants the question is how extreme the variants should be. Obviously the probability that the interval between extreme variants will cover the true value is larger than the probability for a narrow interval. However, if the range is large, the forecast is uninformative. The question how wide the interval between forecast variants should be depends on the question how probable the interval between the variants should be, i.e. how likely it should be that the interval will cover the true future development. Thus an assessment of the width of the interval between variants requires that some indication of the probability be specified. "It is important for users to know the probability of a scenario. For example, a high-fertility scenario with a 20 per cent probability should be taken more serious in policy considerations than one with only 2 per cent probability" (Lutz, Goldstein and Prinz, 1996).
There are other reasons why an indication of the probability of forecast variants is useful. One is that it provides a criterion for achieving consistency between the forecast intervals for fertility, mortality, and migration, in the sense that the differences between the separate intervals should reflect differences in uncertainty between these three components. Forecasts of fertility, mortality, and migration are expressed in different measures: the ultimate number of children per woman, life expectancy at birth, numbers of immigrants and emigration rates, respectively. Thus the width of the intervals between the variants for the three components cannot be compared directly. Probability provides a criterion for comparing the forecast intervals between these indicators, e.g. by requiring that the interval for each component should correspond with a given probability.
Another reason for specifying the probability of a forecast is that it creates greater flexibility in reflecting uncertainty. Deterministic models are based on a choice by the forecaster about whether certain developments will take place or not. They either include or exclude certain parameter values. "Using a probability distribution permits one to express states of knowledge in between these two alternatives" (Daponte et al., 1997).
Even if probabilistic assumptions about future fertility, mortality, and migration are specified, this does not imply that the probability distribution of future population is known. In many countries three forecast variants are produced: low, medium and high. In the low variant fertility, life expectancy and net migration are low in each forecast year. Accordingly in the high variant all three components are high in each year. This implies that the probability of the forecast interval of future population size does not correspond with the probability of the intervals between the low and high values for the separate components, unless there is perfect correlation between the components and between the forecast years. Since we know that this is not true, the probability of the interval for population size is higher than the probability of the separate intervals for the three components. Consequently, the width between the low and high variants may lead to overestimating the degree of uncertainty of the forecasts of population size, at least if the assumptions about the intervals for fertility, mortality, and migration are specified correctly. In contrast, the width between the low and
high variants may lead to underestimating the degree of uncertainty of the old age dependency ratio, because high fertility lowers the ratio, while low mortality raises it (Lee, 1996). Thus for an appropriate indication of the degree of uncertainty of population forecasts we need probabilistic population forecasts. An analytical solution requires many simplifying assumptions. Therefore simulations are used to calculate probability distributions. The simulations are based on assumptions about the underlying distributions. These assumptions are discussed in sections 2 and 3 .

Contrary to population forecasts there is not one common practice of making household forecasts. This is true all the more for assessing the degree of uncertainty. The Dutch household forecasts published by Statistics Netherlands are based on assumptions about changes in the distribution of the population by household position. The main assumptions refer to changes in the age at leaving the parental home, in the percentage of singles and in the percentage of people living together. Therefore the assessment of the uncertainty of the Dutch household forecasts is based on the specification of intervals for these percentages. A high percentage of singles leads to small average household size. If this is combined with the high population variant, this results in a large number of households. It is, however, questionable whether the underlying assumptions are consistent. The high population variant is based on assuming high fertility and high life expectancy. High fertility does not seem to be consistent with assuming a high percentage of single person households, but rather with a high percentage of people living together. Moreover, high life expectancy implies a relatively low percentage of widows and widowers and thus a relatively low percentage of single person households among elderly people. One drawback of the opposite possibility, i.e. the combination of the high population variant with high average household size, is that the difference between the number of households in both variants would be relatively small (e.g. Alders and Manting, 1999). The obvious solution to these problems is the specification of probabilistic household forecasts. The methodology is discussed in section 2.

After the methodological discussion in sections 2 and 3 , sections 4, 5, and 6 discuss the assumptions about the degree of uncertainty of fertility, mortality, and migration respectively underlying the 1998 Dutch population forecasts. Section 7 discusses the assumptions about the uncertainty of changes in household positions underlying the Dutch household forecasts. The main results are discussed in section 8 . The last section provides some concluding remarks.

## 2. Method

### 2.1 Population forecasts

Population forecasts are based on assumptions about future changes in fertility, mortality, and migration. In the Dutch population forecasts assumptions on fertility refer to age-specific rates distinguished by parity, mortality assumptions refer to age- and sex-specific mortality rates, assumptions about immigration refer to absolute numbers, distinguished by age, sex and country of birth, and assumptions on emigration are based on a distinction of emigration rates by age, sex and country of birth.
Based on statistical models of fertility, mortality, and migration, statistical forecast intervals of population size and age structure can be derived, either analytically or by means of simulations. In order to obtain a forecast interval for the age structure of a population analytically a stochastic cohort-component model is needed. Application of such models, however, is very complicated. Analytical solutions require a large number of simplifying assumptions. Examples of applications of such models are given by Cohen (1986) and Alho and Spencer (1985). In both papers assumptions are specified of which the empirical basis is questionable. Instead of an analytical solution, forecast intervals can be derived from simulations. The development of high speed computers has made this possible.

On the basis of an assessment of the probability of the bandwidth of future values of fertility, mortality, and migration, the probability distribution of the future population size and age structure can be calculated by means of Monte Carlo simulations. For each year in the forecast period values of the total fertility rate, life expectancy at birth of men and women, numbers of immigrants and emigration rates are drawn from the probability distributions. Subsequently age-gender-specific fertility, mortality, and emigration rates and immigration numbers are specified. Each draw results in a population by age and gender at the end of each year. Thus the simulations provide a distribution of the population by age and gender in each forecast year.
In recent years probabilistic population forecasts were published for various countries: Austria (Hanika et al., 1997; Lutz and Scherbov, 1998b), Finland (Alho, 1998), Germany (Lutz and Scherbov, 1998a), the Netherlands (de Beer, 1997; Alders and de Beer, 1998), and the United States (Lee and Tuljapurkar, 1994).

For calculating the simulations several assumptions have to be made. First, the type of probability distribution has to be specified. Subsequently assumptions about the parameter values have to be made. The assumption about the mean or median value can be derived from the medium variant. Next assumptions about the value of the standard deviation have to be assessed. Finally assumptions about the covariances between the forecast errors across age, between the forecast years, and between the components have to be specified (see e.g. Lee, 1996).

