Optimal Resource Allocation in Adaptive Survey Designs

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Introduction

How did the recent global financial crisis change the world's economic landscape? What are the repercussions of current economic and political policies on the future societal development, as quantified through indicators such as the unemployment rate, average household income, consumer confidence index? To answer such questions, policy makers have to collect information from the population and summarize it in a meaningful way. This is where survey organizations and statistical bureaus play a crucial role. Collecting information from the entire population requires significant amounts of time and money. Alternatively, a sample survey may be conducted, where only a sample from the specified population is requested to provide information. Using the results from the survey sample, knowledge can be obtained about the population of interest.

1.1 Surveys in practice

Surveys are used all around the world to measure socio-economic status and well-being of people, to test theories, and make policy decisions. However, the different statistics computed from the survey data are of interest only if they accurately describe the corresponding population attribute. Multiple factors play a role throughout the course of a survey from its planning to the final systematization of the results. Some factors may disrupt the framework of statistical inference theory and sampling theory that grant methods to describe accurately (enough) population's characteristics given the survey sample results. Such factors are people's lack of understanding (or interest) as to why they have been selected to participate in a survey, their attitude towards areas such as privacy and confidentiality of personal information, and the influence exerted by the attributes of the survey design on their decision to participate in the survey. Addressing these factors and related social science questions is an integral part of survey research. Tremendous effort has been invested into understanding how human behavior and thought may impact the precision and accuracy of survey statistics and how the

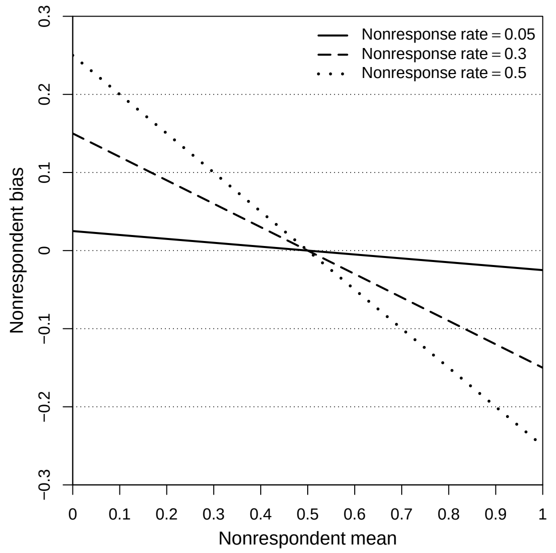


FIGURE 1.1: Level of nonresponse bias for various nonresponse rates and nonresponse means; respondent mean fixed at 0.50.

effects may be reduced or adjusted (see overviews in GROVES ET AL. 2002, LEPKOWSKI ET AL. 2007 and BETHLEHEM ET AL. 2011).

In a perfect world, all sampled population units would be willing to participate in the survey and provide all the requested data. In practical situations, however, information from some sample units is missing due to factors such as those listed above. This is called *nonresponse* and it is one of the most studied errors in the survey literature. Classic inferential properties of sample estimates require these statistics be computed from the *entire* sample. One example statistic is the *sample mean* as an estimator of the population mean. In the presence of nonresponse the sample mean is reduced to the respondent mean, i.e., the sample mean is obtained based only on information coming from the pool of respondents. The deviation of the respondent mean from the full sample mean is called *nonresponse bias* and it is a function of the *nonresponse rate* (i.e., the proportion of nonrespondents in the entire sample) and the difference between the respondent and nonrespondent means. Figure 1.1 (from GROVES AND COUPER 1998) illustrates the consequences of nonresponse rates on the precision of the survey estimates. Given the respondent mean of 0.50, a large nonresponse rate creates significant nonresponse bias, as large as 0.25 on the dotted line. This implies that the full sample mean lies between 0.25 and 0.75, a wide range that offers very little information about the statistic of interest.

As LUNDSTRÖM AND SÄRNDAL (2002) point out, the main challenge in survey statistics is to construct the best possible estimates given data collected from survey respondents and any relevant auxiliary information that one may have about the population units, be they respondents or nonrespondents. To this end, the problem of nonresponse may be viewed from two different angles, namely prevention (reduction) of nonresponse *before* it has occurred and adjustment *after* nonresponse has occurred through special estimation techniques, such as weighting adjustment techniques (see KALTON AND FLORES-CERVANTES 2003). Various options have been developed to reduce nonresponse at each of the two steps (contact and cooperation) involved in obtaining response. For example, to improve contact the survey designer may decide to increase the number of contact attempts on previously uncontacted units, choose different timing for repeated contact attempts (e.g., daytime, weekend, evening calls), or increase data collection length. To enhance cooperation, householder's attention may be boosted through advance letters and incentives, refusers may be re-approached by different interviewers to attempt conversion, the interview mode may be changed or the interview shortened. These options are generally known as *design features*.

Essentially, at any survey organization the available resources must be split between efforts to reduce nonresponse and efforts to get high quality estimates despite nonresponse. Selection of a specific set of design features generates significant changes in the incurred survey costs and the required interviewer capacity. Although limited availability of resources is an issue for all survey organizations, the allocation of resources is not sufficiently addressed in the survey literature. The interaction between costs, design and sample units is best illustrated in the literature on clinical trials. Here, due to the high costs of drug trials, patients receive various treatments within the same trial and treatments may change during the trial in order to adapt to patients' responses.

A direct analogy may not exist yet in the survey world, although practitioners have noted that different sample units (where differences are defined given a list of characteristics such as age, gender, ethnicity) respond differently to the survey design, i.e., they display different contact and cooperation patterns. Additional practical evidence reveals that experienced interviewers tailor their strategy for approaching sample units according to various observations they make regarding the immediate environment and during the first moments of interaction with the sampled person. DILLMAN (2000) also notes that a tailored survey approach may bring several benefits such as increased respondent trust and reduced cost of the response process.

These practical considerations have lead researchers to the development of innovative tools to reduce nonresponse called *tailored survey designs*. Rather similar in concept but different in implementation, two types of tailored designs are presented in the literature. HEERINGA AND GROVES (2006) first introduced *responsive survey designs* followed by *adaptive survey designs*, introduced by WAGNER (2008) and SCHOUTEN ET AL. (2013). The main idea behind a tailored design is to define a set of combinations of design features (sometimes called *survey strategies*) that potentially influence the survey costs and the quality of the estimates. Then, the survey designer has to decide what survey strategies should be assigned to sample units with certain characteristics (e.g., age, gender, ethnicity) such that a balance between costs and quality is achieved. Adaptive survey designs determine the optimal allocation of survey strategies to sample units *before* the

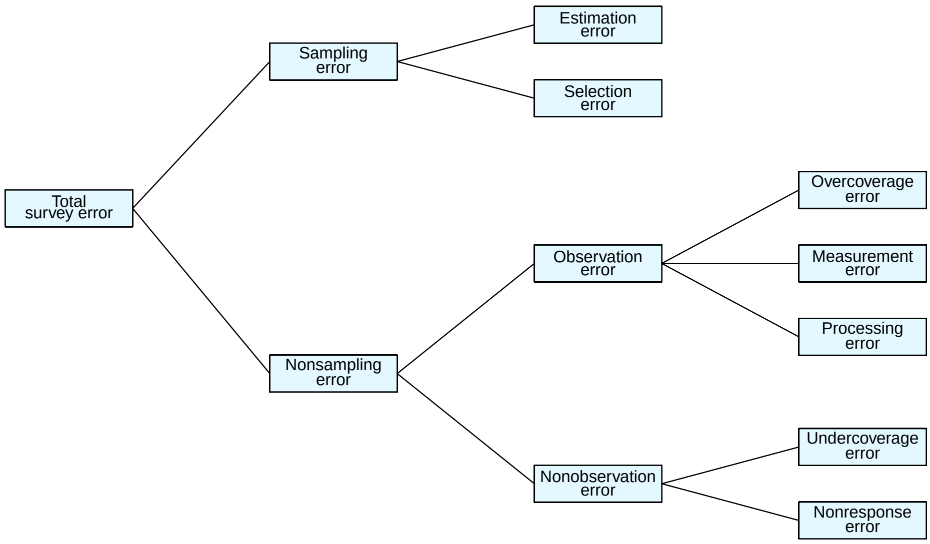


FIGURE 1.2: Types of survey errors.

start of data collection given existence of strong auxiliary information. Responsive designs on the other hand require a learning phase in order to identify the most promising strategies.

Both adaptive and responsive designs need to quantify survey quality in order to define their objective. There is, however, no general definition of survey quality mainly due to the complex interactions between the various survey errors. A full overview on sources of error in surveys in displayed in Figure 1.2 (from BETHLEHEM 1999) and Table 1.1 explains them briefly.

A great deal of existing research focuses on nonresponse while other survey errors such as measurement errors or undercoverage errors are becoming more prominent. Measurement errors occur usually when there is a discrepancy between the true value and the value processed in the survey. These differences may have various justifications such as a respondent not understanding a question or refusing to provide the true answer, a respondent unable to recall the true answer, or an interviewer interfering with the response process. Undercoverage errors occur when elements of the target population do not have a corresponding entry in the sampling frame, i.e., these units could never be contacted. This is an increasing problem in phone surveys where less and less people have publicly available phone numbers. Also online surveys are affected by undercoverage since elderly, low-educated and ethnic minorities have usually less internet access than other population groups (see BETHLEHEM ET AL. 2011). Simultaneous manifestation of such errors leaves no possibility for meaningful mathematical formulations of a composite quality indicator. Hence, addressing multiple survey errors through designs requires new approaches.

Error	Description / Cause
Sampling error	Estimates are based on a sample and not on the complete enumeration of the population.
Nonsampling error	Other phenomena than sampling. Such errors could occur even if the entire population is surveyed.
Estimation error	Every new sample produces a new value of the estimator. It can be managed through the sampling design.
Selection error	Incorrect selection probabilities are utilized in the selection method.
Observation error	Errors during the process of obtaining and recording answers.
Nonobservation error	Errors produced when the intended measurements are not obtained.
Overcoverage error	Elements that do not belong to the target population are included in the survey sample.
Measurement error	A discrepancy between the respondent's true information and the information processed in the survey.
Processing error	Errors produced during data processing.
Undercoverage error	Elements of the target population do not enter the sampling frame.
Nonresponse error	The sampled person does not provide the required information.

TABLE 1.1: Brief description of survey errors.

1.2 Contributions of the thesis

Statistics Netherlands is among the first organizations to consider redesigning their surveys such that tailored survey strategies are implemented. In this context, optimal planning of resources becomes of outmost importance. Carrying out tailored designs raises additional questions. For example, how to design a tailored survey that addresses simultaneously nonresponse and measurement errors? How to generalize a tailored design to widen its applicability? How to employ tailored designs to gain detailed knowledge on interactions between sample units, survey design features and survey resources?

This thesis presents a collection of innovative tools that survey practitioners can use to address the questions above. Its main contribution to the field of survey designs is the innovative application of operations research (OR) techniques to determine the optimal allocation of survey strategies given a quality objective and constraints on the limited resources. By bringing a new application setting into the OR sphere, this research contributes also to the OR practice by warranting the development of new algorithms to solve a class of large-scale problems with binary decision variables and complex nonconvex nonlinear objective functions and constraints. In addition, dynamic learning methods are discussed in the context of limited availability of resources.

Modeling adaptive survey designs as resource allocation problems is a novelty in the field. Survey strategies are assigned to population units such that the available resources are spent efficiently and the response rate (the most commonly used survey quality function) is maximized. The resulting problem is a nonconvex nonlinear binary problem that, when reformulated as a Markov decision problem, can be solved by dynamic programming even for large instances. This first research attempt to address survey designs from the OR perspective is described in detail in Calinescu et al. (2013a).

Modeling adaptive survey designs with multiple quality indicators answers the practical and theoretical concern that response rates may not offer a complete image of survey quality and hence other indicators, such as the response representativity indicator (that measures the resemblance between respondents and nonrespondents given a set of characteristics), should be used additionally. The complex definition of the response representativity indicator does not satisfy the prerequisites of existing algorithms. A two-step algorithm addresses the issue by suitably separating the problem into subproblems. Implementation of parallelization techniques ensures short computational times. This work is presented in Calinescu et al. (2013b).

Addressing nonresponse and measurement errors simultaneously constitutes the first attempt to extend the framework of adaptive designs to address the total survey error. We propose two methods, i.e., summarize measurement errors over the various survey target variables in one indicator and usage of item-dependent indicators. Both methods are investigated using real data from the Dutch Labor Force Survey that produces estimates on the population employment status. While the first approach fits within the framework of the aforementioned two-step algorithm, the second problem calls for a different algorithmic approach. The two methods are described in detail in Calinescu et al. (2013c) and Calinescu and Schouten (2013), respectively.

Addressing uncertainty in the adaptive design optimization parameters is an important step in creating effective survey designs. A first attempt is taken by developing a learning method that updates estimates on response probabilities when new observations become available. The multi-armed bandit framework lends itself to survey design settings where little or no historical data is available at the start of the data collection. Due to the dependence among arms created by the resource constraints, an algorithm that finds the optimal policy does not yet exist. We propose a dynamic programming approach to solve this problem and we show that various adjustments can be made to the state space such that the algorithm is tractable. Implementation of parallelization techniques improves the computational times. The mathematical details are provided in Calinescu and Bhulai (2013).

This thesis is an important stepping stone for the development of survey designs. The mathematical framework presented here constitutes one of the first structured formulations of the complex processes that occur in the survey world. A number of practical aspects are considered in the attempt to construct realistic models. However, the algorithmic tools developed in this sense do not have a limited scope. They could be extended to address similar research questions that arise in other practical settings such as online advertising and clinical trials.

1.3 Overview of the thesis

In this thesis, adaptive survey designs are analyzed from the perspective of resource allocation problems. Chapter 2 sets the theoretical background starting with a brief introduction of resource allocation problems. An overview of the fundamentals of Markov decision theory is presented as a preparation for the algorithmic tools described in the following chapters. After some preliminaries on survey sampling theory, the framework of adaptive designs is described in detail and two simulation examples illustrate the theoretical concepts and notation.

The next three chapters gradually develop a realistic resource allocation model for adaptive survey designs. In Chapter 3 a simple setting is analyzed at first, where the limited availability of resources is summarized through a threshold on the number of contact attempts. The survey strategies to be allocated to population groups consist of various interview modes (face-to-face, phone, etc.) and the corresponding thresholds on the number of contact attempts. The objective is to maximize the response rate and the decision variables denote whether a strategy is assigned or not to a population group. The resulting problem formulation is a nonconvex nonlinear binary problem of potentially large size for a realistic set of input parameters. Although nonconvex mixed-integer nonlinear problems are encountered in several practical situations (e.g., chemical engineering, design of multiproduct batch plants, network-type problems), current optimization tools cannot guarantee global optimality. We show that the resource allocation problem for adaptive designs can be cast as a Markov decision problem and consequently solved by dynamic programming. A first model extension addresses explicit formulations of budget and capacity constraints.

Chapter 4 takes a step further to include multiple quality indicators. It presents the resource allocation formulation for an adaptive design that maximizes the response rate while maintaining the response representativity indicator above a specified threshold. Such an optimal design yields smaller differences between the respondent mean and the full sample mean, thus rendering higher quality sample estimates. However, the definition of the response representativity indicator prevents the application of the resource allocation algorithm described in Chapter 3. To circumvent this issue we propose a two-step approach that suitably separates the original problem into several group subproblems that satisfy the prerequisites of the resource allocation algorithm. Each group subproblem maximizes the group response rate subject to resource constraints, where the right-hand side of these constraints is expressed as fractions of the original budget and capacity. We formulate a master problem that decides what fraction of resources should be supplied to each group such that the overall response rate is maximized, the available budget and capacity are spent efficiently and the response representativity constraint is satisfied. We subsequently discuss state space reductions obtained by elimination of resource allocations that will always be sub-optimal. Furthermore, parallelization techniques are implemented in order to improve the computational time.

In Chapter 5, we expand the resource allocation framework to address both nonresponse and measurement errors. Survey practitioners have noted that specific discrepancies between questionnaire answers and the true answers can be linked to certain answering behaviors. In order to grasp the influence of the answering behavior over all survey variables, we define a new concept called the measurement profile. Then, if the respondent's measurement profile is known to induce undesirable answering behavior, the corresponding survey answers may be excluded from the post-survey analysis. Given the probability that the measurement profile generates undesirable answering behaviors, we adopt two approaches to incorporate this quality indicator in the resource allocation model. First, we investigate maximization of the error-free response rate by discarding the error-prone responses. In the second approach, a constraint on the maximum proportion of respondents that may display the undesired answering behavior can be added to the model. Both resulting models can be addressed using the two-step algorithm with some adjustments. Extensive numerical results on real data coming from the Dutch Labor Force Survey analyze how the three quality indicators (i.e., response rate, response representativity indicator and probability for finding an undesirable answering behavior) influence the optimal design.

Chapter 6 focuses on a specific type of measurement errors, namely the mode effects, that arise in the context of mixed-mode designs. Mode effects occur when the sample unit may offer different answers to the same survey question if asked in different modes. We formulate the problem through the (nonresponse) adjusted mode effect that compares the survey estimate and a "gold standard". The optimization model minimizes the overall population mode effect subject to constraints on availability of resources and differences in mode effects between important population groups. The problem formulation leads to a nonconvex nonlinear problem for which we propose a two-step approach. In the first step we solve a linear programming problem and we submit the optimal solution as a starting point for a local search algorithm to solve the nonlinear problem in the second step. Implementation of the algorithm on real data from the Labor Force Survey reveals that the optimal solution may be influenced by the estimated adjusted mode effects and therefore sufficient historical data should be available to accurately estimate the optimization input parameters.

Strong historical support is required for satisfactory estimation of the optimization input parameters in all models presented in the preceding chapters. The question that immediately arises is what to do when historical data are not available, as for example in the case of launching a new survey. Chapter 7 addresses this issue by incorporating a Bayesian learning method that updates estimates of response probabilities when new observations are available. The problem formulation fits within the multi-armed bandit framework. However, in the presence of constraints on availability of resources, the optimal policy cannot be found easily due to the dependence created among arms in competing for resources. Also called the *budgeted multi-armed bandit*, this problem is generally intractable given the large size of the decision tree. However, under some simplification assumptions, we are able to show that the decision tree can be significantly reduced. Additionally, given the scarce resources, an upper bound on the expected reward can be easily obtained. With careful memory representation of the decision tree and implementation of parallelization techniques the problem becomes tractable and the optimal solution may be obtained via dynamic programming. We note here that this is only a first step in addressing dynamic learning for adaptive survey designs and additional research effort is required to construct a complete and yet feasible mathematical framework.

The work presented in this thesis is notably innovative for the field of survey designs, yielding at the same time complex mathematical formulations that called for the development of new algorithms. Although the empirical studies focus on Dutch surveys, we are confident that the methodologies presented here have laid the basis for new challenging research and survey practice will soon benefit from application of advanced tools in designing surveys.

2

Adaptive survey designs and Markov decision theory: an introduction

This chapter introduces the reader to the field of adaptive survey designs by discussing the ingredients of such designs and their advantages with respect to traditional survey designs. Some preliminaries on survey sampling theory are also provided. We briefly review the framework of resource allocation problems that lends itself to the optimization formulations behind adaptive survey designs. Additionally, we present some elements of Markov decision theory that serve as the theoretical basis for a series of algorithms used in the survey resource allocation optimization. The material discussed in this chapter is fundamental to the remainder of the thesis.

2.1 Resource allocation problems

Resource allocation problems (RAPs) deal with assigning available resources to various activities to meet a specified objective. Such problems translate to seeking an optimal allocation of scarce resources to a number of tasks such that their objective is optimized subject to the given resource constraint. A variety of applications can be modeled as RAPs, e.g., job shop scheduling (allocating time and equipment to work orders such that delivery time is minimized), portfolio optimization (allocating funds to a set of financial instruments to maximize return for a given level of risk), project funding (allocating funds among various projects such that return on investment is maximized).

We start with few definitions before we introduce the standard RAP formulation.

Definition 2.1. *A function $f : \mathcal{X} \to \mathbb{R}$ defined on a convex set $\mathcal{X}$ is called convex if, for any two points $x, y \in \mathcal{X}$ and any $\lambda \in [0, 1]$, the following holds*

$$f\Big(\lambda x + (1-\lambda)\, y\Big) \leq \lambda f(x) + (1-\lambda) f(y).$$

The function is called strictly convex if

$$f\Big(\lambda x + (1-\lambda)\, y\Big) < \lambda f(x) + (1-\lambda) f(y)$$

holds for any $0 < \lambda < 1$ *and* $x \neq y$*. A function* f *is said to be (strictly) concave if* $-f$ *is (strictly) convex.*

Definition 2.2. *A function* $\Phi : \mathbb{R}^n \to \mathbb{R}$ *is called completely additively separable if there exist functions* $\phi_1, \phi_2, \ldots, \phi_n$*, each function of one variable such that*

$$\Phi(\mathbf{x}) = \sum_{i=1}^{n} \phi_i(x_i).$$

Definition 2.3. *In a mathematical program (e.g., (2.1)), a point is feasible if it satisfies all constraints. The feasible region (or feasibility region) is the set of all feasible points. A mathematical program is feasible if its feasible region is not empty.*

The standard RAP for n activities can be stated as follows. Let $\phi_i : \mathbb{R} \to \mathbb{R}$ and $g_i : \mathbb{R} \to \mathbb{R}$ be convex and differentiable functions for $i = 1, \ldots, n$. Take $b \in \mathbb{R}$ be the available resource quantity and $-\infty \leq l_i < u_i \leq +\infty$. Then the optimization problem is given by

$$\begin{aligned} \underset{x}{\text{minimize}}\ \Phi(\mathbf{x}) &:= \sum_{i=1}^{n} \phi_i(x_i) \\ \text{subject to}\ g(\mathbf{x}) &:= \sum_{i=1}^{n} g_i(x_i) \leq b \\ x_i \in X_i &:= [l_i, u_i],\ i = 1, \ldots, n. \end{aligned} \tag{2.1}$$

If the feasible region is bounded then the problem has a finite optimal solution. If all ϕ_i are strictly convex, then the optimal solution is unique.

First formulations of resource allocation problems date back to Gossen (1854) (an English translation in Gossen 1983) that first discussed the optimal allocation of money and time to maximize one's utility. An independent result that leads to the same conclusion is Gibbs' Lemma, given in Gibbs (1876), that formulates the principle for the equilibrium state of chemical substances in actual physical contact with each other. To achieve this equilibrium, the system simultaneously strives for a maximum entropy and a minimum energy. Additional resource allocation formulations arose in traffic models (seeking the best route from origin to destination, see traffic equilibrium principle given in Wardrop 1952), theory of search (e.g., search for a lost object, distribution of destructive effort, response to a sales campaign, drilling for oil, see for example Koopman 1952 and Koopman 1999), weapons allocation game (studied by Danskin (1967) utilizing Gibbs' Lemma).

A great body of literature is devoted to developing fast algorithms for continuous convex nonlinear RAPs such as (2.1) (see extensive literature surveys in Ibaraki and Katoh 1988 and Patriksson 2008). Quite often, however, practical settings are not as simple, i.e., they cannot be fully described by separable convex and differentiable functions, and the RAP formulations lead to integer or mixed-integer problems which have been proven to have an NP-complete worst case complexity (see Ibaraki and Katoh 1988). Algorithms still exist for those applications where convexity in the objective function and/or constraints is satisfied, see, e.g., Katoh et al. (1979), Bretthauer et al. (1999), and Bretthauer and Shetty (1995).

However, the complex operations observed in practice do not display convexity, e.g., due to economies of scale. In this case, additional difficulties are posed by the presence of several local optima and approximation methods are suggested as means to tackle such problems (see Bretthauer et al. 2003, Benson et al. 1990). Calinescu et al. (2013a) offer an exact solution method for a special class of nonconvex resource allocation problems where the decision variables are binary and the objective function is additive. For this class of problems the global optimum can be found in a finite number of iterations by reformulating the problem as a Markov decision problem and applying dynamic programming.

2.2 Markov decision theory

Sequential decision problems involve making decisions at different points in time with the property that future decisions may be influenced by both previous decisions and some stochastic parameters whose values will have been observed at the time future decisions are made (Hadley 1964). Markov decision problems (MDPs) are a subclass of sequential decision problems where any decision rule has the property that it depends on the past history of the process only via the current process state. MDPs are a widely used tool to model decision making under uncertainty. Bellman (1957) formalized the concepts of Markov decision theory and introduced the concept of dynamic programming where the decision making was not only based on immediate decision outcomes but also on the future dynamics of the system. He is also credited for showing that the optimal decision rule to an MDP satisfies the principle of optimality given in Theorem 2.4.

Theorem 2.4 (Ch. 3 in Bellman 1957). *An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.*

This principle allows one to compute the optimal decision rule (or policy) by backward induction starting at the terminal period N (end of decision-making horizon). Of the utmost importance in this process is the so-called *value function*, which summarizes the expected future decision outcomes at any node in the decision tree, assuming that an optimal policy will be followed in the future. Subsequent work by BLACKWELL (1962) and DENARDO (1967) showed that the backward induction method of computing the optimal decision rule can be extended to infinite-horizon MDPs. They showed that the value function in stationary, infinite-horizon, discounted MDPs is the unique solution to a recursive functional equation (the *Bellman's equation*), which is mathematically equivalent to a fixed point of a contraction mapping. Additional important results were obtained by HOWARD (1960) who studied the average cost MDPs and introduced the policy iteration algorithm, one of the first major computational methods in Markov decision theory.

In a Markov decision problem the outcome of each decision rule may be measured in terms of costs, thus the associated decision-making criterion involves minimization of the incurred costs in the system. Two criterion functions are commonly used, namely the *total discounted cost criterion*, that indicates the importance of present costs, and the *long-run average cost criterion*, that reflects the importance of future costs. The outcome of a decision rule may also be measured in terms of rewards received in the system, which leads to a maximization problem. The criterion functions in this case are the *total expected discounted reward* and the *long-run average reward*. The models presented in this thesis describe MDPs in terms of rewards. The remainder of this section gives a formal introduction to MDPs where the criterion function is given by the total expected discounted reward.

Definition 2.5. *A Markov decision process is the tuple* $(\mathcal{X}, \{\mathcal{A}_x | \, x \in \mathcal{X}\}, p, r)$, *where*

- *the set* $\mathcal{X}$ *denotes the discrete state space,*
- *the finite set* $\mathcal{A}_x$ *denotes the set of actions that can be chosen when the process is in state* $x \in \mathcal{X}$,
- $p\,(x, a, y)$ *represents the transition probability function from state* x *to state* y *when action* $a \in \mathcal{A}_x$ *has been chosen,*
- $r(x, a)$ *describes the real-valued reward received in state* x *when action* a *has been chosen.*

The set Π *of policies is defined by all deterministic functions* $\pi : \mathcal{X} \to \mathcal{A}_x$ *that map state* $x \in \mathcal{X}$ *onto an action* $a \in \mathcal{A}_x$. *An illustration is offered in Figure 2.1.*

If the time spent in a given state follows an arbitrary probability distribution, then actions could be taken at any point in time. This setting is described by decision models where the system evolves continuously in time and rewards accumulate continuously in time. If one assumes that the reward is independent of the time spent in a state and it only depends on the state and action chosen at the last decision epoch, then the decision epoch coincides with the transition times, leading to the so-called *semi-Markov decision models* described in PUTERMAN (1994). The models presented in this thesis use only MDPs in discrete time. An extensive presentation of continuous-time Markov decision

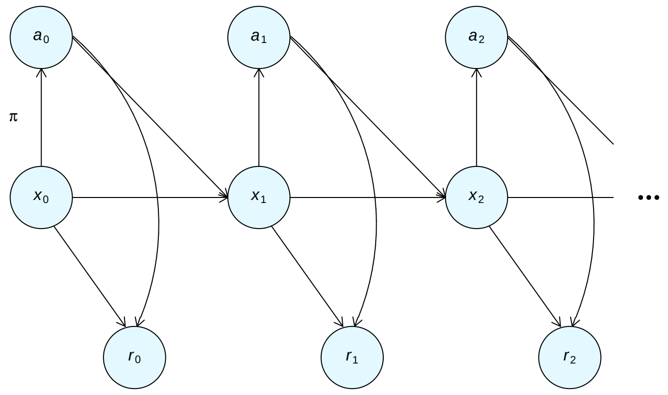


FIGURE 2.1: Markov decision process.

problems is thus beyond the scope of this thesis.

Let X_t denote the random variable for the state that the system is in at time epoch $t \in \mathbb{N}$ and A_t the corresponding action. Under a policy $\pi \in \Pi$, the expected discounted reward when starting in state x is computed as

$$V^{\pi}(x) = \mathsf{E}_x^{\pi} \sum_{t=0}^{\infty} \beta^t r(X_t, A_t), \; x \in \mathcal{X}, \tag{2.2}$$

for a discounting factor $\beta \in (0, 1)$. In other words, $V^{\pi}(x)$ denotes the expected discounted reward when starting in x and following π thereafter. The mapping $V^{\pi} : \mathcal{X} \to \mathbb{R}$ is called the *value function* under policy π. Note that choosing $\beta < 1$ ensures convergence of the infinite sum. The MDP objective is to find a policy $\pi^* \in \Pi$ that maximizes $V^{\pi}(x)$ for all $x \in \mathcal{X}$. Thus, the optimal value function is given by

$$V^*(x) = \sup_{\pi \in \Pi} V^{\pi}(x), \; x \in \mathcal{X}. \tag{2.3}$$

A policy π^* is optimal if it maximizes the value function for every start state, i.e.,

$$\pi^* \text{ optimal} \;\Leftrightarrow\; V^{\pi^*}(x) = V^*(x), \; \forall\, x \in \mathcal{X}.$$

Note that there may exist multiple optimal policies that achieve the optimal value for every start state. To obtain V^*, BELLMAN (1957) formulated the optimality equations given in (2.4) which, if solved iteratively, lead to the optimal value V^*.

$$V^*(x) = \max_{a \in \mathcal{A}_x} \left[r(x, a) + \beta \sum_{y \in \mathcal{X}} p(x, a, y) V^*(y) \right], \; \forall\, x \in \mathcal{X}. \tag{2.4}$$

Bellman's optimality equations state that the value of a state under an optimal policy (i.e., $V^*(x)$) must equal the expected return obtained by taking the best action in the current state. The expected return is computed as the discounted value of the expected next state (i.e., $\beta V^*(y)$), plus the reward obtained by taking action a in the current state (i.e., $r(x, a)$). The value of the next state is averaged over all possibilities, each of which is weighted by the probability of occurring (i.e., $p(x, a, y)$). The optimal policy π^* is given by

$$\pi^*(x) = \arg\max_{a \in \mathcal{A}_x} \left[r(x, a) + \beta \sum_{y \in \mathcal{X}} p(x, a, y) V^*(y) \right].$$

Bellman's equations satisfy the principle of optimality in the sense that whenever the process reaches a state x, the optimal policy selects the same action as an optimal policy in the case that the process started in x.

While there may be many optimal policies, the optimal value function V^* is unique for finite MDPs (i.e., the state space is finite), as proved in PUTERMAN (1994). The main idea is to show that the value function $V(x)$ is equivalent to a contraction mapping with contraction factor β on the Banach space $\mathbb{B}(\mathcal{X})$ of real-valued functions v on $\mathcal{X}$ with max-norms. Then, by Banach's Fixed Point Theorem (see BANACH 1922), there exists a unique function V^* that satisfies the optimality equation and is the limit of recursive application of the contraction mapping. This is formalized in the following.

Definition 2.6. *Maximum norm of a function* $f : \mathcal{X} \to \mathbb{R}$ *is* $||f||_\infty := \sup_{x \in \mathcal{X}} |f(x)|$.

Theorem 2.7 (Ch. 6 in PUTERMAN 1994)**.** *Let U be a Banach space. An operator T is a contraction mapping if there exists $0 \leq \beta < 1$ such that*

$$||Tv - Tu|| \leq \beta ||v - u||, \; \forall \; v, u \in U.$$

Then, by Banach's Fixed Point Theorem, there exists a unique $v^* \in U$ such that $Tv^* = v^*$ and for arbitrary v^0 in U, the sequence $\{v^n\}$ defined by $v^{n+1} = Tv^n = T^{n+1}v^0$ converges to v^*.

Theorem 2.8 (Ch. 6 in PUTERMAN 1994)**.** *Let $V : \mathcal{X} \to \mathbb{R}$ be the MDP value function given by (2.2) and define the operator $T_\beta : V \to V$ as*

$$(T_\beta V)(x) = \max_{a \in \mathcal{A}_x} \left[r(x, a) + \beta \sum_{y \in \mathcal{X}} p(x, a, y) V(y) \right].$$

Then T is a max-norm β-contraction mapping on $\mathbb{B}(\mathcal{X})$.

Theorem 2.9 (Ch. 6 in PUTERMAN 1994)**.** *There exists a unique $v^* \in \mathbb{B}(\mathcal{X})$ that satisfies $Tv^* = v^*$ with T defined in Theorem 2.8 and $v^* = V^*$.*

Iterative application of operator T to obtain V^* results in solving the system of equations

$$V_{t+1}(x) = \max_{a \in \mathcal{A}_x} \left[r(x, a) + \beta \sum_{y \in \mathcal{X}} p\,(x, a, y) V_t(y) \right], \; t \in \mathbb{N}. \tag{2.5}$$

Such iterative computation of the optimal value function is called *value iteration*. Al-

Algorithm 1 Value iteration algorithm

Let $t = 0$ and $V_t(x) = 0$ for all $x \in \mathcal{X}$ and let $\epsilon > 0$ be a small number.
for all $t \in \mathbb{N}$ **do**
  for all $x \in \mathcal{X}$ **do**
    $V_t(x) := \max_{a \in \mathcal{A}_x} \left[r(x,a) + \beta \sum_{y \in \mathcal{X}} p(x,a,y) V_{t-1}(y) \right]$
  end for
  if $\forall x \; |V_t(x) - V_{t-1}(x)| < \epsilon$ **then**
    for all $x \in \mathcal{X}$ **do**
      $\pi(x) = \underset{a \in \mathcal{A}_x}{\arg\max} \left[r(x,a) + \beta \sum_{y \in \mathcal{X}} p(x,a,y) V_{t-1}(y) \right]$
    end for
    Stop
    Return π and V_t.
  end if
end for

gorithm 1 sketches in pseudocode the value iteration algorithm to obtain an ϵ-optimal policy. By choosing ϵ small enough the algorithm stops with a policy that is very close to optimal.