1. Type of probability distribution for each component

For calculating the simulations it is not sufficient to specify the probability corresponding with the bandwidth between low and high values for each component. A complete probability distribution needs to be specified for each component for each forecast year. If a normal distribution is assumed, only two parameters have to be specified: the mean (corresponding with the medium variant) and the standard deviation. Similarly the uniform distribution requires only two parameters to be specified, viz. the minimum and maximum values. If an asymmetrical distribution is assumed, at least one additional parameter indicating the skewness has to be specified. One disadvantage of an asymmetrical distribution is that the mean of the distribution, i.e. the medium variant, does not correspond with the most probable value. This may be confusing for the users of the forecast.
Lutz and Scherbov (1998b) and Alders and de Beer (1998) show that the choice of the probability distribution has an effect on the tails of the distribution of population size rather than on the width of the $50 \%$ or $67 \%$ forecast interval.
In the Dutch population forecasts the forecast errors of the total fertility rate, life expectancy at birth and emigration rates are assumed to be normally distributed. The distribution of immigration is asymmetrical. In order to avoid negative values for separate categories of immigrants, assuming a symmetric distribution would imply that the intervals would be too small. A beta distribution is assumed.
2. Value of standard deviation of each component

For each year in the forecast period assumptions have to be specified about the values of the parameters of the distribution for each component. Usually one set of values is determined for the complete period. The values for the separate forecast years are determined on the basis of the assumption about the pattern of serial correlation. Broadly speaking two approaches can be distinguished. First, the assumptions can refer to the values of the parameters of a time-series model. The value of the standard deviation for each separate forecast year can be derived from this model. Second, assumptions about the standard deviation in the last year of the forecast period year can be specified. Values for the other years in the forecast period can be determined by means of some interpolation method. The latter approach is followed in the Dutch population forecasts. The means of the distributions equal the values in the medium variant. In addition, assumptions are specified about the upper and lower limits of $95 \%$ forecast intervals in
the last year of the forecast period, i.e. the year 2050. The values of the preceding years are determined on the basis of the assumption that the relationship between the standard deviation of the forecast errors and the forecast lead time is similar to the corresponding relationship for the errors of a random walk model.
Section 3 discusses three methods for assessing the values of the standard deviation. Sections 4,5 and 6 discuss the assumptions on the values for fertility, mortality, and migration underlying the 1998 Dutch population forecasts.
3. Correlation across age

The age-specific fertility and mortality rates in a given forecast year can be expected to be positively correlated. If the economic and social situation is favourable for having children, it can be expected that all age-specific fertility rates will be relatively high. For cohort data a negative correlation may be plausible. If the economic situation drives people to postpone having children, age-specific fertility rates at young and old ages may be negatively correlated. Similarly a selection mechanism may cause a negative relationship between mortality rates at young and old ages for the same cohort. Instead of specifying separate distributions for each age-specific fertility, mortality and emigration rate and number of immigrants, in the Dutch population forecasts assumptions are made about the distribution of indicators of the general level of fertility, mortality, immigration, and emigration. The assumptions about the age-specific rates and numbers are based on the assumption of perfect positive correlation across age. Assuming less than perfect positive correlation would result in smaller forecast intervals. Lee (1996) argues that even though correlations across age may be less than perfect, the effect of assuming correlations to equal unity on long range forecasts will be small.
4. Serial correlation

The probability distributions of fertility, mortality, and migration in successive forecast years are correlated. If fertility is very high in one forecast year, it will not be very probable that fertility will be low in the next year. Thus if a high value of fertility is drawn in one forecast year, the probability of drawing a high value in the next year should be higher than that of drawing a low value. In the short run a negative correlation may also be possible. For example, if the number of deaths is relatively high in one year due to a severe winter, the number of deaths may be relatively low in the next year due to a selection mechanism, as many frail people died in the previous year.
Assuming perfect positive serial correlation results in overestimating the degree of uncertainty of total population size. Lutz and Scherbov (1998b) assume perfect autocorrelation as they assume that fertility, mortality, and migration follow standard trajectories in each simulation, which differ only by one random factor by which the values in each year are multiplied (the 'random line' or 'random scenario' approach). They argue that assuming perfect correlation has only a limited effect in the short run. However, Alders and de Beer (1998) show that the effect is not negligible in the long run. In the Dutch population forecasts it is assumed that the serial correlation corresponds with that of the forecast errors of a random-walk model.
5. Correlation between components

The levels of fertility, mortality, immigration, and emigration can be correlated. For example, if immigrants have more children than the native population, an increase in the number of young immigrants may lead to an increase in the fertility rates in later years. Note that even if independence between fertility and mortality rates and migration numbers is assumed, there is no independence of numbers of births and deaths, and numbers of migrants. For example, if immigration is high in a certain year, this will result in larger numbers of births and deaths in later years for given values of fertility and mortality rates.
Keilman (1997) shows that the correlation between errors in fertility, mortality, and migration is only small. In the Dutch population forecasts independence between fertility, mortality and emigration rates and immigration numbers is assumed.

### 2.2 Household forecasts

In contrast with the common use of the cohort-component model for population forecasts in all developed countries, many different methods are used for household forecasts. The methods that are being used in practice range from simple headship rate methods to complex dynamic models (de Beer, 1994). Using a headship rate method the number of households is forecast on the basis of assumptions about future changes in the number of heads of households as a percentage of the population by age and sex. One problem of such methods is that they do not provide insight in the underlying changes in household formation and dissolution. When employing a dynamic model changes in numbers of households are based on assumptions about transitions between household positions. One problem of such models is that they require many assumptions. Usually it is difficult to obtain reliable data on all transitions.
Statistics Netherlands employs a mixed method (de Beer, 1994). It consists of three steps. In the first step the population by marital status is projected on the basis of a multi state model. The assumptions underlying these projections are based on an analysis of data on first marriage rates, divorce rates and remarriage rates that are obtained from population registers. In the second step assumptions are specified about changes in the distribution of the population by household position, distinguished by age, sex, and marital status. Six household positions are distinguished: living at the parental home, living alone, living with a partner, being a single parent, living in an institution, and other. Assumptions are made about the relationship between changes in marital status between successive cohorts and ages, and changes in household position. For example, if the difference of the percentage of married persons between age $x$ and $x$ - 1 for cohort $y$ is lower that that for cohort $y-1$, an assumption is made about the fraction of the resulting increase in the percentage of nevermarried persons that can be attributed to an increase in the percentage of people living alone and an increase in the percentage of people living in a consensual union.

The main assumptions underlying the household forecasts refer to the age of leaving the parental home, to the percentages of people living alone or with a partner and to the number of people living in an institution. For that reason the forecast interval of the future number of households is based on simulations that are based on assumptions about the probability distribution of these household positions. The specification of the multivariate distribution requires assumptions about the entire covariance matrix. In order to restrict the number of assumptions, an alternative approach is followed, based on the specification of conditional probabilities. First an assumption is made about the probability distribution of the number of people living in an institution. Together with the probability distribution of the population by age and sex this yields the population in private households by age and sex. Second, an assumption is made about the probability of changes in the age at leaving the parental home (given that people live in a private household). Third, an assumption is made about the conditional probability (given that people have left home) of changes in the percentage of people living alone. For the sake of simplicity single persons and lone parents are joined together. The rest of the people consists mainly of persons living with a partner. For the year 2050 percentages of the population in the separate household positions are drawn from these probability distributions. The percentages of previous years are assessed by means of interpolation between the latest observed values and the values in 2050. This implies perfect autocorrelation. This does not affect the results, as the percentage distribution of the population by household position in one year does not depend on the distribution in previous years. Furthermore perfect correlation across ages and perfect correlation between men and women is assumed.
Each draw is combined with a draw of the population by age and sex. This yields the number of persons by age, sex, and household position. The probability distribution of the number of households can be calculated simply by assuming that persons living alone and lone parents count for one household, that persons living together with a partner count for a half
household and that children living with their parents and people living in an institution count for no household.