We now discuss briefly finite-horizon MDPs. Assume $N < \infty$ is the decision-making horizon. For finite-horizon MDPs the discount factor does not play the same key role as in the infinite-horizon case. In fact, it does not affect any theoretical results or algorithms, it merely influences the decision maker's preference for policies. Therefore, we consider in the following that $\beta = 1$ and thus we are interested in determining a policy π^* with the largest expected total reward. Let $V_N^\pi(x)$ be the expected total reward over the decision-making horizon if policy π is used and the system starts in state x. Then,

$$V_N^\pi(x) = \mathsf{E}_x^\pi \sum_{t=0}^{N-1} r(X_t, A_t) + \beta^N r_N(X_N).$$

Note that in finite-horizon MDPs, no decision is made at time point N, therefore the reward at this point is only a function of the state. We seek a policy π^* such that

$$V_N^{\pi^*}(x) = \sup_{\pi \in \Pi} V_N^\pi(x), \; x \in \mathcal{X}.$$

The expected total reward of an optimal policy π^* satisfies

$$V_N^{\pi^*}(x) = V^*(x).$$

The optimal policy can be obtained by dynamic programming, i.e., iterating backwards from N over the optimality equations (2.6) for $t \in \{N, \ldots, 0\}$. The algorithm is sketched in pseudocode in Algorithm 2. The following theorem formalizes the main steps of the algorithm through which any optimal policy is evaluated for the entire decision-making

Algorithm 2 Dynamic programming (backward induction algorithm)

Let $t = N$ and $V_t(x) = r_t(x)$ for all terminal states $x \in \mathcal{X}$.
Let $\pi = \{\mu_0, \mu_1, \dots, \mu_{N-1}\}$.
for $t = N - 1$ **to** 0 **do**
  for all $x \in \mathcal{X}$ **do**
    $V_t(x) := \max_{a \in \mathcal{A}_x} \left[r(x,a) + \sum_{y \in \mathcal{X}} p\,(x,a,y) V_{t+1}(y) \right]$
    $\mu_t(x) := \underset{a \in \mathcal{A}_x}{\arg\max} \left[r(x,a) + \sum_{y \in \mathcal{X}} p\,(x,a,y) V_{t+1}(y) \right]$
  end for
end for
Return π and V_0.

horizon and the expected total reward is computed.

$$V_t(x) = \max_{a \in \mathcal{A}_x} \left[r(x,a) + \sum_{y \in \mathcal{X}} p\,(x,a,y) V_{t+1}(y) \right], \; t \in \{0, \dots, N-1\}. \tag{2.6}$$

Theorem 2.10 (Ch. 4 in PUTERMAN 1994). *Let $\pi^* = \{\mu_0^*, \dots, \mu_{N-1}^*\}$ such that each $\mu_t^* : \mathcal{X} \to \mathcal{A}$ satisfies*

$$\mu_t^*(x) \in \underset{a \in \mathcal{A}_x}{\arg\max} \left[r(x,a) + \sum_{y \in \mathcal{X}} p\,(x,a,y) V_{t+1}(y) \right] \; \forall\, x \in \mathcal{X},$$

where

$$V_t(x) = \max_{a \in \mathcal{A}_x} \left[r(x,a) + \sum_{y \in \mathcal{X}} p\,(x,a,y) V_{t+1}(y) \right],$$

with $V_N(x) = r_N(x)$ for the terminal state x. Then π^* is optimal and satisfies

$$V_N^{\pi^*}(x) = \sup_{\pi \in \Pi} V_N^{\pi}(x), \; x \in \mathcal{X}.$$

Multiple optimal policies may exist, each yielding the same expected total reward, if

$$\underset{a \in \mathcal{A}_x}{\arg\max} \left[r(x,a) + \sum_{y \in \mathcal{X}} p\,(x,a,y) V_{t+1}(y) \right], \; \forall\, t \le N, x \in \mathcal{X}$$

contains more than one action. In Algorithm 2 it is assumed that only one action attains the maximum value when the process is in state x at a given time point t.

Note that both value iteration algorithm and dynamic programming require the value function for every state to be stored in the memory for usage at the following step. Such memory requirements are of the order of the state space size $|\mathcal{X}|$. For many practical problems, however, the memory requirements quickly exceed the available capacity and thus the *curse of dimensionality* occurs. Updating the value of every state, one state per iteration, in a fixed order, can be very inefficient since only some updates are necessary. This observation gave rise to various heuristic search algorithms that consider an update only when necessary. Thus, storing unnecessary steps in the memory is avoided, see for example, Hansen and Zilberstein (2001), Bonnet and Geffner (2003) and Sanner et al. (2009). The main idea behind these algorithms is to generate a greedy policy and perform a series of heuristic searches until all the states the policy can reach have converged. Updating the value of the visited states is performed during the search. However, these algorithms must be employed with care in order to guarantee optimality. By studying the graphical structure of the problem, i.e., the intrinsic order in which states are reached, Dai et al. (2011) develop two promising algorithms that break the structure of the MDP into strongly connected components. The optimal policy is found given the topological order of the components and by removing state transitions that are irrelevant for reaching the objective.

2.3 A brief introduction to sample surveys

Traditionally, surveys collect information about persons, households and business by complete enumeration or census of the entire population of interest. However, this approach displays numerous disadvantages such as large expenditures (large number of employed interviewers, large computer facilities, personnel to process the collected information) and low data processing speed (with a negative impact on the timeliness of the results). A simple solution to address these issues is by conducting a *sample survey* where only a sample of the specified population is surveyed. Thus, the information collection costs are reduced, by surveying only a small part of the entire population, and the processing speed is increased. Additionally, more complex surveys may be conducted, where highly trained personnel or specialized equipment is required that is generally limited in availability.

However, survey samples were received with reluctance since it was not clear how researchers may be able to state results about the whole population given conclusions from a survey sample. Statistical inference theory and sampling theory describe methods of sample selection and estimation such that precise enough estimates are produced at lowest cost (see extensive information on sampling techniques in Cochran 1977). To judge the precision of a sampling procedure, one must examine the frequency distribution generated for the estimate if the procedure is applied again and again to the same population. In practice, the sample sizes are chosen such that confidence intervals are of specified size (usually assuming normality). Hence, if one knows the mean and variance of the distribution, then the entire frequency distribution becomes known.

In order to use the survey sample to make inferences about the population, the sample must satisfy the *probability sampling* property, in other words, the selection method that samples from the population must use a random process. By probability sampling, all individual units receive their appropriate probability π of being selected in the sample. Then, given a population characteristic μ, an estimator $\hat{\mu}$ of μ obtained through a sampling method is called unbiased if

$$\mathsf{E}(\hat{\mu}) = \sum_{i=1}^{n} \pi_i \hat{\mu}_i = \mu,$$

where $\hat{\mu}_i$ is the estimate of μ obtained from sample unit i. In other words, if the mean value of $\hat{\mu}$ taken over all possible samples provided by the given sampling method is equal to the population value μ then $\hat{\mu}$ is an unbiased estimator of μ. The exact error $(\hat{\mu} - \mu)$ cannot be known since μ is not known. However, one can compute its mean and standard deviation (also called standard error) and provide a confidence interval for the value of μ.

Various errors prevent sample surveys from benefiting from unbiased estimators. Nonresponse is one of the most studied causes of error in surveys and it occurs when a sample unit does not provide the requested information. As a consequence, the probability distribution of the values of the estimate is distorted and the confidence interval around μ may be erroneous. If m is the mean of the biased estimate $\hat{\mu}$ and $\sigma_{\hat{\mu}}$ its standard deviation, then for biases caused by nonresponse it is not possible to guarantee an upper limit on $\frac{m-\mu}{\sigma_{\hat{\mu}}}$ that is small (Cochran 1977).

These aspects have motivated survey researchers to develop methods to adjust for possible nonresponse bias, such as weighting adjustment techniques (see Kalton and Flores-Cervantes 2003 for an overview), or to prevent nonresponse from occuring by re-approaching nonrespondents. For this case, Hansen and Hurwitz (1946) proposed the callback approach, where a sample of nonrespondents is selected and re-approached by specially trained interviewers. Also, Kersten and Bethlehem (1984) proposed the basic-question approach, that tries to obtain nonrespondent's cooperation by offering a shorter questionnaire containing only a few basic questions.

Despite the extensive research effort, nonresponse and other survey errors create bias in survey estimates. More recently, survey researchers have turned towards investigating the relationship between survey design features (the survey interview mode, the number of visits until established contact) and presence of nonresponse, in the context of a list of characteristics of the population units. The main reason for this stream of research is that different population groups behave differently with respect to the characteristics to be surveyed, i.e., nonresponse is selective. This gave rise to adaptive and responsive survey designs where attempts to reduce nonresponse are taken by learning respondent and nonrespondent behavior patterns and tailor the survey design accordingly. The next section discusses these new designs in more detail.

2.4 Adaptive survey designs

In most surveys all sample units receive the same treatment and the same design features apply to all selected persons and households. However, different people respond differently to a given treatment. Therefore, tailoring the survey design could help improve response rates and reduce nonresponse selectivity. Adaptive survey designs (ASDs) allow different persons or households to receive different treatments. These treatments may be defined before the survey starts, but may also depend on data that is observed during data collection, i.e., paradata (information about the survey data collection process, e.g., details of the answering process, interviewer observations about the neighborhood and the respondents, the performance of interviewers themselves). A general introduction to ASDs is given by Wagner (2008) and Schouten et al. (2013).

ASDs find their origin in the literature on medical statistics where treatments are varied beforehand over patient groups but also depend on the responses of patients, i.e., on measurements during data collection, see for example Heyd and Carlin (1999) and Murphy (2003). Generally, a number of stages is identified at which the patient status is evaluated and treatments may change. In medical statistics, such treatment designs are called *adaptive clinical trials* (see Bretz et al. 2009). In survey statistics, however, a generally accepted terminology may still be lacking. Schouten et al. (2013) introduce *static* and *dynamic* ASDs, where both design types tailor treatments by taking into account prior information on interactions between populations groups and the available treatments. Dynamic designs consider paradata (information that becomes available during data collection) as additional source of information for treatment tailoring. The allocation of tailored treatments balances optimally survey quality and costs and for static designs it is set before the data collection starts. For dynamic designs, only the set of tailored treatments is known before the data collection starts while the allocation itself is determined during the data collection when paradata become available. Heeringa and Groves (2006) discuss *responsive* designs where the data collection is split in phases such that promising and effective treatments can be identified during data collection given a tradeoff between costs and quality for a specified list of quality indicators. The subsequent phases employ the selected treatments, see for example Mohl and Laflamme (2007), Laflamme and Karaganis (2010) and Tabuchi et al. (2009). Responsive designs are motivated by survey settings where little information is available beforehand about the sample and/or about the effectiveness of treatments. From the second phase on such information becomes available and responsive designs become similar to ASDs, with the distinction that in previous design phases part of the sample has already responded.

ASDs, in their most general framework, have the following ingredients. A vector X of *covariates* (age, gender, ethnicity) is available for each population unit from external sources of data. A set of *design features* such as the interview mode (face-to-face, phone, web), number of mode attempts, interviewer skills are available. Combinations of design features form *survey strategies*. A survey strategy can, for example, be expressed as

$$s = (\text{web, phone, 6 calls}), \tag{2.7}$$

where two design features are applied, namely the interview mode and the number of mode attempts. The treatment prescribed by strategy (2.7) indicates that the sample unit should first be sent a web questionnaire and, in case of no response, the unit should be approached by phone with a maximum of 6 call attempts to establish contact. Each population unit is assigned a strategy in the ASD from the set S of survey strategies. The *empty strategy* $\emptyset$, that implies that the population unit is not sampled, is explicitly included in the set S. However, an ASD may also be separated from the sampling design which signifies that the allocation of strategies applies to a given sample and $\emptyset \notin S$.

For dynamic designs, a second vector of covariates $\tilde{X}$ may exist for a sampled unit that reflects characteristics observed during data collection, i.e., paradata. Examples of paradata include information about the response process (the number of contact attempts, the sequence of outcomes of contact attempts), the interviewer assessment of the probability to respond or be contacted, the state of the dwelling or the neighborhood, and the presence of an intercom. The important distinction between X and $\tilde{X}$ is the level of availability, with $\tilde{X}$ available only for the sampled units.

Let $\rho(s, x)$ be the response probability of a unit carrying characteristics $X = x$ and that is assigned strategy $s \in S$. It is assumed that $\rho(s, x)$ is available from historic data, i.e., previous versions of the same survey and/or surveys with similar topics and designs. Similarly, $c(s, x)$ denotes the expected costs to approach a unit with characteristics $X = x$ via strategy s. Let $p(s, x)$ be the allocation probability of strategy s to a population unit with characteristics $X = x$. It should hold that

$$\sum_{s \in S} p(s, x) = 1,$$

i.e., all units are assigned a strategy. In general, allocation probabilities take values in $[0, 1]$, in other words, units with the same characteristic values x may be (randomly) assigned to different strategies. For instance, for strategy s in (2.7), only 20% of the web nonrespondents may be re-approached by phone. Thus, flexibility in meeting quality levels or cost constraints is increased. For dynamic designs, the corresponding quantities, i.e., response probabilities, expected costs and allocation probabilities, would be formulated at the $(x, \tilde{x})$ level for each sample unit with $X = x$ and $\tilde{X} = \tilde{x}$.

The survey strategy set S together with covariates X (and $\tilde{X}$ for dynamic designs, respectively), estimates on response probabilities $\rho(s, x)$ and costs $c(s, x)$ and allocation probabilities $p(s, x)$ (respectively, $p(s, x, \tilde{x})$ for dynamic designs) define an ASD. Given $\rho(s, x)$ and $c(s, x)$ for all $s \in S,\ X = x$ and a survey quality definition Q, then the ASD optimization problem can be formulated as

$$\begin{aligned} \max_{\mathbf{p}} \;& Q(\mathbf{p}) \\ \text{s.t.} \;& C(\mathbf{p}) \leq C_{max}, \end{aligned}$$

where $\mathbf{p} = \big(p(s,x)\big)_{s\in S,\, X=x}$ represents the vector of allocation probabilities, $Q(\mathbf{p})$ the quality function, $C(\mathbf{p})$ the cost function and C_{max} a threshold on maximum survey costs. Hence, the optimization decision variables are in fact the allocation probabilities $\mathbf{p}$ and their values in the optimal solution achieve an optimal balance between survey costs and quality given the selected strategy set S and definitions of $Q(\mathbf{p})$ and $C(\mathbf{p})$.

It is important to stress that also for dynamic designs the optimization is carried out only once, before survey data collection starts, and it is not repeated during data collection. The optimization parameters are estimated from historic survey data that include paradata. Thus, for a new data collection, the candidate strategies for units in stratum x are the same up to the moment paradata $\tilde{x}$ is observed. For clarification, take the following example.

Example 2.1. (Example 13.1 from Bethlehem et al. 2011). For a given survey the data collection is scheduled for four weeks. The starting interview mode is phone and X = (age, gender) is known for all sample units. After two weeks, the response status of each sample unit is assessed, i.e., respondent, refusal, noncontact, which forms the paradata vector $\tilde{X}$. For the remaining two weeks, the interview mode can be one of the following, paper, web, phone. Hence, the ASD has three strategies, namely s_1 = {phone, paper}, s_2 = {phone, web} and s_3 = {phone, phone}. If allocation of strategies depends solely on X, for example old persons receive s_1, young males s_2 and young females s_3, then the design is static. On the other hand, if allocation of strategies depends also on the response status, for example s_1 is allocated to sample units who refused, s_2 to young persons who have not been contacted and s_3 to old persons listed as noncontact, then the design is dynamic.

The topic of survey quality functions holds a key point for both adaptive and responsive designs. A quality function should map the survey sample characteristics X, paradata $\tilde{X}$ if available, and answers to survey items to a scalar value that can be interpreted and optimized. The most commonly used quality function is the response rate, i.e., mean response probability, given by

$$Q(\mathbf{p}) = \sum_{x,s} q(x)\, p(s,x)\, \rho(s,x),$$

where $q(x)$ represents the distribution of X in the population. However, literature argues, see, e.g., Schouten et al. (2009) and Groves (2006), that using only the response rate as a quality function is not sufficient since it is a poor predictor of nonresponse bias. There is an ample debate among survey researchers regarding a general definition of survey quality. A great body of literature focuses on nonresponse bias, see, e.g., Bethlehem et al. (2011) for an overview, while other researchers find measurement errors more concerning since they increase response bias, see Lyberg et al. (1997) and Dillman (2007). Nevertheless, any tailored survey design should take into account both measurement and nonresponse error, especially, when the survey mode is one of the candidate design features. This may prove more difficult to investigate and implement since it is not clear how to measure the combined effect of nonresponse and measurement errors. This becomes particularly daunting for a survey with multiple key variables for which measurement errors could differ in magnitude and direction, i.e., increase on some variables and decrease on others. Nonetheless, literature has recently shown interest in this area, see Olson (2007), Olson (2012) and Fricker and Tourangeau (2010). Moreover, there is some experience in practice with tailored treatments, see e.g., Laflamme and Karaganis (2010), Peytchev et al. (2010) and Luiten and Schouten (2013) and few attempts to extend ASDs to address measurement error, see Calinescu et al. (2013b), and variance, see Beaumont and Haziza (2011).

Cost functions, although not as challenging as the survey quality definition, require further consideration. Derivation of cost components is complicated when a survey organization runs many surveys in parallel. The interaction between surveys makes it hard to separate costs per survey, especially when strategies are tailored. Moreover, when only a relatively small number of population units are assigned to a face-to-face interview, then traveling costs may be assumed to remain unchanged as the addresses are clustered with addresses from other surveys. However, the validity of the cost estimates influences the outcome of the optimization. Therefore, it is important to monitor data collection closely and build suitable indicators for strategies.

Next to cost parameters and quality functions, the other important ingredient of ASDs is the set of response probabilities for the various strategies. Such quantities need to be known from past surveys, preferably the same survey or otherwise a similar survey. Literature on household surveys gives an extensive list of models for response that include design features. The common denominator in all models is that response probabilities are estimated based on a number of assumptions about the true nature of the nonresponse missing-data mechanism. In general, such models are simplifications. Consequently, anticipated response probabilities have a standard error, and may even be biased themselves when they are based on similar, but different surveys. In the optimization, this uncertainty can be accounted for by allowing response probabilities to be random variables rather than fixed quantities. Then, sensitivity analyses and evaluations of the robustness of the optimization can provide insight into the variation of quality and costs when the survey is conducted multiple times under the same circumstances.

For a better exposition of the ASD framework, we present two examples of resource allocation optimization for ASDs, namely Example 2.2 studies a static adaptive design and Example 2.3 a dynamic adaptive design.

Example 2.2. Consider a population of size 2,000 that is clustered in two groups given $X = (\text{age} \leq 35, \text{ age} > 35)$. Their respective proportions in the population are given by $q(x) = (0.5, 0.5)$. The strategy set $S = \{s_1, s_2, s_3, s_4\}$, with

$$s_1 = (\text{web, 1 reminder}) \qquad s_2 = (\text{web, no reminders, face-to-face, 3 visits})$$
$$s_3 = (\text{face-to-face, 6 visits}) \qquad s_4 = \emptyset,$$

that is, the survey design features chosen for tailoring are the interview mode and the number of mode attempts. Strategies s_1 and s_3 describe a unimode survey, i.e., only one interview mode is employed throughout the entire data collection, while s_3 describes a mixed-mode design where web nonrespondents are approached in face-to-face within the same data collection. However, given a limited length of the data collection, only 3 visits are allowed to establish contact in face-to-face. The design in strategy s_1 offers nonrespondents one reminder about the survey request. The cost components are as follows

$$\text{administration of web questionnaire} = 5€ \qquad \text{web reminder} = 2€$$
$$\text{one face-to-face visit} = 15€ \qquad \text{one face-to-face interview} = 20€.$$

The maximum budget available is $B = 30{,}000$. From historical data we can estimate the response probabilities $\rho(s, x)$ and the group strategy costs assuming that the entire group is assigned the respective strategy (see Table 2.1). As expected, the response probabilities are highest for the mixed-mode strategy. Moreover, this strategy is cheaper than s_3 which suggests that s_3 may not appear in the optimal allocation. With $p(s, x)$ the allocation probability for group g to strategy s, the quality objective function, the weighted response rate, is formulated as follows

$$Q(\mathbf{p}) = \sum_{x,s} q(x)\, p(s, x) \rho(s, x).$$

For simplicity, we assume that there is enough interviewer capacity to carry out the survey regardless of the chosen strategy. The resource allocation problem for the above formulated ASD is how to allocate strategies to groups such that there are no budget overruns and the response rate is maximized. Note that since no paradata are taken into account to define the groups, the present adaptive design is static.

Table 2.2 presents the optimization results, with $p^*(s, x)$ the optimal solution. The objective value at optimum is 0.713, i.e., a maximum of 71.3% response rate can be obtained given survey strategy S and a budget of 30,000. The budget value is large enough to address all population units. Moreover, the young group is fully assigned to strategy s_2 that has the highest response probability. Only 27.1% of the old group is assigned to s_2 and the remainder to s_1. The reason is that, although the group is more

	age ≤ 35				age > 35			
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4
$\rho\,(s,x)$	0.432	0.789	0.684	0	0.574	0.806	0.663	0
$c\,(s,x)$	6320	20094	20535	0	6120	20103	23132	0
$p^*\,(s,x)$	0	1	0	0	0.729	0.271	0	0

TABLE 2.1: Resource allocation optimization in static adaptive survey designs: input parameters and optimal solution.

Budget	age ≤ 35				age > 35				$Q(\mathbf{p}^*)$
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	
5,000	0	0	0	1	0.817	0	0	0.183	0.235
10,000	0.614	0	0	0.386	1	0	0	0	0.420
20,000	0.451	0.549	0	0	1	0	0	0	0.601
30,000	0	1	0	0	0.729	0.271	0	0	0.713
40,000	0	1	0	0	0.014	0.986	0	0	0.796
40,200	0	1	0	0	0	1	0	0	0.797

TABLE 2.2: Resource allocation optimization in static adaptive survey designs: optimal solutions for various budget levels.

responsive, it is also more expensive in s_2. By varying the value of B, we gain more information on the strategy allocation process (see Table 2.2). Note that $B = 40{,}200$ is the budget level for which maximal response rate is achieved since both groups are assigned to the most responsive strategy.

Example 2.3. (Section 3 in SCHOUTEN ET AL. 2013) Consider a sample of size $n = 2{,}000$ and a total capacity of $M = 80$ interviewers. During a first phase of data collection, interviewers make assessments on the sample unit's probability to participate in the survey if contacted again. The assessment is made on a three point scale, (*easy*, *medium*, *difficult*), where *easy* corresponds to a high probability to participate. Note that no assessment is available if the sample unit has not been contacted. This type of information will constitute the paradata for our dynamic design. After the first phase of data collection, sample units are divided into respondents, nonrespondents and noncontacts. To decrease the nonresponse bias, we decide to re-approach refusers using a different strategy. The strategy for noncontacts is not altered since there is no assessment available. The design feature we consider in this example is the interviewer skill. Based on their historic performance, interviewers can be grouped in *good* and *less good.* Hence, $S = \{\text{good}, \text{less good}\}$. Similarly to Example 2.2, we choose $X = (\text{age} \leq 35,\ \text{age} > 35)$, with respective proportions $q(x) = (0.5, 0.5)$. The quality objective is given by

$$R = 1 - 2\sqrt{\sum_{x,s} q(x)\big(\rho\,(s,x) - \bar{\rho}\big)^2},$$

where $\bar{\rho}$ represents the overall response rate. This quality indicator, known as the representativity indicator (see Schouten et al. 2009), aims at quantifying the quality of response composition given X. If the measured standard deviation of response probabilities across groups is small, then the response is balanced with respect to the set of characteristics X which lowers the chance for bias in the survey estimates. Thus, the problem at hand is assigning interviewers to refusers such that the R-indicator is maximized. Note that $R \in [0,1]$, with 1 the maximum level of representativity.

Let $q\,(\tilde{x},x)$ denote the conditional probability that a sample unit from age group x is of type $\tilde{x}$, where $\tilde{x} \in \{\text{easy, medium, difficult}\}$. Furthermore, let $\lambda(\tilde{x},x)$ be the probability that a sample unit of type $\tilde{x}$ from age group x is a refuser. If a person is not a refuser, then $\mu(\tilde{x},x)$ is the probability that the person either was a respondent after the first phase or becomes a respondent when he/she was a noncontact after the first phase. Let p_s be the proportion of interviewers with skill $s \in S$. Obviously, $0 \le p_s \le 1$ and $p_{s_1} + p_{s_2} = 1$. We assume that each interviewer can handle at most $c = 30$ refuser cases in the second phase of the survey. The probability that a refuser of type $\tilde{x}$ from group x will respond if contacted by an interviewer of skill s is denoted by $\rho\,(s,x,\tilde{x})$ and it is again assumed to be known from previous surveys. Let $\big(p\,(s,x,\tilde{x})\big)_{s,x,\tilde{x}}$ be the set of decision variables, where $p\,(s,x,\tilde{x})$ represents the probability that a sample unit of type $\tilde{x}$ will be assigned to an interviewer of skill s given that he/she belongs to group x. In other words, we allow for a random assignment of sample units to the two interviewer groups. In this example, we have constraints on the interviewer occupation rates, formulated as

$$n \sum_{x,\tilde{x}} q(x)\, q(\tilde{x},x)\, p\,(s,x,\tilde{x})\, \lambda(\tilde{x},x) \le M\, p_s\, c, \;\; \forall\; s \in S.$$

The response probability for a unit from group x is derived as

$$\rho\,(s,x) = \sum_{\tilde{x}} q(\tilde{x},x) \left[\big(1-\lambda(\tilde{x},x)\big)\mu(\tilde{x},x) + \lambda(\tilde{x},x) \sum_{s} p\,(s,x,\tilde{x})\, \rho(s,x,\tilde{x}) \right]$$

and forms the input to the R-indicator together with $\bar{\rho} = \sum_{s,x} q(x)\, \rho\,(s,x)$. With the input parameters from Table 2.3 and $p_{s_1} = 0.25$, the optimization yields an R-indicator value of 0.827. The optimal solution $p^*(s,x,\tilde{x})$ is also given in Table 2.3. All but one of the decision variables are either 0 or 1, i.e., the interviewer allocation is mostly non-probabilistic. The exception is the subpopulation of young persons with medium response probability assessment. For a result comparison, assume that interviewers are randomly allocated to refusers. Then, the value of the R-indicator equals 0.749, lower than our objective value. The optimal assignment, thus, leads to a considerable increase in the R-indicator. The response rates are, respectively, 72.0% and 70.1% for the optimal and the random assignment. If we increase the number of interviewers, while the other parameters remain fixed, then for any interviewer capacity higher than $M = 84$, the R-indicator does not improve anymore. The reason is that both interviewer groups are sufficiently big to handle the entire sample and the capacity constraint is no longer

		age ≤ 35			age > 35		
		easy	medium	difficult	easy	medium	difficult
$q(\tilde{x}, x)$		0.2	0.3	0.5	1/3	1/3	1/3
$\mu(\tilde{x}, x)$		0.85	0.8	0.76	0.95	0.93	0.91
$\lambda(\tilde{x}, x)$		0.5	0.6	0.7	0.2	0.3	0.4
$\rho(s, x, \tilde{x})$	good	0.8	0.6	0.4	0.9	0.7	0.5
	less good	0.7	0.5	0.3	0.8	0.6	0.4
$p^*(s, x, \tilde{x})$	good	1	0.83	1	0	0	0
	less good	0	0.17	0	1	1	1

Table 2.3: Resource allocation optimization in dynamic adaptive survey designs: input parameters and optimal solution.

prohibitive. The R-indicator value for $M = 84$ is equal to 0.830 and the corresponding response rate is 72.1%. Note that since the objective is to maximize the R-indicator, the optimization problem translates to striving for an optimal balance across group response rates by assigning interviewers with higher skill to more difficult refusers. If the objective function changes to maximizing the response rate, the optimal solution would converge to assigning only *good* interviewers to all cases.

3

The survey resource allocation problem

Resource allocation problems (RAPs) deal with assigning available resources to various activities to meet a specified objective. A variety of applications can be modeled as RAPs, e.g., job shop scheduling (allocating time and equipment to work on orders such that delivery time is minimized), portfolio optimization (allocating funds to a set of financial instruments to maximize returns for given level of risk), project funding (allocating funds among various projects such that return on investment is maximized). A great body of literature is devoted to developing fast algorithms for continuous convex nonlinear RAPs (see an extensive survey in Patriksson 2008). Quite often, however, the RAP formulation leads to an integer or mixed-integer problem which has been proven to have an NP-complete worst case complexity (see Ibaraki and Katoh 1988). Algorithms still exist for those applications where convexity in the objective function and/or constraints is satisfied, see, e.g., Katoh et al. (1979), Bretthauer et al. (1999), and Bretthauer and Shetty (1995). However, the increasingly complex operations observed in practice do not display always convexity, e.g., due to economies of scale. In this case, additional difficulties are posed by the presence of several local optima and, as means to tackle such problems, approximation methods are suggested (see Bretthauer et al. 2003, Benson et al. 1990).

Survey designs can also be modeled as a RAP by translating the various design features (i.e., survey mode, timing of call attempt, interviewer skill) in terms of costs and quality. Consequently, design features can be assigned to survey sample units, i.e., in an adaptive design (see Schouten et al. 2013), such that quality is maximized and costs of these resources meet a budgetary constraint. However, even for a simple setting, such a formulation leads in general to a nonconvex mixed-integer nonlinear problem (NCMINLP) which is a difficult problem to solve from a mathematical programming perspective. In this chapter we introduce the notation for the RAP in the context of adaptive survey designs. We show that the problem can be reformulated as a Markov decision problem and by applying dynamic programming, the global optimum is obtained in a finite number of iterations. Our algorithm (further referred to as the RAP algorithm) is applicable to a class of NCMINLPs where the decision variables are binary (i.e., 0-1) and the problem structure satisfies the additivity property presented in Section 3.2.

3.1 Problem formulation

Consider a survey sample consisting of N units that can be clustered into homogeneous groups based on characteristics, such as age, gender, and ethnicity (information that can be extracted from external sources of data). Let $\mathcal{G} = \{1, \ldots, G\}$ be the set of homogeneous groups with size N_g for group $g \in \mathcal{G}$ in the survey sample. The survey fieldwork is divided into time slots, denoted by the set $\mathcal{T} = \{1, \ldots, T\}$, at which units in a group can be approached for a survey. The survey itself can be conducted using certain interview modes, such as a face-to-face, phone, web/paper survey; the set of different modes is denoted by $\mathcal{M} = \{1, \ldots, M\}$. At each time slot $t \in \mathcal{T}$ one can decide to approach units in group $g \in \mathcal{G}$ for a survey using mode $m \in \mathcal{M}$. A survey request ends with success if two steps are achieved, i.e., successful contact and participation by answering the questionnaire. From historical data group-dependent contact probabilities $p_g(t, m)$ and participation probabilities $r_g(t, m)$ can be estimated, which we consider as given quantities in our problem. Note that from historical data it can also be observed that certain time slots (e.g., morning, evening) have an influence on the availability of the unit and the willingness to respond. Therefore, to employ most of the available information, the contact and participation probabilities are modeled at the level of time slots for each group as well rather than the mode only.

Let $x_g(t, m) \in \{0, 1\}$ be the decision variable to model whether units in group g are approached for a survey at time t using mode m. Note that at any time t only one mode can be employed to approach a group, yielding the constraint

$$\sum_{m \in \mathcal{M}} x_g(t, m) \leq 1. \tag{3.1}$$

For the sake of simplicity, we make the following assumptions.

Assumption 3.1. *Participation and contact probabilities are independent of each other for all g, t, m.*

Assumption 3.2. *Contact and cooperation probabilities for all g, m are independent of the history of contact attempts up to time t.*

The former assumption is a weak assumption, although in practice a certain degree of correlation can be observed between contact and participation. However, guidelines on how to quantify such correlation are not clearly given in the literature. The latter

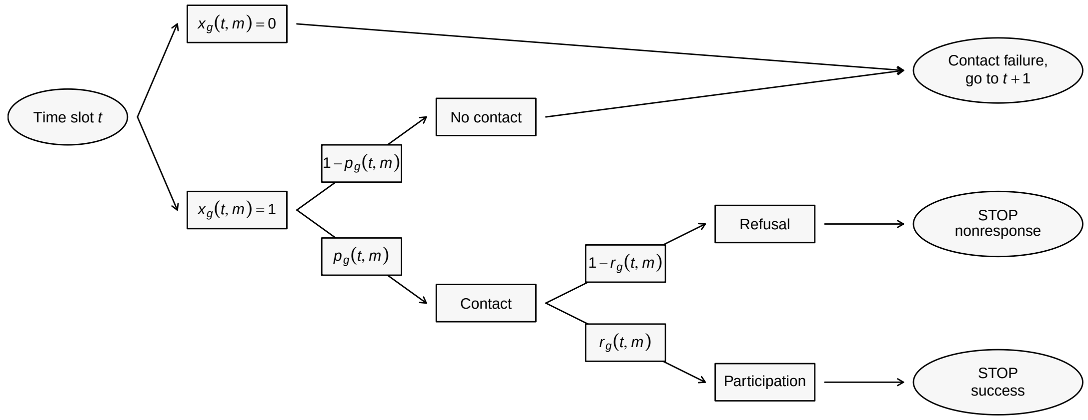


FIGURE 3.1: Sequence of events for a given survey approach.

assumption may be a strong assumption for survey modes such as telephone or face-to-face. Sample units that are harder to reach may have different characteristics and at-home patterns than those that are easy to reach. As a result, units that are not contacted after several attempts may have different contact and cooperation probabilities at time t via mode m than those that are first attempted at time t in mode m. Under Assumption 3.2, fitting to real survey data will likely lead to an overestimation of contact probabilities at earlier attempts and an underestimation at later attempts. The assumption can be relaxed by including timing and results of previous attempts, which would create a cumbersome model.