## 3. Assumptions about parameters of distributions

The main assumptions underlying the probability distribution of future population relate to the standard deviation of the distributions of future fertility, mortality, and migration. The values of the standard deviations can be assessed in three ways:

1. An analysis of errors of past forecasts.
2. Model-based estimate of forecast errors.
3. Expert judgement.

Ad 1. An analysis of errors of past forecasts.
The probability of a forecast interval can be assessed on the basis of a comparison with the errors of forecasts published in the past. On the assumption that the errors are approximately normally distributed - or can be modelled by some other distribution - and that the future distribution of the errors is the same as the past distribution, these errors can be used to calculate the probability of forecast intervals of new forecasts. Keilman (1990) examines the errors of forecasts of fertility, mortality, and migration of Dutch population forecasts published between 1950 and 1980. He finds considerable differences between the errors of the three components. For example, errors in birth rates grow three times as fast as errors in life expectancy. Furthermore, he examines how strongly forecast errors increase with the length of the forecast period, to what extent errors vary between periods and whether errors of recent forecasts are smaller than those of older forecasts, taking into account the effect of differences in the length of the forecast period. One problem in comparing old and new forecasts is that they refer to different periods. Thus a simple comparison of forecast errors of old and new forecasts does not tell whether or not forecast accuracy has increased. For that reason Keilman (1990) uses an APC (Age Period Cohort)-type model for estimating the separate effects of the length of the forecast period (the 'age effect'), the - social and demographic conditions during the - period to be forecast (the 'period effect'), and the jumpoff year (the 'cohort-effect'). One problem in assessing these separate effects is that interactions may play an important role and that it not obvious how these interactions should be implemented in the model. Keilman's model recognises that some periods are more 'difficult' to forecast than others. For that reason his model includes a period effect. The question what is 'difficult' to forecast, however, is not independent of the type of forecast. For example, developments in a period in which fertility rates hardly change would be forecast accurately by a forecast based on constant levels of fertility rates, but not by a forecast based on assuming constant changes in fertility rates (unless fertility rates did not change in the period before the forecast was made). Thus a period in which rates do not change much is not necessarily more 'easy' to forecast than a period with changing rates. If in the period before the forecast was made, rates changed in one direction, a subsequent period with rates changing in the same direction would have been more 'easy' to forecast than a period with constant rates.
One problem in using errors of past forecasts for assessing the degree of uncertainty of new forecasts is that the sample of ex post forecasts tends to be biased towards the older ones, as for recent forecasts the accuracy cannot yet be checked except for the short run (Lutz, Goldstein and Prinz, 1996). Forecast errrors for the very long run result from forecasts made a long time ago. Both the variability of fertility, mortality, and migration and the methodology of the forecast may have changed. Hence it is questionable whether the assumption that the future distribution of errors is the same as the past distribution is valid. For example, the sharp decline of fertility after 1965 in many European countries caused large forecast errors. In assessing the uncertainty of new forecasts the question is how likely it is that large errors such as those of the 1960s may occur again in the future, for after 1975 fluctuations in fertility have been much smaller. We will come back to this in section 4.

Ad 2. Model-based estimate of forecast errors.
Instead of assuming that future forecast errors will be similar to errors of past forecasts, one may attempt to estimate the size of future forecast errors on the basis of the assumptions underlying the methods used in making new forecasts. If the forecasts are based on an extrapolation of observed trends, ex ante forecast uncertainty can be assessed on the basis of the time-series model used for producing the extrapolations. If the forecasts are based on a stochastic time-series model, the model produces not only the point forecast, but also the probability distribution. For example, ARIMA (Autoregressive Integrated Moving Average)models are stochastic univariate time-series models that can be used for calculating the probability distribution of a forecast (Box and Jenkins, 1970). Alternatively, structural time series models can be used for this purpose (Harvey, 1989). The latter model is based on a Bayesian approach: the probability distribution may change as new observations become available. The Kalman filter is used for updating the estimates of the parameters. One problem in using stochastic models for assessing the probability of a forecast is that the probability depends on the assumption that the model is correct. Obviously the validity of this assumption is uncertain, particularly in the long run. If the point forecast of the time series model does not correspond with the medium variant, the forecaster does apparently not regard the time series model as correct. Moreover, time series forecasting models were developed for short horizons, and they are not generally suitable for long run forecasts (Lee, 1996). Alternatively, the form of the time series model can be based on judgement to constraint the long-run behaviour of the point forecasts so they are in line with the medium variant of the official forecast (Tuljapurkar, 1996).
Keilman and Pham (1998) specify a VAR model (i.e. a multivariate ARIMA model) for the total fertility rate, the mean age at childbearing and the variance in the age at childbearing. The point forecast of that model is rather close to the medium variant of the total fertility rate in the Norwegian population forecasts. The forecast interval based on the VAR model is much wider than the interval between the low and high variants of the Norwegian forecasts. Keilman and Hetland (1999) conclude that the high-low interval of the official forecasts corresponds with a $20 \%$ interval.
Rather than identifying an appropriate time-series model and analytically deriving forecast intervals from the selected model, empirical forecast errors can be assessed by means of calculating the forecast errors of a simple baseline projection. Alho (1998) notes that the point forecasts of the Finnish population forecasts are similar to projections of simple baseline projections, such as assuming a constant level of fertility rates and a constant rate of change of age-specific mortality rates. If these baseline projections are applied to past observations, forecast errors can be calculated. The relationship of these forecast errors with the length of the forecast period can be used to assess forecast intervals for new forecasts.

## Ad 3. Expert judgement.

In assessing the probability of forecast intervals on the basis of either an analysis of the errors of past forecasts or an estimate of the size of model-based errors, it is assumed that the future will be like the past. Instead, the probability of forecasts can be assessed on the basis of experts' opinions about the possibility of events that have not yet occurred. For example, the uncertainty of long-term forecasts of mortality depends on the probability of technological breakthroughs that may have a substantial impact on survival rates. The uncertainty of long-term forecasts of migration depends on the probability of major shifts in international economic relations, e.g. the possibility of an economic boom in Africa. Even though these developments may not be assumed to occur in the most likely variant, an assessment of the probability of such events is needed to determine the uncertainty of the forecast. More generally, an assessment of ex ante uncertainty requires assumptions about the probability that the future will be different from the past. If a forecast is based on an extrapolation of past trends, the assessment of the probability of structural changes which may cause a reversal of trends cannot be derived directly from an analysis of historical data and therefore requires judgement of the forecaster. Lutz, Goldstein and Prinz (1996) argue that subjective distributions are to be preferred to a time-series approach, because "structural
changes and unexpected events are likely to happen." Lutz, Sanderson, Scherbov and Goujon (1996) assess the probability of forecasts on the basis of opinions of a group of experts. The experts are asked to indicate the upper and lower boundaries of 90 percent forecast intervals for the total fertility rate, life expectancy, and net migration up to the year 2030. Subjective probability distributions of a number of experts are combined in order to diminish the danger of individual bias.
When the probability of forecasts is assessed on the basis of expert judgement it is important to take into account possible sources of bias. On the basis of an analysis of Dutch population forecasts Keilman (1990) concludes that after a steady movement of trends in a consistent direction, the confidence of forecasters increases. For example, in the 1980-based Dutch population forecasts the interval between the low and high variants of fertility was rather narrow. Keilman concludes that "this reflects the optimism among the forecasters about the predictability after a period of stable trends." By the same token the relatively wide interval between the low and high variants in the 1975-based forecasts can be considered as a reaction to the sharp decline of fertility rates in the preceding years.
In making assumptions about the standard deviation of forecast errors for each component one has to take into account that the way uncertainty increases with the forecast lead time differs between components. In the short run migration tends to be the most uncertain component, as migration usually shows very large fluctuations in successive years. However, the degree of uncertainty only increases to a limited extent with the forecast lead-time as increases tend to be followed by decreases, and vice versa. For fertility the uncertainty in the short run is smaller than that for migration, as the number of births is much less strongly changing between one year and the next. However, uncertainty strongly increases with the forecast horizon due to the uncertainty about the size of long-run changes in the trend. Finally, the uncertainty about mortality is much smaller than that about fertility or migration, as mortality rates tend to change only gradually.