When a successful contact is established and the unit agrees to participate, the survey ends with success, i.e., response is obtained. By the independence assumption, this happens with probability $p_g(t,m)r_g(t,m)$. However, if the unit refuses participation after successful contact, the unit is not considered for a future survey approach; this happens with probability $p_g(t,m)\big(1-r_g(t,m)\big)$. Only in the case that the unit is not contacted successfully, the unit can be considered for a future survey approach (see Figure 3.1); this happens with probability $1-p_g(t,m)$.

Thus, if the unit is approached again at time t' using mode m', then the probability of a successful approach is $\big(1-p_g(t,m)\big)p_g(t',m')r_g(t',m')$, and the probability of a contact failure is $\big(1-p_g(t,m)\big)\big(1-p_g(t',m')\big)$. In general, the probability that a contact fails up to time t' is denoted by $f_g(t')$ given by

$$
\begin{aligned}
f_g(t') &= \prod_{t=1}^{t'} \prod_{m\in\mathcal{M}} \left[x_g(t,m)\big(1-p_g(t,m)\big) + \big(1-x_g(t,m)\big)\right] \\
&= \prod_{t=1}^{t'} \prod_{m\in\mathcal{M}} \left[1-x_g(t,m)p_g(t,m)\right].
\end{aligned}
$$

Note that this is a highly nonlinear expression in the decision variables, which can be recursively computed by

$$\begin{aligned} f_g(t') &= \prod_{m \in \mathcal{M}} \left[x_g(t',m)\big(1 - p_g(t',m)\big) + \big(1 - x_g(t',m)\big) \right] f_g(t'-1) \\ &= \prod_{m \in \mathcal{M}} \left[1 - x_g(t',m) p_g(t',m) \right] f_g(t'-1), \end{aligned} \tag{3.2}$$

using the fact that $f_g(0) = 1$. Using this definition, the response rate for group g, ρ_g, can then be computed by

$$\rho_g = \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m). \tag{3.3}$$

The clustering of the N units usually results in groups that are not of the same size or importance. Therefore, the response rates for the groups are usually weighted by a factor w_g (e.g., $w_g = N_g/N$ is taken in practice). Hence, the objective of the decision maker becomes to maximize the overall response rate

$$\begin{aligned} \bar{\rho} &= \sum_{g \in \mathcal{G}} w_g \rho_g \\ &= \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m), \end{aligned} \tag{3.4}$$

by setting the decision variables $x_g(t,m)$ optimally. The decision variables are subject to constraints, though, due to scarcity in resources. In practice, due to resource management constraints, the number of times that a group can be approached by mode m is limited to $\bar{k}_g(m)$ times, leading to the constraint

$$\sum_{t \in \mathcal{T}} x_g(t,m) \leq \bar{k}_g(m). \tag{3.5}$$

By combining the objective (3.4) with the constraints (3.1), (3.2) and (3.5), we can draft our optimization problem as a binary programming problem in the following manner.

$$\begin{aligned} \max\ & \bar{\rho} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m) \\ \text{s.t. } & \sum_{t \in \mathcal{T}} x_g(t,m) \leq \bar{k}_g(m),\ \forall g \in \mathcal{G},\ \forall m \in \mathcal{M}, \\ & \sum_{m \in M} x_g(t,m) \leq 1,\ \forall g \in \mathcal{G},\ \forall t \in \mathcal{T}, \\ & f_g(t) = \prod_{m \in \mathcal{M}} \left[x_g(t,m)\big(1 - p_g(t,m)\big) + 1 - x_g(t,m) \right] f_g(t-1), \forall g \in \mathcal{G}, t \in \mathcal{T} \\ & f_g(0) = 1,\ \forall g \in \mathcal{G}, \\ & x_g(t,m) \in \{0,1\},\ \forall g \in \mathcal{G},\ \forall t \in \mathcal{T},\ \forall m \in \mathcal{M}. \end{aligned} \tag{3.6}$$

Problem (3.6) represents the adaptive survey design problem in which survey features with significant influence on the quality of the survey are balanced. In our model, the features are the *interview mode*, the *number of allowed attempts*, and *number of time slots*. The solution of the problem is, however, not trivial as can be seen from the following remark.

Proposition 3.1. *The objective function is a nonconvex nonlinear function, and the constraints do not form a convex polytope either.*

Proof. The proof rests on the following remarks. Nonlinearity appears given the definition of contact failure probability $f_g(t)$ in (3.2). This term is present in both the objective function and the constraints. Nonconvexities occur mainly because of the binary decision variables that make the functions discontinuous. □

Suppose, however, that we would relax the integer condition on the decision variables, i.e., $x_g(t,m) \in [0,1]$, $\forall g,t,m$. We have the following result

Proposition 3.2. *The objective function is nonconvex on* $[0,1]^{G \cdot T \cdot M}$.

Proof. Take $G = 1$, $M = 1$ and $T = 2$ in (3.4) in Definition 2.1 of convexity. In this case, $w_g = 1$ thus we disregard it from the calculations. Suppressing indices for g and m, the response rate in this case $\bar{\rho}$ can be written as

$$\begin{aligned}\bar{\rho} = f(\boldsymbol{x}) &= x(1)\,p(1)\,r(1) + [1 - p(1)\,x(1)]\,x(2)\,p(2)\,r(2)\\ &= ax(1) + bx(2) - cx(1)x(2),\end{aligned}$$

with $\boldsymbol{x} = \big(x(1), x(2)\big)'$, $a = p(1)\,r(1)$, $b = p(2)\,r(2)$ and $c = p(1)\,p(2)\,r(2)$. Rewriting the convexity definition for our $f(\boldsymbol{x})$ with $\boldsymbol{x} = \big(x(1), x(2)\big)'$ and $\boldsymbol{y} = \big(y(1), y(2)\big)'$ and grouping the terms accordingly we obtain the following

$$c\lambda(1-\lambda)[x(1)y(2) + x(2)y(1) - x(1)x(2) - y(1)y(2)] \geq 0.$$

Take $\boldsymbol{x} = (0,0)'$ and $\boldsymbol{y} = (1,1)'$ and we reach a contradiction. Since the inequality should hold for any two points $\boldsymbol{x}, \boldsymbol{y} \in [0,1]^T$ this concludes the proof. □

Similarly we can prove that $f(\boldsymbol{x})$ is not concave either. As such, presence of nonconvexities makes our problem non-tractable from a mathematical programming perspective, even for small-sized problems (e.g., 1 group and 4 time slots). In the next section, we develop an algorithm that is able to derive optimal solutions by aggregating information in the adaptive survey design problem.

3.2 Adaptive survey design policies

In this section, we reformulate the adaptive survey design problem such that the problem becomes numerically tractable. We show that our problem satisfies the Bellman optimality principle (see Bellman 1957) and can therefore be solved by dynamic programming. In order to do this, note that at any time t, it is sufficient to know the probability of contact failure up to time t, $f_g(t-1)$, instead of the complete configuration $x_g(t', m)$ for

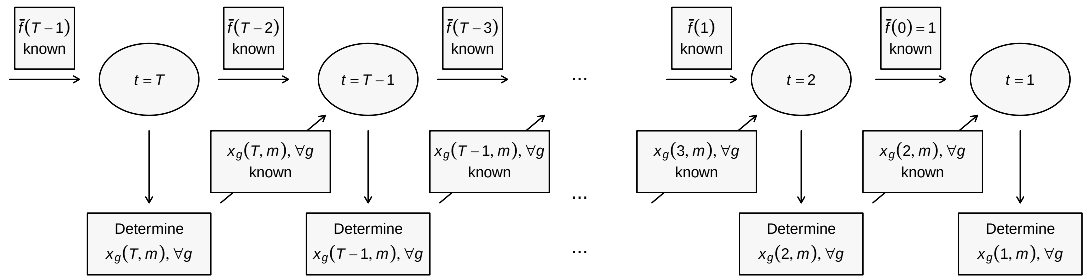


FIGURE 3.2: Sequence of time slot decisions backwards in time.

$t' < t$ for all g. Denote by $\vec{f}(t) = \big(f_1(t), \ldots, f_G(t)\big)$ the vector storing the probability of contact failure up to time t. Hence, given $\vec{f}(T-1)$, the decision at time T is obvious when one also keeps track of the number of times that mode m has been used for each group g. Since the decision at time T is completely determined, one can then calculate the optimal decisions at time $T-1$, and continue working back towards the first time epoch (see Figure 3.2). By keeping track of the time, the contact failure probability, and the utilization of the different modes, the problem becomes completely Markovian and the problem can be cast as a Markov decision problem.

Let the state space of the Markov decision problem be denoted by

$$\mathcal{S} = \mathcal{T} \times [0,1]^G \times \{0,1,\ldots\}^{G \cdot M},$$

where $s = \big(t, \vec{f}, K\big) \in \mathcal{S}$ has components t, denoting the time at which the process resides, $\vec{f}$ the probability of contact failure up to time t, and $K = \big(k_g(m)\big)_{g\in\mathcal{G}, m\in\mathcal{M}}$ denoting that mode m can still be used $k_g(m)$ times for group g.

Remark 3.1. In the following, we suppress the time index t in the notation of $\vec{f}$ and its components f_g since it is already provided in s.

The action space $\mathcal{A}_s$ is given by

$$\mathcal{A}_s = \left\{ \big(a_g(m)\big)_{g\in\mathcal{G}, m\in\mathcal{M}} \,|\, a_g(m) \in \{0,1\}, a_g(m) \leq k_g(m), \sum_{m\in\mathcal{M}} a_g(m) \leq 1 \right\},$$

where $a_g(m)$ denotes the available action for group g using mode m. More specifically, given the state space s the process is in, choosing an action translates to choosing whether to approach ($a_g(m) = 1$) or not ($a_g(m) = 0$) provided that there are attempts left. If the number of attempts has been exhausted, the only allowed action is not to approach. The transition probability p is given by

$$p(s,a,s') = \begin{cases} 1, & \text{if } s' = \Big(t+1, \vec{f}' = \big(f'_g\big)_{g\in\mathcal{G}}, K' = \big(k'_g(m)\big)_{g\in\mathcal{G})m\in\mathcal{M}}\Big), \\ 0, & \text{otherwise,} \end{cases}$$

where $f'_g = \prod_{m\in\mathcal{M}} \big[1 - a_g(m)p_g(t,m)\big] f_g$, and $k'_g(m) = k_g(m) - a_g(m)$. The rewards r

are given by

$$r(s,a) = \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} w_g a_g(m) f_g p_g(t,m) r_g(t,m).$$

The tuple $(\mathcal{S}, \mathcal{A}, p, r)$ completely defines the Markov decision problem (see also definition in Section 2.2). The value function for $s = (t, \vec{f}, K)$ is given by

$$\begin{aligned} V(s) &= \max_{a \in \mathcal{A}_s} \Big[r(s,a) + \sum_{s' \in \mathcal{S}} p(s,a,s') V(s') \Big] \\ &= \max_{a \in \mathcal{A}_s} \Big[\sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} w_g a_g(m) f_g p_g(t,m) r_g(t,m) \; + \\ &\quad V\Big(t+1, \big(\prod_{m \in \mathcal{M}} \left[1 - a_g(m) p_g(t,m)\right] f_g \big)_{g \in \mathcal{G}}, \big(k_g(m) - a_g(m)\big)_{g \in \mathcal{G}, m \in \mathcal{M}} \Big) \Big]. \end{aligned} \tag{3.7}$$

Note that for all $s = (T+1, \vec{f}, K)$ we have $V(s) = 0$ since (3.6) is a finite-time horizon problem. Bellman (1957) showed that we can find the optimal policy by iterating backwards from T over the value function (see also Section 2.2). We can thus solve the Markov decision problem $(\mathcal{S}, \mathcal{A}, p, r)$ by backward recursion over (3.7). The optimal policy, i.e., the optimal values of the decision variables, can be recovered by tracking back the already performed calculations. We find the value of the optimal solution, i.e., the weighted response rate, in $V(s_0)$, with s_0 the initial state given by $t = 1$, $\vec{f} = \big(f_g = 1\big)_{g \in \mathcal{G}}$, and $K = \big(\bar{k}_g(m)\big)_{g \in \mathcal{G}, m \in \mathcal{M}}$.

Remark 3.2. The formulation of the value function $V(s)$ is convenient since at any time epoch t the weighted response $\bar{\rho}$ can be expressed as a cumulative function of response obtained at every epoch $t' \leq t$. That is, recursively, we can write the following for all t,

$$\begin{aligned} \bar{\rho}_t &= \sum_{g \in \mathcal{G}} \sum_{t'=1}^{t} \sum_{m \in \mathcal{M}} w_g \, f_g(t'-1) \, x_g(t',m) \, p_g(t',m) \, r_g(t',m) \\ &= \sum_{t'=1}^{t} \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} w_g \, f_g(t'-1) \, x_g(t',m) \, p_g(t',m) \, r_g(t',m) \\ &= \sum_{t'=1}^{t-1} \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} w_g \, f_g(t'-1) \, x_g(t',m) \, p_g(t',m) \, r_g(t',m) \; + \\ &\quad \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} w_g \, f_g(t-1) \, x_g(t,m) \, p_g(t,m) \, r_g(t,m) \\ &= \bar{\rho}_{t-1} + \rho_t, \end{aligned}$$

where by ρ_t we understand the weighted response obtained at epoch t and $\bar{\rho}_0 = 0$. We will further refer to this property as the additivity property.

Remark 3.3. The RAP algorithm only needs T iterations, and in each iteration $2^{G \cdot M}$ actions need to be considered. Hence, for values of realistic size, the algorithm is computationally feasible and it guarantees global optimality.

3.3 Budget and capacity constraints

In the previous section, we formulated the adaptive survey design problem in which the focus was on the quality of the survey results modeled by maximizing the weighted response rates. However, the model formulation is sufficiently flexible to include other features as well, such as budgetary constraints or capacity restrictions. In this section, we discuss how these features can be integrated within this framework.

First, we consider a constraint on the budget. Every time a sample unit is approached for a survey, costs are incurred for the effort. These costs mainly depend on the interview mode and also on the outcome of each approach. Denote by $b^s(m)$ the costs that are incurred by using mode m with a successful outcome. For the costs that are incurred by mode m that results in a failure, we distinguish two types of costs: $b^{f_c}(m)$ when the failure occurs due to failure of contact, and $b^{f_r}(m)$ when the failure occurs due to failure to participate. Let B be the total budget that is available for the survey. An approach at time t using mode m bears the following costs

$$p_g(t,m)\big[r_g(t,m)b^s(m) + \big(1 - r_g(t,m)\big)b^{f_r}(m)\big] + \big(1 - p_g(t,m)\big)b^{f_c}(m).$$

In general, the costs $b_g(t,m)$ at time t using mode m depend on the contact failures before time t. These costs can be written as follows

$$\begin{aligned} b_g(t,m) = x_g(t,m)f_g(t-1)\Big[&p_g(t,m)\big[r_g(t,m)b^s(m) \\ &+ \big(1 - r_g(t,m)\big)b^{f_r}(m)\big] + \big(1 - p_g(t,m)\big)b^{f_c}(m)\Big], \end{aligned} \tag{3.8}$$

with $f_g(t)$ given by (3.2). Hence, using this definition, the budgetary constraint that needs to be added to problem (3.6) is given by

$$\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} N_g b_g(t,m) \le B. \tag{3.9}$$

A capacity constraint can be addressed in a manner analogous to the constraint on the budget. Let C be the available capacity, measured by the number of interviewer hours available to survey the sample. Similar to the cost structure, the required capacity depends on the interview mode and the outcome of each approach. Denote by $c^s(m)$, $c^{f_c}(m)$, and $c^{f_r}(m)$ the capacity utilized when the approach is successful, or has failed due to contact failure, or failed due to participation failure, respectively. Following the same steps as above, the capacity constraint to be added to problem (3.6) is given by

$$\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} N_g c_g(t,m) \le C, \tag{3.10}$$

with $c_g(t,m)$ defined as

$$\begin{aligned} c_g(t,m) = x_g(t,m)f_g(t-1)\Big[&p_g(t,m)\big[r_g(t,m)c^s(m) \\ &+ \big(1 - r_g(t,m)\big)c^{f_r}(m)\big] + \big(1 - p_g(t,m)\big)c^{f_c}(m)\Big]. \end{aligned} \tag{3.11}$$

Remark 3.4. We have chosen to model the budgetary constraint and the capacity restriction as a global constraint over all the groups. However, it is quite easy to divide the budget B into budgets B_g for each group g, and then have a constraint per group. A similar remark holds for the capacity restriction as well.