These three methods do not exclude each other. Rather these methods may complement each other. For example, an analysis of errors of past forecasts may be more useful for short- and medium-term forecasts than for long-term forecasts, since there are only few data on errors for the long run and, moreover, these errors result from forecasts that were made some decades ago on the basis of considerably less methodological and substantial knowledge than is available at present. One way of trying to overcome this problem is to extrapolate forecast errors by means of a time-series model (De Beer, 1997). If the increase of forecast errors with the length of the forecast period follows a regular pattern, the size of forecast errors for the long run can be projected on the basis of forecast errors of recent forecasts for the short run. Thus estimates of ex ante forecast errors can be based on an extrapolation of ex post errors. Alternatively the use of past forecast errors for assessing the probability of short-term forecast intervals can be complemented by expert judgement for assessing the uncertainty of long-term forecasts.
One problem in assessing the probability distribution of household forecasts is that there are hardly historic forecast errors. Moreover, only limited time-series data tend to be available. Thus the first two methods are hardly feasible for household forecasts. Therefore the assumptions underlying the probability distribution of households are mainly based on judgement.

## 4. Fertility

In the Dutch population forecasts the assumptions about future changes in fertility are based on a distinction by parity. Accordingly the assumptions about the standard deviation of forecast errors are based on a distinction by parity. The assumptions relate to the upper and lower limits of a $95 \%$ interval. This implies that it is assumed that it is very unlikely that a forecast that is higher than the upper limit or lower than the lower limits will come true. The last year of the forecast period is 2050 . The assumptions underlying the medium variant refer to cohort fertility. In the long run age-specific fertility rates are assumed to remain
constant. As a consequence the total fertility in 2050 equal the average number of children of women born around 2020. Simulations show that the forecast interval for the TFR in 2050 equals the interval for cohort total fertility in 2020. Keilman and Hetland (1999) draw the same conclusion on the basis of Norwegian data. Therefore assumptions are specified about the width of the interval of the number of children for cohort 2020.

One problem in analysing forecast errors of historic forecasts is that there are no forecast errors for a period of 50 years. Therefore the standard deviation of forecast errors for the long run is assessed on the basis of a projection of forecast errors for shorter time horizons. The way forecast errors of the TFR develop with increasing forecast lead time can be described by a random-walk model. Errors of forecasts of the TFR are used rather than forecasts of cohort fertility since there are not sufficient forecasts of cohort fertility. This model projects a $95 \%$ forecast interval for 2050 of $\pm 1.1$ children. This is partly based on the large forecast errors of the forecasts made in the 1960s and early 1970s. One question is whether similarly large errors can be expected to occur in the future. This question cannot be answered on purely statistical grounds. Judgement should play a role. Judgement can be based on analysis of fertility rates by parity. The large forecast errors in the 1960s and 1970s occurred when fertility was at a considerably higher level than at present. Fertility rates of third and fourth births were much larger. Similar forecast errors in the same direction seem very unlikely in the future. Fertility rates of parity 3 can hardly decline to the same extent as in the past, since they are already rather low. This would imply that fertility rates of parity 1 and 2 would have to decline much stronger than they did in the past. It would e.g. imply that a vast majority of women would remain childless. The Dutch Fertility and Family Surveys (FFS), however, indicates that only a small minority of young generations of women want to remain childless. Thus very low fertility levels would imply that very many women could not realise their expectations. Similar large errors in an upward direction do not seem likely either, because of the changed position of women. The strong increase in the labour force participation rates of women does not make it very likely that the percentage of women having three or four children will increase strongly.

The assumptions about the forecast interval of the average number of children are based on the distribution by parity. Note that if there is no perfect correlation between the forecast errors for the separate parities, the $95 \%$ forecast interval for the total number of children does not equal the sum of the $95 \%$ intervals for the separate parities. However, if the correlation is strong, albeit not perfect, the difference with the $95 \%$ interval is only small. Since the assumptions refer to the order of magnitude of the probabilities rather than the exact level, the difference may be ignored in specifying the assumptions. Thus it is assumed that the forecast interval of the total number of children equals the sum of the assumed forecast intervals for the separate parity-specific fertility rates.

## First children

In the Netherlands about $90 \%$ percent of women born in 1945 had at least one child. For younger generations the percentage of women who had their first child before age 30 has decreased strongly. According to the medium variant of the 1998 Dutch population forecasts the percentage of young women having their first child before age 30 will be only half that of the generation born in 1950. However, above age 30 the differences between successive cohorts will become much smaller. According to the medium variant $80 \%$ of women of future generations will have at least one child. For the upper limit of the 95\% forecast interval $90 \%$ seems a reasonable assumption. The percentage cannot be much higher because of infecundity. It is estimated that about $5 \%$ of couples cannot have children because of infecundity. Moreover, not all women will find a partner. Furthermore dissolution of a partnership may lead to childlessness. Consequently it is assumed to be unlikely that childlessness will be lower than 10\%. This percentage is high compared with historic values. Assuming that $90 \%$ of women would have at least one child would imply that either fertility above age 30 would increase very strongly or that there would be an increase of fertility at young ages, in contrast with the trend of the last decades.
A reasonable assumption for the lower limit of the $95 \%$ interval seems a value of $60 \%$. It seems unlikely that childlessness would be over $40 \%$. Successive fertility surveys, held by Statistics Netherlands, indicate that the vast majority of women wants to have children. For example, in the 1998 Fertility and Family Survey less than 10 percent of women with a low level of educational attainment and less than 20 percent of highly educated women who were born in the second half of the 1960s expect to remain childless. A level of childlessness of $40 \%$ would imply that either a large percentage of women would not realise their expectations, e.g. because increasingly women would face problems in combining children and a paid job or because the attitude to having children would change dramatically, e.g. because young women would give higher priority to a career than to raising children or because for many women postponement would result in ultimate childlessness. Of course, such a development is not impossible but it does not seem very likely either. It would imply that the percentage of women having children would decrease even more than it did during the last decades. The value of $60 \%$ is lower than the current period total fertility rate for first births. This level will only be reached by young cohorts if there would come a sudden end to the increasing trend in the percentage of women having their first child above age 30.

## Second children

About $80 \%$ of women born in 1945 had two children or more. According to the medium variant of the 1998 Dutch population forecasts $60 \%$ of women of future generations will have at least two children. This implies that $20 \%$ will have only one child. For the upper limit of the $95 \%$ interval $80 \%$ of women having at least two children seems a reasonable assumption. This is equal to the level for women born in the second half of the 1930s. Compared with the upper limit of $90 \%$ for first births, this would imply that almost $90 \%$ of women having a first child would also have a second child. According to the medium variant this would be $75 \%$. A higher percentage does not seem likely as there will always be women who have only one child, e.g. because of the dissolution of their relationship or because they have their first child at a relatively high age. The lower limit of the $95 \%$ interval is assumed to be $40 \%$. In view of the strong preference for a family size of two children which appears from the Dutch FFS a lower percentage does not seem likely. Compared with the lower limit of $60 \%$ for first births it would imply that only two thirds of women having a first child would also have a second child. Even in a country like Italy, where the period fertility rates have reached a very low level in the 1990s, the fertility rates for parity 2 did not decline below $40 \%$.

## Third children

The percentage of women having at least three children declined sharply between cohorts 1935 and 1945 from about $45 \%$ to less than $25 \%$. According to the medium variant of the Dutch forecast $20 \%$ of women of future generations will have at least three children. This implies that one third of women having a second child will also have a third child. The lower
limit of the $95 \%$ interval is assumed to equal $5 \%$. Obviously the percentage cannot be much lower. Compared with the lower limit of the interval for second births this would imply that only one out of eight women having a second child would also have a third child. This is considerably lower than the percentages observed in the past. The upper limit is assumed to be $40 \%$. In comparison with the upper limit for second births, this implies that half of women having a second child will have a third child also. A higher percentage does not seem likely. Even in Sweden, where the number of births of third children increased sharply in the early 1990s, the period fertility rate for parity three did not reach this level. Moreover, this peak was only temporary. In the mid 1990s fertility rates in Sweden dropped sharply.

## Four or more children

The fertility rate for fourth and higher order births equals 0.10 for women born after 1945. Note that since this figure refers to an addition of birth orders 4,5 , etc. this cannot be interpreted as the fraction of women having at least four children. According to the medium variant the fertility rate for birth order 4+ equals 0.10 . The lower limit of the $95 \%$ interval is assumed to equal 0.05 . Obviously the value cannot be much lower, taking into account that this is the addition of birth orders 4 and higher. The upper limit of the $95 \%$ interval is assumed to equal 0.20 . This does not seem to be very high compared with past developments, but it does not seem very likely that the high level of fertility of generations born before the Second World War will be reached again. In combination with a high level of labour force participation of women it is not very likely that a high percentage of women will have four or more children. One possible cause of a high level of fertility could be the increase in the foreign population, but that effect will probably be only small. True, the percentage of foreign women in the childbearing ages is increasing, but the level of fertility of foreign-born women has been decreasing strongly.

## Total fertility

Women born in 1945 had two children on average. According to the medium variant of the 1998 Dutch population forecasts the average number of children of future generations will equal 1.7. If the $95 \%$ forecast intervals for the separate birth orders are added up, the resulting interval for total fertility ranges from 1.1 to 2.3 (figure 1). A higher value than 2.3 would imply that the level of fertility of women born before the Second World War would be approached (for example, women born in 1935 had on average 2.5 children). Because of the changed situation of women (higher level of educational attainment, higher labour force participation, changed sex roles within the family) this seems rather unlikely. Also a lower value than 1.1 does not seem very likely. It would imply that either women will by no means have the number of children they want or that the attitudes towards having children would have to change drastically. True, developments during the last decades have shown that the level of fertility can change sharply. But the decrease concerned mainly a decrease in the number of third and following children. A comparable decrease starting from the current level would imply a much more fundamental change. It would not imply that women would have less children, but rather that considerably more women will not have children at all (even a strong increase in the percentage of women having only one child would not be sufficient). The time series model of forecast errors discussed above would result in a $95 \%$ interval for the TFR in 2050 ranging from 0.6 to 2.8 children. This seems an unlikely wide margin. The lower limit would imply that only a minority of women would have children. The upper limit would imply that a vast majority of women would have three or more children. The probability of both developments is extremely small. Even for a $95 \%$ interval this margin seems to be much too wide.
Thus on the basis of judgement it was decided to assume a smaller interval in the Dutch population forecasts. The interval 1.1-2.3 corresponds reasonably well with the $90 \%$ forecast interval for Germany, ranging from 1.0 to 1.9 in 2030, which is assumed by Lutz and Scherbov (1998a). Keilman and Hetland (1999) assume a much wider interval for Norway. On the basis of an ARIMA model they calculate a 95\% interval of the TFR in 2050 ranging from 0.5 to 6 . The authors acknowledge that this seems unlikely. Therefore they make
additional calculations assuming different maximum values for the TFR. If they assume that the TFR will not exceed 4 , the $95 \%$ interval in 2050 ranges from 0.5 to 3.5 . If they assume a maximum value of 3 , the interval ranges from 0.5 to 2.8 . Clearly the width of the interval depends heavily on the assumption about the maximum value of the TFR. Thus, even though Keilman and Hetland use a time-series model, their forecast interval is mainly determined by judgement.

## 5. Mortality

The development of life expectancy at birth for men and women in the Netherlands can be described by a random walk model with drift (Lee and Tuljapurkar, 1994, model mortality in the United States as a random walk with drift too). The $95 \%$ forecast interval produced by this model for the year 2050 equals 12 years. The time series of errors of historic forecasts of life expectancy can be modelled as a random walk model (without drift). This model projects that the $95 \%$ forecast interval for the year 2050 equals 8 years. In order to decide which interval seems more appropriate judgement has to play a role. Judgement ought to be based on an analysis of the processes underlying changes in life expectancy.