The maximum number of attempts $\bar{k}_g(m)$ is replaced by the budgetary constraint and the capacity limitation. Hence, the binary programming problem now becomes

$$\begin{aligned}
\max\ \ & \bar{\rho} = \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m) \\
\text{s.t.}\ \ & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} N_g b_g(t,m) \le B, \\
& \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} N_g c_g(t,m) \le C, \\
& \sum_{m\in M} x_g(t,m) \le 1,\ \forall g\in\mathcal{G},\ \forall t\in\mathcal{T}, \\
& f_g(t) = \prod_{m\in\mathcal{M}} [x_g(t,m)(1-p_g(t,m)) + 1 - x_g(t,m)] f_g(t-1), \forall g\in\mathcal{G}, t\in\mathcal{T}, \\
& f_g(0) = 1,\ \forall g\in\mathcal{G}, \\
& b_g(t,m) = x_g(t,m) f_g(t-1)\Big[p_g(t,m)\big[r_g(t,m)b^s(m) + \big(1-r_g(t,m)\big)b^{f_r}(m)\big] \\
& \qquad + \big(1-p_g(t,m)\big)b^{f_c}(m)\Big],\ \forall g\in\mathcal{G},\ \forall t\in\mathcal{T},\ \forall m\in\mathcal{M}, \\
& c_g(t,m) = x_g(t,m) f_g(t-1)\Big[p_g(t,m)\big[r_g(t,m)c^s(m) + \big(1-r_g(t,m)\big)c^{f_r}(m)\big] \\
& \qquad + \big(1-p_g(t,m)\big)c^{f_c}(m)\Big],\ \forall g\in\mathcal{G},\ \forall t\in\mathcal{T},\ \forall m\in\mathcal{M}, \\
& x_g(t,m) \in \{0,1\},\ \forall g\in\mathcal{G},\ \forall t\in\mathcal{T},\ \forall m\in\mathcal{M}.
\end{aligned} \tag{3.12}$$

In order to incorporate the budgetary constraint and the capacity restriction in the Markov decision problem, we need to add the state variables b and c for both the budget and the capacity, respectively. In each state $s = (t, \vec{f}, b, c)$, these variables denote the budget and the capacity that are left for the rest of the survey. At time t, the budget and the capacity after taking an action $a_g(m)$ are decreased by $\sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} a_g(m)b_g(t,m)$ and $\sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} a_g(m)c_g(t,m)$, respectively. This can only be done as long as the budget and the capacity remain non-negative. This requirement is added to the action set. Hence, the Bellman equations become

$$\begin{aligned}
V(s) = \max_{a\in\mathcal{A}_s} \Bigg[& \sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} w_g a_g(m) f_g p_g(t,m) r_g(t,m)\ + \\
& V\Bigg(t+1, \Big(\prod_{m\in\mathcal{M}} \big[1 - a_g(m)p_g(t,m)\big] f_g\Big)_{g\in\mathcal{G}}, \\
& b - \sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} a_g(m)b'_g(t,m), c - \sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} a_g(m)c'_g(t,m)\Bigg)\Bigg],
\end{aligned} \tag{3.13}$$

Mode	Probability	t_1	t_2	t_3	t_4	t_5	t_6	$\bar{k}_g(m)$
Face-to-face	$p_g(t,m)$	0.3	0.4	0.8	0.2	0.3	0.7	2
	$r_g(t,m)$	0.9	0.7	0.3	0.8	0.8	0.6	
Phone	$p_g(t,m)$	0.4	0.5	0.9	0.4	0.4	0.8	4
	$r_g(t,m)$	0.8	0.5	0.7	0.6	0.4	0.6	

TABLE 3.1: Input data for group g.

with

$$\begin{aligned}\mathcal{A}_s = \{a_g(m) \,|\, a_g(m) \in \{0,1\}, &\sum_{m\in\mathcal{M}} a_g(m) \leq 1,\\ b - \sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} a_g(m) b'_g(t,m) \geq 0, &\text{ and } c - \sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} a_g(m) c'_g(t,m) \geq 0\},\end{aligned}$$

in which we defined $b'_g(t,m)$ and $c'_g(t,m)$ given s to be

$$\begin{aligned}b'_g(t,m) = &f_g p_g(t,m)\big[r_g(t,m)b^s(m) + \big(1 - r_g(t,m)\big)b^{f_r}(m)\big]\\ &+ \big(1 - p_g(t,m)\big)b^{f_c}(m)\end{aligned}$$

and

$$\begin{aligned}c'_g(t,m) = &f_g p_g(t,m)\big[r_g(t,m)c^s(m) + \big(1 - r_g(t,m)\big)c^{f_r}(m)\big]\\ &+ \big(1 - p_g(t,m)\big)c^{f_c}(m).\end{aligned}$$

Remark 3.5. The addition of constraints affects the complexity of the problem in terms of the state space size. However, for realistic values, the problem remains computationally feasible.

3.4 Numerical examples

The previous sections dealt with the theoretical models to solve the problem of resource allocation within adaptive survey designs. In this section, we give two numerical examples to illustrate our methodology.

Our first example shows that the solution of the basic unconstrained model is indeed optimal, although counterintuitive upon first glance at the input parameters. Consider a survey sample in which all units belong to the same group g. The set of available interview modes is $\mathcal{M} = \{\text{face-to-face, phone}\}$. The survey fieldwork is divided in $T = 6$ time slots. Table 3.1 gives the contact and participation probabilities $p_g(t,m)$ and $r_g(t,m)$ estimated from historical data and the maximum number of attempts $\bar{k}_g(m)$.

Note that there is a clear preference for contact at time slots t_3 and t_6 for both interview modes. For participation, on the other hand, there is more than 50% probability for positive participation except for an attempt by *face-to-face* at t_3 and by *phone* at t_5. Therefore, it is not obvious what time slots should be chosen in order to maximize the total response. Hence, the optimal solution is hard to derive from intuition. Using the RAP algorithm from Section 3.2, we obtain the solution Setting 1 in Table 3.2.

	t_1	t_2	t_3	t_4	t_5	t_6	Response rate
Setting 1	F2F	F2F	Ph	Ph	0	Ph	0.753
Setting 2	Ph	F2F	Ph	Ph	F2F	Ph	0.736
Setting 3	F2F	F2F	Ph	Ph	F2F	Ph	0.755

TABLE 3.2: Optimal solution for group g.

Mode	Probability	t_1	t_2	t_3	t_4	t_5	t_6	$k_{g_2}(m)$
Face-to-face	$p_{g_2}(t,m)$	0.8	0.6	0.4	0.6	0.4	0.2	1
	$r_{g_2}(t,m)$	0.9	0.7	0.6	0.8	0.5	0.3	
Phone	$p_{g_2}(t,m)$	0.7	0.6	0.5	0.6	0.5	0.4	2
	$r_{g_2}(t,m)$	0.8	0.6	0.5	0.6	0.4	0.2	

TABLE 3.3: Input data for group g_2.

Let us analyze this solution. It looks surprising that for the first time slot *face-to-face* is chosen and not *phone*, although the immediate reward is higher for *phone*. However, considering the formula given in (3.2) for the group average response, we see that the lower the contact probability for the first time slot, the higher the future reward. Also, the participation probability $r_g(t_1, \text{F2F})$ is higher than $r_g(t_1, \text{Ph})$. The situation changes when $r_g(t_1, \text{F2F}) < r_g(t_1, \text{Ph})$. For example, take $r_g(t_1, \text{F2F}) = 0.7$. As expected, the new optimal solution (see Table 3.2, Setting 2) uses *phone* as first approach interview mode.

The structure of the solution given in Table 3.2, Setting 1 is motivated by the choice of $\bar{k}_g(m)$. From t_3 onward $k_g(\text{F2F}) = 0$, therefore *phone* is the only interview mode left available. Thus, the choice for time slots t_3, t_4, and t_6 is logical. However, taking action 0 at t_5 again looks counterintuitive. Since there are enough attempts left for *phone* and there are no budget or capacity constraints, it feels natural to choose for an attempt to approach. The explanation lies in the value of the objective function that is higher in this case (0.753 compared to 0.752 if the unit is approached).

The optimal solution in Setting 1 does not employ all attempts available for *phone*. Therefore, we cannot obtain a different solution if we increase the number of attempts for this mode. On the other hand, if we increase the number of attempts to 3 for *face-to-face*, then the average response improves (see Table 3.2, Setting 3). The structure of the optimal solution does not change much from the original setting. The only difference appears at t_5 where this time there are enough attempts for *face-to-face*, and selecting this mode leads to higher response.

Our second example depicts the optimization mechanism for two groups in the presence of budgetary and capacity constraints. Consider again the setting from the previous example, where Table 3.1 has the input data for group g_1. Table 3.3 gives the corresponding input data for group g_2.

Approaching group g_2 for the survey follows a more intuitive behavior, e.g., high partici-

Time slot	t_1	t_2	t_3	t_4	t_5	t_6	Response rate
Mode	F2F	Ph	0	Ph	0	0	0.821

TABLE 3.4: Optimal solution for group g_2.

Group \ Time slot	t_1	t_2	t_3	t_4	t_5	t_6	Response rate
g_1	F2F	F2F	Ph	F2F	F2F	Ph	0.757
g_2	F2F	F2F	F2F	F2F	F2F	Ph	0.851

TABLE 3.5: Optimal solution for two-group optimization for $B = 4{,}000$.

pation probabilities correspond to high contact probabilities. In the case of single group optimization, the optimal solution for group g_2 (see Table 3.4) starts with the choice of *face-to-face* as interview mode at t_1, since this results in the highest immediate reward. The same argument governs the entire structure of the solution.

Now consider a sample of $N = 2{,}000$ units that can be clustered in two groups given age, i.e., *young* and *old*. The proportion of the two groups in the survey sample is $w = (0.62, 0.38)$. A total budget $B = 4{,}000$ monetary units is available to survey the sample units using two modes, i.e., $\mathcal{M} = \{\text{face-to-face,phone}\}$. For simplicity we assume that one attempt costs one monetary unit regardless of the employed survey mode. Tables 3.1 and 3.3 give the estimates for contact and cooperation probabilities for the two groups, where g_1 denotes the young group and g_2 the old group, respectively. For the sake of simplicity we assume that capacity is unlimited. The overall response rate in this case is 0.793 and the optimal solution for the two groups is given in Table 3.5.

The costs incurred with this solution amount to 2,841 units for g_1 and 1033 units for g_2. The remaining budget could be an indication that the group response rates have attained their maximum, given the input probabilities. An easy approach to confirm such a hypothesis is to optimize for $B > 4{,}000$. The solution does not change which leads to the conclusion that $B = 3{,}873$ units is sufficient to collect the maximum response from the two groups. Evidently, dropping the constraint on the number of attempts has created a larger feasible region. This in turn leads to a higher response rate, 0.793 compared to 0.779 obtained if weighting the group response rates from Tables 3.2, Setting 1 and 3.4 with the corresponding values in w. The increase of 3.8% in the response rate could be explained by the relatively high budget. Figure 3.3 depicts the evolution of the response rate for various levels of budget.

Let us take a look at the changes in the optimal solution (see Table 3.6) that cause the two steep jumps in the response rate. As expected, group g_2 receives more effort since it yields a higher response per attempt than group g_1. This is particularly interesting in the case of $B = 1{,}250$ where the young group is not at all surveyed whereas the old group receives enough monetary units to yield its maximum response rate. From a cost

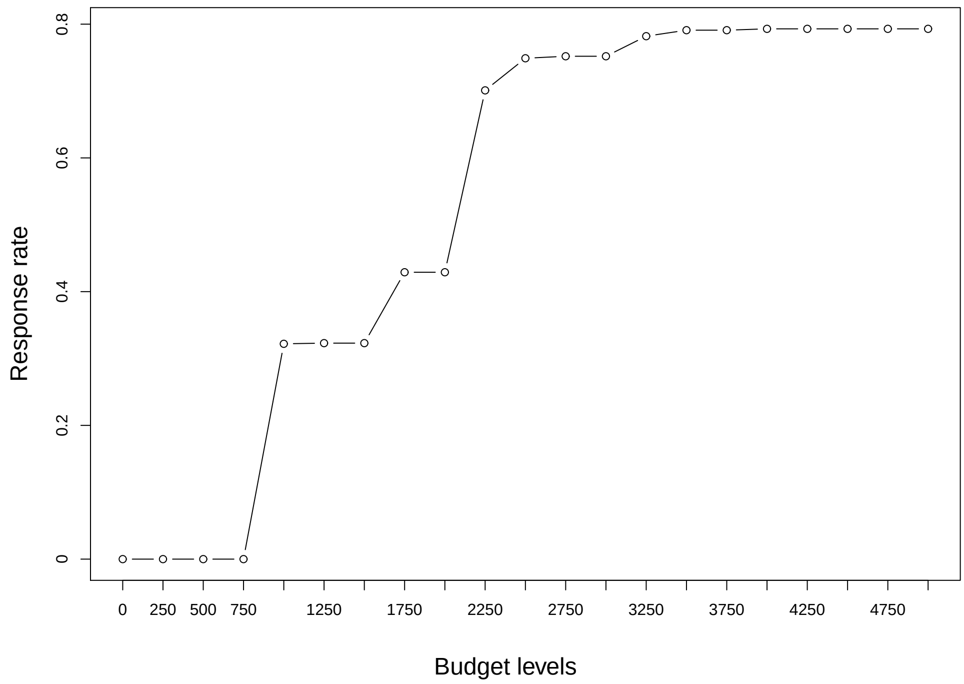


FIGURE 3.3: Response rate evolution for various budget levels.

perspective there is no difference between approaching the group at time t_1 or later. The reason that group g_1 is not approached at time slot t_1 is the corresponding response probability. For example, for B = 2,250 at t_3 there is no difference between the two modes in the cost for an attempt. The yielded response however is higher when using *phone*.

Figure 3.3 shows that it is sensible to analyze the optimal solution for various budget levels. Survey designers can gain useful information from comparing the response rate gains for small budget changes. For example, a budget increase of 11% from 2,250 to 2,500 leads to an expected 6.9% more response. On the other hand, a similar budget increase from 2,500 to 2,750 leads to only 0.4% additional expected response.

The RAP algorithm is implemented in `C++`. Table 3.7 presents the computational times for the two-group example. All run times are for an Intel Xeon L5520 processor with 4 cores. The run times increase with the increase in the budget. An increase in the budget expands the feasible region with points that yield a response rate at least as high as the previous feasible region. Therefore, additional time is spent on exploring the new points. The significant drop in the runtime for B = 4,000 can be explained by the fact that, at this point, the sequence of actions that yields maximal group response rates is feasible. The algorithm converges then very quickly to this point.

Other software tools such as Xpress, Maple and R were used in the attempt to solve the RAP for adaptive survey designs as a mathematical program. However, presence of

B	Group	Time slot						Group	Response
		t_1	t_2	t_3	t_4	t_5	t_6	response rate	rate
1,000	g_1	0	0	0	0	0	0	0	0.322
	g_2	F2F	F2F	0	F2F	Ph	0	0.849	
1,250	g_1	0	0	0	0	0	0	0	0.323
	g_2	F2F	F2F	F2F	F2F	F2F	Ph	0.851	
1,750	g_1	0	0	Ph	F2F	F2F	Ph	0.692	0.429
	g_2	0	0	0	0	0	0	0	
2,250	g_1	0	0	Ph	0	0	0	0.63	0.701
	g_2	F2F	0	0	F2F	0	0	0.816	
2,500	g_1	0	0	Ph	0	F2F	Ph	0.688	0.749
	g_2	F2F	F2F	0	F2F	Ph	0	0.849	
2,750	g_1	0	0	Ph	F2F	F2F	Ph	0.692	0.752
	g_2	F2F	F2F	F2F	F2F	F2F	Ph	0.851	
3,250	g_1	F2F	0	Ph	0	0	Ph	0.745	0.782
	g_2	F2F	F2F	0	F2F	0	0	0.842	
3,500	g_1	F2F	0	Ph	F2F	F2F	Ph	0.754	0.791
	g_2	F2F	F2F	F2F	F2F	F2F	Ph	0.851	
4,000	g_1	F2F	F2F	Ph	F2F	F2F	Ph	0.757	0.793
	g_2	F2F	F2F	F2F	F2F	F2F	Ph	0.851	

TABLE 3.6: Optimal solution two-group optimization for various values of B.

Budget	1,000	1,250	1,750	2,250	2,500	2,750	3,250	3,500	4,000
Runtime (seconds)	16.2	17	52.7	54.2	55	55.5	55.9	55.8	19

TABLE 3.7: Computational times.

nonconvexity prohibited convergence to the global optimum. We omit presentation of computational times for these tools since the optimal solution was only a local optimum.

3.5 Concluding remarks

This chapter dealt with the formulation of the RAP in the context of adaptive survey designs. While survey organizations across the world are confronted with the same issues of decreasing survey quality with increasing costs, the literature in the field does not address resource allocation models. A good reason for this is the intricacy of processes involved in a survey design. Moreover, great responsibility comes with designing large surveys with potentially great impact on economic policies such as the Labor Force Survey that estimates population's employment status.