Changes in life expectancy at birth are the result of changes in mortality for different age groups. In assessing the degree of uncertainty of forecasts of life expectancy it is important to make a distinction by age as the degree of uncertainty of future changes in mortality differs between age categories. The effect of the uncertainty about the future development of mortality at young ages on life expectancy at birth is only small, because of the current, very low levels of mortality at young ages. On the basis of the current age specific mortality rates, $95 \%$ of live born men and about $96.5 \%$ of women would reach the age of 50 . Clearly the upper limits are not far away. According to the medium variant of the 1998 Dutch population forecasts the percentage of men surviving to age 50 will rise to $97.1 \%$ in 2050 and the percentage of women to $97.5 \%$. A much larger increase is not possible. A decrease does not seem very likely either. That would e.g. imply that infant mortality would increase, but there is no reason for such an assumption. The increase in the foreign population could have a negative effect, since the infant mortality rates for the foreign population are considerably higher than those for the native population. However, it seems much more likely that infant mortality rates for the foreign population will decline than that they would become increasingly negative. Furthermore the effect on total mortality is limited. Another cause of negative developments at young ages could be new, deadly diseases. The experience with AIDS, however, has shown that the probability that such developments would have a significant impact on total mortality in the Netherlands (in contrast with e.g. African countries) does not seem very large. A third possible cause of negative developments at young ages would be a strong increase in accidents or suicides. However, there are no indications of such developments. Thus it can be concluded that the effect of the uncertainty about mortality at young ages on the uncertainty about the future development of life expectancy at birth is limited.
In the medium variant of the Dutch forecasts it is assumed that the main cause of the increase in life expectancy at birth is that more people will become old rather than old people becoming still older. This implies that it is assumed that the survival curve will become more rectangular. This assumption is based on an analysis of recent changes in age-specific mortality rates. The development of mortality rates for the eldest age groups in the 1980s and 1990s has been less favourable than for the middle ages. Another reason for assuming rectangularisation of the survival curve is that expectations about a large increase in the maximum life span seem very speculative, and even if they would become true, it is questionable whether their effect would be large during the next 50 years or so. A very strong progress of life expectancy can only be reached if life styles would change drastically or if medical technology would generate fundamental improvements (and health care would be
available for everyone). Experts who think that a life expectancy at birth could reach a level of 100 years or higher usually do not indicate when such a high level could be reached. Assuming a tendency towards rectangularisation of the survival curve implies that uncertainty on the future percentage of survivors around the median age of dying is relatively high. If the percentage of survivors around that age would be higher than in the medium variant (i.e. if the median age would be higher), the decrease in the slope of the survival curve at the highest ages age will be steeper than in the medium variant. Thus the deviation from the medium variant at the highest ages will be smaller than around the median age. This implies that the degree of uncertainty of forecasts of life expectancy at birth mainly depends on changes in the median age of dying rather than on changes in the maximum life span. According to the medium variant of the 1998 population forecasts the percentage of survivors at age 85 in the year 2050 will be slightly under $50 \%$ for men and slightly over $50 \%$ for women. For that reason the degree of uncertainty of the mortality forecast is based on the assessment of a forecast interval at age 85. On the basis of that interval, the interval for the entire survival curve is assessed.
According to the medium variant assumptions on the age-specific mortality rates for the year 2050, $43 \%$ of men will survive to age 85 . According to the present mortality rates, slightly over $20 \%$ of men would reach age 85 . The lower limit of the $95 \%$ forecast interval for the year 2050 is based on the assumption that it is very unlikely that the percentage of survivors in 2050 will be lower than the current percentage. Thus for the lower limit it is assumed that one out of five men will survive to age 85 . This would imply that the median age of dying equals 78 years. Obviously, negative developments are conceivable, e.g. due to unhealthy life styles or new diseases, but it does not seem likely that these negative developments would predominate positive effects of improvement in technology and living conditions during a very long period of time. The upper limit of the forecast interval is based on the assumption that it is very unlikely that more than two thirds of men will become older than 85 years. This would imply that the percentage of survivors would increase twice as strongly as in the medium variant. The median age at dying would increase to 89 years. The current percentage of men becoming older than 89 years is only 13. A higher median age at dying than 89 seems very unlikely.
As discussed above, in the medium variant it is assumed that the gender difference will become smaller. This implies that life expectancy of women will increase less strongly than that of men. This is in line with the observed development since the early 1980s.
Consequently the probability that future life expectancy of women will be lower than the current level is higher than the corresponding probability for men. According to the medium variant $56 \%$ of women will survive to age 85 on the basis of the mortality rates for 2050. The lower limit of the $95 \%$ forecast interval is based on the assumption that it is very unlikely that less than one third of women will survive to age 85 . That would correspond with a median age at dying of 81 years. This equals the level reached in the mid-1970s. This could become true e.g. if there would be a strong increase in mortality by lung cancer and coronary heart diseases due to an increase in smoking. The upper limit of the forecast interval is based on the assumption that three quarters of women will reach age 85. This would imply that half of women would become older than 92 years. This is considerably higher than the current percentage of 15 . It does not seem very likely that the median age would become still higher On the basis of these upper and lower limits of the $95 \%$ forecast interval for percentages of survivors at age 85 , forecast intervals for percentages of survivors at the other ages are assessed, based on the assumption that for the youngest and eldest ages the intervals are smaller than around the median age (figure 2). The intervals for the survival curves are based on assumptions about the intervals for age-specific mortality rates. The latter intervals are based on the assumption that the age patterns of changes in mortality rates in the upper and lower limits correspond with the age pattern in the medium variant. These assumptions result in a $95 \%$ forecast interval for life expectancy at birth in 2050 of 12 years. For men the interval ranges from 74 to 86 years and for women from 77 to 89 years (figure 3). This interval corresponds with the interval based on the random walk with drift model of life
expectancy at birth mentioned before. The intervals for the years up to 2050 are assessed on the basis of the random walk model.
The intervals are slightly narrower than the intervals for Germany specified by Lutz and Scherbov (1998a). They assume that the width of the $90 \%$ interval equals 10 years in 2030. This is based on the assumption that the lower and upper limits of the $90 \%$ interval of the annual increase in life expectancy at birth equal 0 and 0.3 years respectively. This would imply that the width of the $90 \%$ interval in 2050 equals about 15 years.

## 6. Migration

In the Dutch population forecasts assumptions about the size of future net migration are based on separate assumptions about immigration and emigration (emigration includes the so-called net administrative corrections, which is assumed to consist mainly of undocumented emigration).
The assumptions about immigration are specified as absolute numbers. Immigrants are distinguished by country of birth. Twelve (groups of) countries of birth are distinguished: Turkey, Morocco, Suriname, Netherlands Antilles and Aruba, Indonesia and the Netherlands, Africa (excluding Morocco), Asia (excluding Indonesia and Japan), Latin America, European Economic Area, other European countries (mainly Central and East Europe) and other nonEuropean countries (North America, Oceania and Japan).
For each category assumptions are specified about the $95 \%$ interval in 2050. For some groups the intervals are asymmetric. For example, according to the medium variant the immigration of people born in Turkey will decline to 3 thousand. A symmetric forecast interval would imply a very narrow width. Therefore it is assumed that the interval between the upper limit and the medium variant exceeds that between the lower limit and the medium variant.

Because a detailed discussion of the assumptions about the forecast intervals for all categories of immigration would take too much space, we only give three examples. First, in the medium variant of the 1998 Dutch population forecasts it is assumed that the annual number of immigrants born in Morocco will decline gradually (figure 4). The reason for this assumption is that it is expected that family reunification will decline since most families of Moroccans residing in the Netherlands are already reunited. In addition, it is assumed that marriage migration will decline because increasingly young Moroccans residing in the Netherlands will choose a Moroccan partner already living in the Netherlands. The lower limit of the $95 \%$ interval is assumed to be almost zero. The upper limit is assumed to equal 11 thousand. The maximum level so far was about 10 thousand. It is assumed that it is unlikely that immigration will be higher than that level during a long period, because of the increasingly restrictive immigration policies. Each time immigration from countries like Morocco increases significantly, there is a reaction of policy.
Second, people born in the Netherlands Antilles and Aruba are Dutch citizens. They are free to migrate to the Netherlands. The development of immigration from the Antilles and Aruba shows a cyclical pattern, which is related to the economic situation on the islands. The upper limit of the $95 \%$ interval is assumed to equal 11 thousand, one and a half times as much as the maximum level observed so far. A much higher value does not seem likely, al least not in the long run, because of the limited population size of the islands. The lower limit is assumed to equal one thousand. Zero immigration seems unlikely, because Antilleans are Dutch citizens. This limits the possibilities to impose restrictive immigration rules for this group. Third, immigration from Asia has increased strongly from about 10 thousand in the early 1990s to 20 thousand in the late 1990s. One major cause was the increase in the number of asylum seekers from countries like Iraq, Iran, and Afghanistan. In the medium variant only a slight increase is assumed, because of the increasingly more restrictive policies towards asylum seekers. Because immigration of Asia has risen strongly, the upper limit of the 95\% interval is assumed to be considerably higher than the maximum value observed so far, thus allowing for a further increase. However, it is assumed that the rate of increase will level off,
because an unlimited growth would inevitably lead to political reactions. Consequently the upper limit is assumed to equal almost 50 thousand in the year 2050.

In addition to the assumptions about the forecast intervals for the separate groups of immigrants, an assumption needs to be specified about the correlation between these groups. Perfect correlation would imply a $95 \%$ interval for total immigration ranging from 32 thousand to 247 thousand in 2050 (this is simply the sum of the intervals for the separate groups of immigrants). However, an analysis of past developments shows that there is no perfect correlation. If all categories of immigration are modelled as a random walk model, it turns out that the interval of total immigration equals $60 \%$ of the addition of the intervals of the separate categories. On the basis of this result, in the 1998 Dutch population forecasts it is assumed that the $95 \%$ forecast interval of total immigration in 2050 ranges from 67 thousand to 202 thousand (figure 5). The medium variant equals 125 thousand.

Assumptions about emigration are based on emigration rates by age, sex and country of birth. The level of the emigration rates depends on the duration of residence in the Netherlands. New immigrants have a higher emigration rate than people who have been living in the Netherlands for a number of years. For that reason in the medium variant it is assumed that there is a relationship between the emigration rate and the share of new immigrants in the size of the separate groups of foreign-born persons. In the medium variant it is assumed that the emigration rates change by $50 \%$ of the change in the size of immigration as a percentage of the size of the foreign-born population for each group. This assumption is based on sensitivity analyses of a simple simulation model. Since the share of new immigrants in the size of the non-native population is expected to decline (because total immigration is assumed to be nearly constant in the forecast period), in the medium variant emigration rates are assumed to decrease. The upper limit of the $95 \%$ interval is based on the assumption that the level of the emigration rates changes by $0 \%$ of the change in the relative size of immigration. This boils down to assuming constant emigration rates. The lower limit assumes that the change in the level of the emigration rates equals $100 \%$ of the change in the percentage of immigrants. For the three groups of immigrants discussed above these assumptions imply that the $95 \%$ interval of the number of emigrants in 2050 ranges from 1.3 thousand to 4 thousand for Moroccans, from 2 thousand to 6 thousand for Antilleans, and from 4 thousand to 23 thousand for Asians.
This procedure implies that there is correlation between the size of emigration and immigration. Thus in assessing forecast intervals for net migration no additional assumptions about the correlation between immigration and emigration have to be specified. The 95\% forecast interval of net migration in 2050 ranges from -6 thousand to 60 thousand (figure 6).

## 7. Household positions

The medium variant of the Dutch household forecasts assumes that the proportion of young people living at the parental home will remain constant. The proportion of people living alone, including lone parents, is assumed to increase, especially at the middle ages. The proportion of elderly men living alone will stabilise, whereas the proportion for women will decrease. This decrease is the result of the increase in life expectancy and, in particular, the decrease in the gender gap in mortality. Consequently, couples will survive longer. Since the age at which young persons leave the parental home will not change, the age at which they start to live alone or with a partner will change neither.

## The population in institutional households

In the medium variant it is assumed that the number of persons in institutions does not change very much. Since most people living in institutions are older than 70 years and the total number of people aged 70 years or over will increase in the next decades, the proportion of elderly people living in an institution will decrease. Government policy is aimed
at stimulating people to live on their own as long as possible rather than to expand the capacity of institutions for the elderly.
The upper limit of the $95 \%$ forecast interval is based on the assumption that the percentage of elderly people living in an institution will not change, in contrast with the decreasing proportion in the medium variant. It seems unlikely that the proportion will increase because of restrictive policies and because elderly people themselves increasingly prefer to live on their own. The lower limit of the interval is based on the assumption that the role of collective institutions will be taken over by mixed types of institutions in which elderly people retain their independence, but which guarantee that sufficient care will be available when needed.

## Living at the parental home

The main assumption underlying the projection of the number of children living with their parents concerns changes in the median age at leaving home. Nowadays, at age 23 just $50 \%$ of men still live at the parental home. For women the median age is 21 years. The medium variant of the Dutch household forecasts assumes these median ages to remain constant till 2050. The lower limit of the $95 \%$ forecast interval in 2050 for the median age of leaving home of women is assumed to be 18 years (figure 7). Regarding the age limits in the Dutch educational system and the high density of educational institutions, it seems unlikely that before age 18 more than $50 \%$ of women will have left the parental home. The upper limit of the $95 \%$ forecast interval is 24 years. This may happen if for instance economical circumstances for the young are poor during a long period of time or housing for the young will be limited. For men the width of the interval equals that of women: the median age ranges between 20 and 26 years.

## Living alone

The assumptions about the percentage of people living alone (including lone parents) refer to persons who have left the parental home. The upper limit of the $95 \%$ forecast interval in 2050 is assumed to equal the percentage in the medium variant multiplied by 1.5 , the lower limit equals the percentage multiplied by 0.5 . The medium variant assumes that in 2050 almost $60 \%$ of 18 to 22 year old men who do not live at the parental home will live alone. The upper limit of the $95 \%$ interval assumes that it is very unlikely that this proportion will exceed $90 \%$. The lower limit is assumed to be $30 \%$, a percentage which is much lower than observed nowadays. For women the percentages are $75 \%$ and $25 \%$ respectively. In 2050 the medium variant assumes that 25 to $30 \%$ of the 30 to 70 year old men and women will live alone, against 15 to $20 \%$ nowadays. In view of the ongoing individualisation in the Netherlands illustrated by increasing proportions of people living alone - it seems unlikely that proportions of persons living alone will stabilise or even decrease in the next decades. It is therefore assumed that chances are very small that proportions for 30 to 70 year old men and women will be less than $15 \%$ (see figure 7). For people older than 70 years the increasing life expectancy plays an important role. Because couples survive longer the medium variant assumes that especially proportions of women living alone will decrease. It is assumed that chances are small that the proportions will exceed the current proportions. That would mean that the effect of the increasing life expectancy and the diminishing gender gap of mortality would be counteracted by changes in attitudes towards living with a partner at elderly ages or by very high divorce rates.

## Living with a partner

All those who do not live at the parental home anymore and who do not live alone (with or without children), live with a partner (or live in a multi-person household as a non-family member). Because the corresponding proportions must add up to $100 \%$ the assumptions about living at the parental home and living alone already determine the proportion of persons living with a partner. The assumptions specified for the percentage of people living alone imply that it is very unlikely that in 2050 more than $90 \%$ of people aged between 30 and 70 will live with a partner (see figure 7). The lower limit of the $95 \%$ forecast interval implies that only $60 \%$ of the people in this age group will live with a partner.

## 8. Results

According to the medium variant of the 1998 Dutch population forecasts population size of the Netherlands will continue to increase during the first three to four decades of the 21st century. Maximum population size will equal almost 17.5 million. This implies that population size will grow by more than 1.5 million. The lower limit of the $67 \%$ forecast interval will be above 16 million during most years in the first half of 21st century (figure 8). Thus if the underlying assumptions about the uncertainty of fertility, mortality, and migration are correct, it is more likely that population size in the first half of the 21st century will be larger than current population size (slightly below 16 million) than that population size will decline below the current level. Similarly it is much more likely that population size will be lower than 19 million than that it will reach a level of 20 million, the population size projected in the mid 1960s. The probability that population size will exceed 20 million is less than 5 percent. Note that the intervals refer to separate years. The probability that population size will be higher than the upper limit of the interval in any year is, of course, larger than the probability for an individual year.
Figure 9 shows that the uncertainty of future population size increases strongly with the forecast horizon. The width of the $67 \%$ interval as a percentage of the medium variant increases more strongly than linearly. In 2050 the interval between the lower and upper limit equals almost $20 \%$ of the medium variant. Figure 10 shows that the modus of the distribution in 2050 has much lower probability than that in 2020.
The $67 \%$ interval of the number of households in 2050 ranges between 7.5 million and 9 million. This implies that the width of the interval amounts to almost $20 \%$ of the medium variant. Thus the relative width of the forecast interval of the number of households equals that of total population size. The interval differs clearly from deterministic high and low variants, based on assuming small (large) average household size in the high (low) variant. That would imply that the width of the interval for households would exceed that for population size. Alternatively, assuming large (small) household size in the high (low) variant would result in a much smaller interval. The probabilistic household forecast provides a better indication of the degree of uncertainty of the forecast than deterministic variants.