We start by analyzing a simpler version of the problem, where we focus only on survey mode, number of allowed attempts and time slot as design features that can be tailored to the subpopulations. In this setting, optimizing the resource allocation translates to choosing a sequence of time slots such that the response rate is maximized given the contact and participation probabilities for each group, each time slot, and all available interview modes. The history of past actions that has to be considered at each step when choosing an action is a complex nonlinear term in the objective function. Therefore, the problem formulation leads to a NCMINLP which is non-scalable and non-tractable even at a small scale.

Although currently there are no standard methods to solve such a problem we present an algorithm that solves the problem exactly by exploiting the structure of the survey design problem, i.e., additivity of the objective function across time slots. The idea is to use a Markovian decision formulation of the problem, in which the state space is extended such that the contact failure probability is included in the state. Thus, there is no need to store the entire configuration of past actions. Via dynamic programming the new formulation is solved to optimality.

The model can easily be extended to accommodate budget and capacity constraints. The main advantages of this method are guaranteed optimality and short computational times. Thus, the model can be successfully used as a basis for representation of more complex practical settings. The following chapter discusses some necessary adjustments for settings where the additivity property is no longer satisfied.

4

The survey resource allocation problem for multiple quality indicators

The previous chapter studied the RAP in the context of adaptive survey designs where the response rate (i.e., weighted response across population groups) is maximized subject to constraints on budget and capacity availability. As illustrated also in the second example in Section 3.4, groups with low response probabilities may not be approached when resources are scarce. When maximizing the response rate, resources get directed towards collecting response from sample units that have a high response-cost ratio while disregarding those that have lower response probabilities and/or can cause higher costs. This approach is termed “going for the lowest hanging fruit” due to the little effort invested in obtaining such response. The negative impact of such an approach on the quality of a survey is the possibility of introducing bias in survey estimates when respondents are different from nonrespondents. Best known as *nonresponse bias*, the problem of bias in survey estimates due to nonresponse has kept survey researchers busy for many years (for a synopsis see Groves et al. 2002, Groves and Peytcheva 2008 and Bethlehem et al. 2011). As explained above, response rates alone can be deceiving indicators of the survey quality. Consequently, maximizing the response rate is not a sufficient condition for a high-quality survey. Schouten et al. (2009) introduce the representativity indicator, a measure of similarity of the respondent sample to the initial sample, that could help assess overall survey quality.

In this chapter, we extend the RAP formulation to address the nonresponse bias by imposing a minimum level of representativity of the respondent sample. However, the new constraint, by the definition of the representativity indicator, does *not* possess the additivity property which prohibits application of the RAP algorithm described in Chapter 3. Nevertheless, there is an approach to solve the problem exactly. We devise a two-step algorithm that maximizes the group response rate given available resources in the first step and balances the distribution of resources in the second step such that a representative respondent sample is obtained. Numerical experiments on real data, coming from the Dutch Labor Force Survey, are presented in Section 4.3.

4.1 Problem formulation

Despite the continuous effort of researchers, survey methodology has yet to present a general indicator for survey quality. Currently, the survey response rate is still largely used due to its comparability across surveys. However, in the recent years, other quality indicators have been developed to supplement the imperfect information about nonresponse bias provided by the response rates. Such an indicator is the representativity indicator (R-indicator) described in SCHOUTEN ET AL. (2009) that aims at quantifying the quality of response composition across a set of characteristics, such as age, gender, and ethnicity. Such characteristics should be available for both respondents and nonrespondents which means that external sources of data, e.g., government registries, should be employed. If a respondent sample is as close as possible to a simple random sample of the survey sample, or in other words, if respondents and nonrespondents cannot be distinguished with respect to the selected set of characteristics, then the respondent sample is considered representative. As a consequence, the response is balanced with respect to the set of characteristics which lowers the chance for bias in the survey estimates. Using the notation in Section 3.1 we express the R-indicator as

$$R = 1 - 2\sqrt{\sum_{g \in \mathcal{G}} w_g(\rho_g - \bar{\rho})^2}, \tag{4.1}$$

where ρ_g is the response rate for group g and $\bar{\rho}$ is the weighted response rate over groups, and sample units are clustered in groups given the selected set of characteristics. The R-indicator judges the quality of response by measuring the standard deviation of response probabilities across groups. When the standard deviation is low, the representativity is high. Since the standard deviation can be minimally 0, i.e., ρ_g is constant over all groups, it follows that the R-indicator can be maximally 1. In order to compute the minimal value note that for a given value of $\bar{\rho}$ the maximum variation in response is obtained by letting $\bar{\rho}N$ of the response probabilities be equal to 1 and $(1-\bar{\rho})N$ be equal to 0, where N represents the population size (see also COBBEN 2009). We can then write the following

$$\sqrt{\sum_{g \in \mathcal{G}} w_g(\rho_g - \bar{\rho})^2} \leq \sqrt{\bar{\rho}(1-\bar{\rho})}.$$

It it easy to see that the right-hand side of the inequality is a concave function on $[0,1]$ that attains its maximum at $\frac{1}{2}$ for $\bar{\rho} = \frac{1}{2}$. Thus,

$$\sqrt{\sum_{g \in \mathcal{G}} w_g(\rho_g - \bar{\rho})^2} \leq \frac{1}{2}.$$

It follows that

$$R \geq 1 - 2 \cdot \frac{1}{2} = 0,$$

that is, the minimal value of the R-indicator is 0. In order to obtain a representative respondent sample, the effort for collecting response has to be balanced across the sample. That is, resources should be directed towards groups that are underrepresented in the respondent sample. Thus, attempts to collect more response would also provide a more balanced response. If high response representativity is coupled with a high response rate $\bar{\rho}$, then the level of quality in the survey is also assumed to be high.

Bounding the R-indicator from below provides means to guarantee better quality of the response. Any value below the selected threshold would suggest that additional effort has to be invested in underrepresented groups. Imposing a minimum level for the R-indicator comes down to formulating the following constraint

$$1 - 2\sqrt{\sum_{g \in \mathcal{G}} w_g (\rho_g - \bar{\rho})^2} \geq \alpha, \tag{4.2}$$

with $\alpha \in [0, 1]$. In practice, $\alpha \geq 0.5$. In the context of the RAP formulation, such a constraint will balance the distribution of resources such that all groups have a similar contribution to the response. We will further refer to this constraint as *the representativity constraint*. We can now formulate the extended RAP for survey designs as follows

$$\begin{aligned}
\max \;\; & \bar{\rho} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} w_g \, f_g(t-1) \, x_g(t,m) \, p_g(t,m) \, r_g(t,m) \\
\text{s.t.} \;\; & \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} N_g b_g(t,m) \leq B, \\
& \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} N_g c_g(t,m) \leq C, \\
& \sum_{m \in M} x_g(t,m) \leq 1, \;\; \forall g \in \mathcal{G}, \;\; \forall t \in \mathcal{T}, \\
& 1 - 2\sqrt{\sum_{g \in \mathcal{G}} w_g (\rho_g - \bar{\rho})^2} \geq \alpha, \\
& x_g(t,m) \in \{0,1\}, \;\; \forall g \in \mathcal{G}, \;\; \forall t \in \mathcal{T}, \;\; \forall m \in \mathcal{M},
\end{aligned} \tag{4.3}$$

where we simply add the representativity constraint to the problem presented in (3.12). Note that $f_g(t)$, $b_g(t,m)$ and $c_g(t,m)$ are defined as in (3.12), but their definition is omitted here for clarity.

The representativity constraint bounds the difference between group response rates ρ_g and the weighted response rate $\bar{\rho}$, quantities that cannot be known until after the optimization of resource allocation finishes. Hence, the representativity constraint does not possess the additivity property (presented in Remark 3.2) and the problem cannot be cast directly as a Markov decision problem. This prohibits the application of the RAP algorithm. Furthermore, the structure of the constraint and the presence of nonconvexities in the objective function are reasons to avert the application of existing tools. Additionally, convex approximations, though theoretically convenient, are not recommended for practical reasons since any optimality gap could lead to additional errors in the survey estimates. However, there is an approach to solve the problem exactly. In the following section we present an algorithm that handles the representativity constraint iteratively, given group response rates ρ_g optimized through the RAP algorithm.

4.2 The two-step algorithm

The previous section introduced the extended RAP for survey designs where a constraint on representativity of the respondent sample was added to the RAP formulation in (3.12). Note that the objective function is not convex and the feasible region is not convex either. Hence, most of the existing algorithms are not applicable since their convexity prerequisite is not fulfilled. Moreover, for realistic sizes of the problem, e.g., $T = 10$, $M = 3$, $G = 5$, branch-and-bound algorithms displayed long computational times or even failed to execute.

The approach we propose does not solve the problem directly but iteratively. Note that in the absence of the representativity constraint the problem can be fully decomposed at group level. This can be achieved by removing the summation over $g \in \mathcal{G}$ in both objective function and constraints. Namely, the group response rate ρ_g is maximized subject to constraints on the group budget B_g and capacity C_g. This is possible since, as remarked in Section 3.3, the budget B (and capacity C, respectively) can easily be divided into budgets B_g (capacities C_g) for each group g. Thus, we obtain a reduced version of the initial RAP that can be solved using the RAP algorithm. With the optimal values for ρ_g, $\forall\, g \in \mathcal{G}$ we can now compute $\bar{\rho}$ and verify whether the representativity constraint is satisfied. What still remains to discuss is the division of the budget and capacity over groups. To address this issue we build an additional optimization problem that maximizes the overall response rate $\bar{\rho}$ subject to the representativity constraint and the distribution of resources, e.g., budget and capacity, among the G groups. Our algorithm thus iterates over possible resource distributions and at each iteration it performs two optimization steps. At step 1 it solves G group RAPs using the RAP algorithm and the current resource distribution. At step 2 it verifies the representativity constraint given group response rates ρ_g obtained at step 1 and if it is satisfied then it updates the value of $\bar{\rho}$ if the current resource distribution yields a higher response rate.

Denote by λ_g and β_g the fractions of budget and capacity that each group $g \in \mathcal{G}$ is allocated. In the group-level problem, the right-hand side of the budgetary and capacity constraints will then be replaced by $\lambda_g B$ and $\beta_g C$, respectively. We can then consider the group-level problem as a complex function with variables λ_g and β_g that maps values of the two variables to the group response rate ρ_g. The formulation of the group-level problem is given by

$$\begin{aligned} \rho_g(\lambda_g, \beta_g) = \quad \max \quad & \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m) \\ \text{s.t.} \quad & \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} N_g b_g(t,m) \leq \lambda_g B, \\ & \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} N_g c_g(t,m) \leq \beta_g C, \\ & \sum_{m\in M} x_g(t,m) \leq 1, \ \forall t \in \mathcal{T}, \end{aligned} \tag{4.4}$$

with $f_g(t)$ given by (3.2) and $b_g(t, m)$, $c_g(t, m)$ given by equations (3.8) and (3.11). The optimal solution of (4.4) is obtained using the RAP algorithm.

The second step of our algorithm addresses the response representativity constraint. We formulate a second optimization problem (further referred to as the *master problem*) that uses the obtained group response rate $\rho_g(\lambda_g, \beta_g)$ as input and maximizes the overall response rate subject to the response representativity constraint (4.2) and requirements on distribution of budget and capacity among groups. The decision variables are λ_g and β_g, $\forall g \in \mathcal{G}$. The master problem can be drafted as follows

$$\begin{aligned} \max \quad & \bar{\rho} = \sum_{g\in\mathcal{G}} w_g\, \rho_g(\lambda_g, \beta_g) \\ \text{s.t.} \quad & \sum_{g\in\mathcal{G}} \lambda_g \leq 1, \sum_{g\in\mathcal{G}} \beta_g \leq 1 \\ & 1 - 2\sqrt{\sum_{g\in\mathcal{G}} w_g \big[\rho_g(\lambda_g, \beta_g) - \bar{\rho}\big]^2} \geq \alpha, \\ & \lambda_g, \beta_g \in [0,1], \ \forall g \in \mathcal{G}, \\ & \rho_g(\lambda_g, \beta_g) = \text{solution of the group-level problem}, \forall g \in \mathcal{G}, \end{aligned} \tag{4.5}$$

where we explicitly rewrite the objective function and the R-indicator as functions of $\rho_g(\lambda_g, \beta_g)$.

The two-step algorithm starts by solving G group-level problems for an initial pair of values for λ_g and β_g, for all $g \in \mathcal{G}$. We discuss later the choice of the starting point. With the obtained group response rates $\rho_g(\lambda_g, \beta_g)$, we move to the second step. If the response representativity constraint is not met, we proceed to the next iteration of the algorithm, where G group-level problems are solved for new values of λ_g and β_g. The algorithm continues until the optimum is found.

However, given the complex relationship between objective function $\bar{\rho}$ and decision variables λ_g and β_g, coupled with the dependence of the response representativity constraint on the objective function value at the current point (λ_g, β_g), finding a global optimum will require tremendous computational effort.

To circumvent this problem, we discretize the initial domain $[0, 1]$ for both λ_g and

β_g, for all $g \in \mathcal{G}$. Thus, the domain of values becomes a multidimensional cloud of points, whose density depends on the discretization step. When a new iteration of the two-step algorithm is performed, a new point is selected from the cloud. Once all the points have been visited, the algorithm finishes. The solution thus obtained is a point $(\lambda_g, \beta_g)_{g\in\mathcal{G}}$ that yields the maximum value for $\bar{\rho}$ and meets the constraints of the master problem (4.5). Thus, given a cloud of points obtained through some discretization step of the initial domains of values, the two-step algorithm is guaranteed to achieve the best solution in the cloud.

However, this extensive search could prove computationally infeasible. In the following we show that the domain of values $[0, 1]$ can be significantly reduced, i.e., a significantly large number of points from the cloud can be discarded, which makes the problem tractable also for realistic sizes.

Proposition 4.1. *λ_g is bounded from below by $\lambda_g^{min} = \min_{m\in\mathcal{M}} \frac{1}{B} N_g b_g(t, m)$ and from above by $\lambda_g^{max} = \frac{1}{B} \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} N_g b_g^*(t, m)$ for $b_g^*(t, m)$ computed given the optimal solution from (4.4) with $B_g = B$.*

Proof. Note that for the first time slot t where $x_g(t, m) = 1$, the entire group g must be approached for survey (i.e., N_g sample units). This generates $N_g b_g(t, m)$ costs for each $m \in \mathcal{M}$. The minimum necessary budget for an approach is given by $B_g^{min} = \min_{m\in\mathcal{M}} N_g b_g(t, m)$. Let $\lambda_g^{min} = \frac{B_g^{min}}{B}$, the proportion of budget that is necessary to cover these minimum costs for an approach. Any λ_g lower than λ_g^{min} will result in a group response rate of 0 since it is too expensive to approach group g. Therefore, we can bound λ_g from below by λ_g^{min} since lower values do not bring better solutions.

Note that from a level λ_g^{max}, allocating additional budget to group g does not bring further improvement in the group response rate. In other words, given the length of the survey fieldwork and the input probabilities (i.e., contact and participation), $\lambda_g^{max} B$ is the necessary budget to achieve maximum response rate for group g. One can compute this maximum budget level as follows. Replace $\lambda_g B$ in (4.4) by B and solve the corresponding problem instance. Let B_g^{max} be the total costs incurred by the optimal solution thus obtained. Take $\lambda_g^{max} = \frac{B_g^{max}}{B}$ to be the fraction of total budget necessary to recover the maximum response rate from group g. Therefore, we can bound λ_g by λ_g^{max} since any other larger value will not bring a better solution. □

Thus, we can reduce the initial domain of values $[0, 1]$ to $[\lambda_g^{min}, \lambda_g^{max}]$. Analogously, the domain of values for β_g can be adjusted to $[\beta_g^{min}, \beta_g^{max}]$. To keep the master problem feasible we allow for the trivial solution (i.e., $\lambda_g = 0,\ \beta_g = 0,\ \forall g \in \mathcal{G}$), although it is an unrealistic solution. The algorithm steps are illustrated in pseudocode in Algorithm 3 where only the budget constraint is considered.