The forecast intervals for the separate age groups indicate that the degree of uncertainty varies strongly between age categories. The uncertainty of the forecasts of the size of the youngest age groups (i.e. the age groups that are born in the forecast period) is much larger than that of other age groups (figure 11). The intervals for the middle age groups are very small, even in the long run.
Note that the intervals refer to separate age categories. The interval for an aggregate age category does not equal the summation of the intervals for separate age groups. Thus the interval of the old age dependency ratio does not equal the summation of the intervals of ages 65 or over divided by the summation of the intervals for the working ages. The interval for each aggregate age group and for each relative figure cannot be calculated directly on the basis of the intervals for the individual ages but has to be computed from the database containing the results of the simulations.

## 9. Concluding remarks

On the basis of assumptions about the probability distributions of future fertility, mortality, and migration, a probability distribution of the future population size and age structure can be calculated. Even though the 'true' probability is not known, as a probability distribution of the future is a forecast, a probability distribution seems to be the best way in which forecasters can communicate the uncertainty of forecasts to the users. A probability distribution allows users to make their own choices. A forecaster does not claim that certain events will or will not take place, but that some events are more likely than others. It is up to the user to decide
how much risk he is prepared to take, i.e. whether or not he will take events that are not very likely, but that nevertheless are not impossible, into account in making his decisions. This is particularly important since population forecasts are used for a variety of purposes. The group of users is heterogeneous. "All users do not want the same forecast" (Keyfitz, 1977). One user needs a conservative forecast, wanting to guard against the possibility that the realisation will be lower than the forecasts, another needs a high forecast. "Only the user can know how much he stands to lose through a projection being wrong in one direction or the other. This loss function may be strongly asymmetric." (Keyfitz, 1977). The loss function indicates how much loss a user would suffer with a particular departure of the forecast from the true population. Only if the user is given the population forecast in the form of a probability distribution, the user can decide how he can minimise the total expected loss. If the loss function is asymmetric, it may be advisable for the user not to use the most probable, medium variant, but another value (Tuljapurkar, 1996). The choice depends on the weighing of the possible losses due to forecast errors and their probability. If the probability of the medium variant is considerably higher than the probability of other variants, the user should only choose another variant if the loss function is extremely asymmetric. If, on the other hand, the probability of alternative variants differs only slightly, the user may decide to use another variant if the asymmetry is not extreme.

Since the uncertainty of forecasts increases with the length of the forecast period, one question is whether beyond a certain point in time we are too unsure to specify values of fertility, mortality, and migration. One reason is that the state of the world may change strongly in the long run, which may cause drastic changes in demographic behaviour. Another reason is that the impact of the current age structure on future population growth decreases in the long term. For example, the current age structure has an important effect on the development of the number of births in the next 20 to 30 years, because most future parents are already born. But beyond that time children will be born to parents who are not yet born at the moment the forecast is made. Consequently the uncertainty of forecasts of births is large beyond some 30 years. Forecasts of the number of deaths beyond 30 years in the future are less uncertain than forecasts of births, as most people dying then are already part of the current resident population.
When assessing deterministic forecast variants, the forecaster has to decide beyond which time horizon he thinks it is not warranted to specify assumptions about probable values of fertility, mortality, and migration. Calculations beyond that time, e.g. assuming constant rates of fertility, mortality, and migration, may be useful, as they show the consequences of the current age structure for population growth in the long run. But these calculations are illustrations rather than forecasts of the most probable future. If the values of fertility, mortality, and migration in the extended period are kept constant in all variants, the bandwidth between the variants does not indicate the degree of uncertainty. This may be misleading for the users. Another cause of confusion may be that the time beyond which forecasts are very unsure differs between fertility, mortality, and migration. Thus the period for which values are kept constant may be different for the three components. This may be confusing for the users, because it implies that for the population size and age structure there is no clear-cut divide between the period for which the results can be interpreted as forecasts of the likely future and the period for which the figures are only the result of calculations based on an arbitrary assumption, viz. constant values of fertility, mortality, and migration.
If forecast intervals are based on an assessment of the probability, the increase in the degree of uncertainty in the long term is reflected in the increase in the forecast interval. If the forecasts for the very long term are very uncertain, the interval should be very wide. Obviously in the very long run the interval may become so wide, that it is not informative for the user. However, it is up to the user to decide at which point the forecast interval is not informative for his purpose.

The assessment of the probability of a forecast is based on assumptions of which the validity is uncertain (Lutz, Sanderson and Scherbov, 1996). Judgement plays an important part.
Therefore it is important that the forecaster not only gives an indication of the probability of his forecast, but that he gives an explicit account of the assumptions underlying the assessment of the probability. The user needs this information in order to be able to assess whether the forecaster is optimistic or pessimistic. Armstrong (1985) states that experts tend to be too optimistic about the accuracy of their own forecasts. Ayton (1998) gives an overview of studies showing that people suffer from an over-confidence bias.

Assumptions about the degree of uncertainty of future fertility, mortality, and migration can be based on time series models, on an analysis of errors of historic forecasts, and on judgement. This paper describes the way the assumptions underlying the forecast intervals of the Dutch population and household forecasts are specified. Even though time series models and analyses of historic forecast errors play an important role in the specification of the assumptions, judgement based on an analysis of the underlying processes has been the determining factor.
In the 1998 Dutch population forecasts it is assumed that the width of the $95 \%$ forecast interval for fertility in 2050 equals 1.2 children per woman, the interval for life expectancy at birth equals 12 years and the interval for net migration equals 66 thousand. Furthermore it is assumed that forecast errors of fertility, mortality, and migration are serially correlated, that for each component there is perfect correlation across age and that the three components are independent. Simulations based on these assumptions lead to a $67 \%$ forecast interval of total population size of 3.2 million in 2050. This amounts to $19 \%$ of the medium variant. The odds are one out of six that population size in 2050 will be more than $10 \%$ larger than in the medium variant and also one out of six that population size will be more than $9 \%$ smaller than in the medium variant.
On the basis of assumptions about the degree of uncertainty about changes in the distribution of the population by household position, $67 \%$ intervals of the number of households can be calculated. The 67\% interval of the number of households in 2050 ranges from 7.5 million to 9 million. The interval amounts to $18 \%$ of the medium variant. Thus the relative interval for the number of households is almost equal to the interval for population size. The width of this interval is smaller than that between deterministic high and low variants, in which high (low) population growth is combined with small (large) average household size.

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## 1. Total fertility rate (TFR), medium variant and $95 \%$-forecast interval


2. Survival curve in 2050, medium variant and $95 \%$-forecast interval

3. Life expectancy at birth, medium variant and $\mathbf{9 5 \%}$-forecast interval


4. Number of immigrants by country of birth, medium variant and $\mathbf{9 5 \%}$-forecast interval



5. Total number of immigrants, medium variant and $\mathbf{9 5 \%}$-forecast interval

6. Net migration, medium variant and $\mathbf{9 5 \%}$-forecast interval

7. Population by household position, 2050, medium variant and $\mathbf{9 5 \%}$-forecast interval; as percentage of population in private households

Persons in institutional households (as percentage of total population)



Living at the parental home



Living alone



Living with a partner


8. Population size (in millions), medium variant and $67 \%$-forecast interval, 1 January

9. Width of $67 \%$-forecast interval of total population size by length of forecast period as a percentage of the medium variant

10. Probability distribution of the Dutch total population (in millions)



$11.67 \%$-forecast intervals for the age structure of the Netherlands, 2020 and 2050