Algorithm 3 Two-step algorithm for resource allocation in survey designs

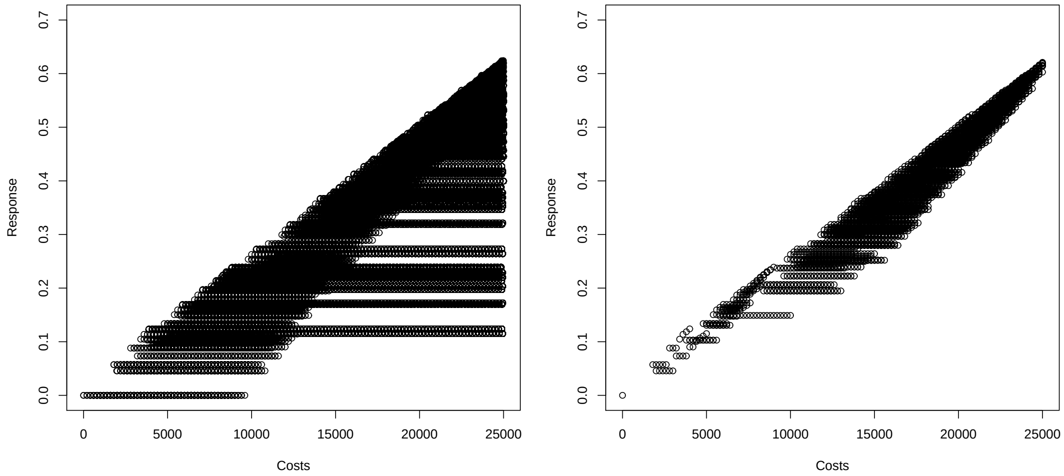
INITIALIZE
$\bar{\rho}^{current} \leftarrow 0$
$\left(x_g^{current}(t,m)\right)_{g,t,m} \leftarrow 0$
START
for all $\lambda_g \in [\lambda_g^{min}, \lambda_g^{max}]$ **do**
  solve (4.4) for λ_g and obtain optimal solution $\left(x_g^*(t,m)\right)_{g,t,m}$ ▷ Step 1
  if $\sum_{g\in\mathcal{G}} \lambda_g = 1$ and $R(\lambda_g) > \alpha$ **then** ▷ Step 2
    compute $\bar{\rho}$
    if $\bar{\rho} > \bar{\rho}^{current}$ **then**
      $\bar{\rho}^{current} \leftarrow \bar{\rho}$
      $\left(x_g^{current}(t,m)\right)_{g,t,m} \leftarrow \left(x_g^*(t,m)\right)_{g,t,m}$
    end if
  else
    jump to next iteration
  end if
end for
END
SOLUTION ▷ Best solution found
$\left(x_g^*(t,m)\right)_{g,t,m} \leftarrow \left(x_g^{current}(t,m)\right)_{g,t,m}$
$\bar{\rho}^* \leftarrow \bar{\rho}^{current}$

From numerical experiments, it follows that discretizing the reduced domains of values may trim off large sections of the cloud (more than 80% of points are discarded). Figure 4.1 compares projections of the multidimensional cloud of points before and after adjustment, for an instance of the problem where $G = 3$, $M = 2$, $T = 10$, the available budget is $B = 25{,}000$, capacity is unlimited (i.e., the capacity constraint does not play a role in defining the feasible region) and $\alpha = 0.5$. In this particular instance, 96.5% of the points are discarded. Figure 4.1a displays the realized response rate $\bar{\rho}$ when $\sum_{g\in\mathcal{G}} \lambda_g B$ is spent (i.e., only feasible combinations of λ_g were plotted), discretized with a discretization step of magnitude 10^{-3}. Figure 4.1b also shows the realized response rate $\bar{\rho}$ when $\sum_{g\in\mathcal{G}} \lambda_g B$ is spent, where this time the adjusted interval $[\lambda_g^{min}, \lambda_g^{max}]$ was discretized with a discretization step of magnitude 10^{-3}.

Remark 4.1. Given the notable cloud reductions obtained by adjusting from $[0,1]$ to $[\lambda_g^{min}, \lambda_g^{max}]$, we have not considered additional space reduction methods.

Numerical experiments show that the objective function in (4.5) does not display spikes (see again Figure 4.1). From here we can derive

Conjecture 4.1. *The discretization step need not be very small.*



(A) Initial set of discretized values of λ_g (B) Adjusted set of discretized values of λ_g

FIGURE 4.1: Visualization of the objective value as a function of the incurred costs.

For example, if we take two consecutive values of λ_g from the set of discretized values and the corresponding values of the response rate for group g are the same, then any value of λ_g in between the selected values will yield the same contribution of group g to the objective function $\bar{\rho}$. Therefore, we do not need to consider these intermediate values.

For additional accuracy, the algorithm can be applied again for $\lambda_g \in [\lambda_g^{optim}, \lambda_g^{max}]$ and $\beta_g \in [\beta_g^{optim}, \beta_g^{max}]$, for a decreased discretization step. However, from our numerical experiments, the gain in the objective function value is not significant. For example, for the problem instance considered above, the increase in the objective value is only of 1% when we decrease the magnitude of the discretization step to 10^{-4} but the runtime increased by 60%. Practitioners would consider the additional computational time rather damaging than useful given the relatively small increase in the response rate.

We can now discuss the choice of the starting point. Note that if $\sum_{g \in \mathcal{G}} \lambda_g^{max} \leq 1$, $\sum_{g \in \mathcal{G}} \beta_g^{max} \leq 1$ and the response representativity constraint is met, the global optimum is found and the resource allocation problem is solved, without applying the algorithm. Thus, choosing $(\lambda_g^{max}, \beta_g^{max}), \forall g \in G$ as starting point could shortcut the entire algorithm and yield instantaneously the optimal solution. When any of the constraints above are not met, the starting point of the algorithm is $(\lambda_g^{min}, \beta_g^{min})$. In case also $\sum_{g \in \mathcal{G}} \lambda_g^{min} > 1$ or $\sum_{g \in \mathcal{G}} \beta_g^{min} > 1$, the trivial solution is chosen as a starting point.

The group-level problems solved in the first step are independent of each other since the overall constraints are active only in the master problem. As a result, implementation of parallel computing techniques becomes possible, which improves significantly the computational times and can accommodate large-scale problems. Adjusting the domain of values for λ_g and β_g brings another important reduction in the computational times

Mode	Probability	Group		
		15-25	26-55	56-65
$m = 1$	$p_g(1)$	0.261	0.303	0.434
	$r_g(1)$	0.594	0.651	0.69
$m = 2$	$p_g(2)$	0.392	0.367	0.461
	$r_g(2)$	0.594	0.651	0.690

TABLE 4.1: Estimated probabilities for contact and cooperation (LFS 2008).

and thus the enumeration of values for realistic problems can be handled.

The two-step algorithm takes advantage of the Markov formulation in handling the main nonlinear component of the problem (the contact failure probability $f_g(t)$) in the first step and addresses the response representativity constraint in the second step.

Remark 4.2. Other survey quality indicators can be included in the RAP formulation in a similar manner to the response representativity constraint. Chapter 5 offers an example in this direction by adding a constraint on the maximum accepted degree of manifestation of measurement errors in ASDs.

4.3 Numerical examples

The Labor Force Survey (LFS) is one of the most important surveys in any country. It aims at estimating the unemployment rate for the population that is legally allowed to work. Our case study builds on data from the Dutch LFS from 2008, with a survey sample of size $N = 10{,}000$. From this survey we estimate the input parameters for our model, i.e., the contact and cooperation probabilities.

The Dutch LFS targets people with age between 15 and 65. We consider age as our criterion for clustering the sample units. Note that in the Netherlands access to external sources for information such as age, ethnicity, house value is allowed. Therefore, criteria for clustering can be multiple, leading to an increased degree of homogeneity within the cluster. However, for illustration purposes, we restrict to age and we split the sample in three groups, namely $\mathcal{G} = \{15-25, 26-55, 56-65\}$. The proportion of the three groups in the sample is $w = (0.196, 0.624, 0.18)$. The set of modes is $\mathcal{M} = \{0, 1, 2\}$, where 0 denotes that the sample unit is not approached. The survey fieldwork is divided in $T = 10$ time slots. The contact and cooperation probabilities are given in Table 4.1.

In practice, establishing contact with the sample unit uses most of the resources. Therefore, survey organizations pay close attention to understanding the factors that drive the contact rate. For this reason, we would like to capture the influence of the two modes on the contact rate by keeping the cooperation probabilities independent of the mode. For simplicity, we consider that the cost of a survey approach is independent of the mode and outcome, i.e., $b^s(m) = b^{f_r}(m) = b^{f_c}(m) = 1$. Additionally, we assume that enough

Budget \ α	0	0.5	0.7	0.75	0.8	0.85	0.9
2,500	5.8 (0.756)	5.8 (0.756)	5.8 (0.756)	5.8 (0.815)	4.6 (0.815)	0 (1)	0 (1)
5,000	13.4 (0.621)	13.4 (0.621)	10.3 (0.73)	5.7 (0.756)	4.6 (0.815)	0 (1)	0 (1)
7,500	20.4 (0.464)	18.9 (0.514)	14.9 (0.769)	14.9 (0.769)	4.6 (0.815)	0 (1)	0 (1)
10,000	26.8 (0.59)	26.8 (0.59)	25.2 (0.938)	25.2 (0.938)	25.2 (0.938)	25.2 (0.938)	25.2 (0.938)
12,500	31.9 (0.653)	31.9 (0.653)	31.1 (0.802)	31.1 (0.802)	31.1 (0.802)	28 (0.889)	25.2 (0.938)
15,000	38.6 (0.615)	38.6 (0.615)	37.7 (0.839)	37.7 (0.839)	37.7 (0.839)	34.6 (0.876)	25.2 (0.938)
17,500	44.1 (0.768)	44.1 (0.768)	44.1 (0.768)	44.1 (0.855)	43.9 (0.855)	43.9 (0.855)	42.2 (0.917)
20,000	50.5 (0.718)	50.5 (0.718)	50.5 (0.718)	50 (0.804)	50 (0.804)	49.8 (0.919)	49.8 (0.919)
22,500	56.5 (0.884)	56.5 (0.884)	56.5 (0.884)	56.5 (0.884)	56.5 (0.884)	56.5 (0.884)	56.2 (0.934)
25,000	62.4 (0.924)	62.4 (0.924)	62.4 (0.924)	62.4 (0.924)	62.4 (0.924)	62.4 (0.924)	62.4 (0.924)
27,500	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)
30,000	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)	64.2 (0.939)

Table 4.2: Overview optimization results.

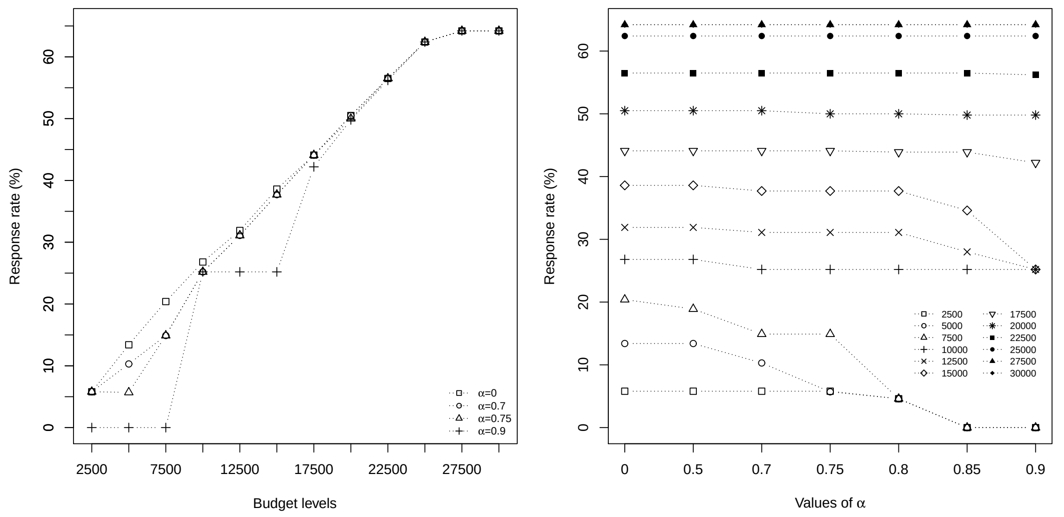
capacity is available. The optimal solution of the current problem will display the choice of the survey mode (or no approach) at each time slot such that constraints are met and the overall response rate is maximized.

Remark 4.3. The simplifying assumptions described above may seem improbable from a practical point of view. The numerical setup can be easily extended to address time and mode dependent costs, contact and cooperation probabilities. However, for illustration purposes, we prefer, a simplified example for illustration purposes.

We consider a varying budget with values between $B_{min} = 2{,}500$ and $B_{max} = 30{,}000$ and a minimum level of response representativity $\alpha \in \{0, 0.5, 0.7, 0.75, 0.8, 0.85, 0.9\}$. Table 4.2 presents the response rate (in %) and the corresponding R-indicator in brackets obtained for various combinations of budget and α values.

From Table 4.2 we see that for $B = 27{,}500$ and $B = 30{,}000$ the response rate and R-indicator do not change. Moreover, further increase in the budget level does not bring additional improvement. It follows that, given the input parameters, at most 64.2% of the sample is estimated to respond to the survey. The total costs necessary to obtain this level of response amount to approximately 26,011. In the case of $\alpha = 0$, the solution for $B > 26{,}011$ is the same as the solution to the unconstrained problem (i.e., maximize the response rate when budget is unlimited).

The R-indicator levels for $\alpha = 0$ can provide a good hint as to whether the optimal



(A) Response rate as a function of budget (B) Response rate as a function of α

FIGURE 4.2: Impact of the representativity constraint on the response rate.

solutions change for higher values of α. For high levels of budget (e.g., $B \geq 25{,}000$), the R-indicator is already above the 0.9 threshold. Hence, the optimal solution will remain the same for any $\alpha \leq 0.9$. For $B < 25{,}000$ on the other hand, once α rises above the level the R-indicator has for $\alpha = 0$, the optimal solution shifts towards increasing the effort to obtain response from more reluctant groups while the more responsive groups are abandoned due to the low budget. Thus, the overall response rate decreases. The response rate reaches 0 for those cases where $\alpha > 0.8$ and $B \leq 7{,}500$. For $B < 1{,}800$ the response rate will be 0, regardless of the value of α since this is the minimum budget necessary to perform a survey approach.

The differences in the response rates are also depicted in Figure 4.2. Here, the response rate is displayed as a function of the available budget (in Figure 4.2a) and as a function of α, the minimum required level of representativity (in Figure 4.2b).

Table 4.3 displays the optimal solutions for the indicated values of budget and levels of response representativity. The pattern in the optimal solutions can be guessed from the input parameters. Given the mode independence in the cooperation probabilities, the mode with a higher contact rate (i.e., mode 2) will be preferred. Investigating the group contact probabilities, we remark that age group 56-65 is expected to have a great influence on the response rate, since it is the most responsive group, and age group 26-55 has a great impact on costs, given that it is the largest group in the sample. As a result of its low contact and cooperation probabilities, age group 15-25 can be viewed as the reluctant group of the sample.

All optimal solutions per group, further referred to as group policies, display the same pattern in the mode choices, namely use survey mode 2 or do not approach. The number

Input	Group	Time slot										Response
		1	2	3	4	5	6	7	8	9	10	rate (%)
$B = 20{,}000$ $\alpha = 0$	15-25	2	0	0	0	0	0	0	0	0	0	23.3
	26-55	2	2	2	2	0	0	0	0	0	0	54.7
	56-65	2	2	2	2	2	0	0	0	0	0	65.9
$B = 25{,}000$ $\alpha = 0$	15-25	2	2	2	2	2	2	0	0	0	0	56.4
	26-55	2	2	2	2	2	2	2	0	0	0	62.4
	56-65	2	2	2	2	2	2	2	2	2	2	68.9
$B = 30{,}000$ $\alpha = 0$	15-25	2	2	2	2	2	2	2	2	2	2	59
	26-55	2	2	2	2	2	2	2	2	2	2	64.4
	56-65	2	2	2	2	2	2	2	2	2	2	68.9
$B = 20{,}000$ $\alpha = 0.8$	15-25	2	2	0	0	0	0	0	0	0	0	37.4
	26-55	2	2	2	0	0	0	0	0	0	0	48.6
	56-65	2	2	2	2	2	2	2	2	0	0	68.5
$B = 20{,}000$ $\alpha = 0.85$	15-25	2	2	0	0	0	0	0	0	0	0	37.4
	26-55	2	2	2	0	0	0	0	0	0	0	48.6
	56-65	2	2	2	0	0	0	0	0	0	0	58.2
$B = 20{,}000$ $\alpha = 0.9$	15-25	2	2	0	0	0	0	0	0	0	0	37.4
	26-55	2	2	2	0	0	0	0	0	0	0	48.6
	56-65	2	2	0	0	0	0	0	0	0	0	50

TABLE 4.3: Optimal solution.

of survey approaches (or calls) per group is determined mostly by the potential of that group. If the group is responsive and not expensive (e.g., age group 56-65), the number of calls tries to reach its maximum (i.e., 10). If the group is expensive and not responsive (e.g., age group 15-26), the number of calls goes to 0.

The situation changes when the representativity constraint becomes influential. In this case, group policies with 0 calls are not accepted. In other words, the number of calls to responsive groups has to be decreased in order to preserve resources to call the more reluctant groups. This shift in policies is particularly visible in the case of age group 56-65 for $B = 20{,}000$ when $\alpha \in \{0.8, 0.85, 0.9\}$. The group policy for $\alpha = 0.8$ indicates a total of 8 calls. When α is increased, the group response rates have to become more balanced. This leads to a decreased number of calls for group 56-65 in order to shift resources towards the more expensive groups. As a consequence, the overall response rate goes down (see Table 4.2). The number of calls to the age group 26-55 remains the same due to either insufficiency in resources (i.e., the resources that are spared by decreasing the number of calls to group 56-65 are insufficient to cover additional calls to this group) or an increased unbalance in the group response rates would occur that would violate the representativity constraint.

4.4 Conclusions

In this chapter, we have addressed the RAP for adaptive survey designs when additional constraints are posed on the quality of the obtained response. The need for simultaneous monitoring of multiple indicators of survey quality comes from the imperfect information the response rate offers about the bias in survey estimates (see again GROVES

ET AL. 2002, BETHLEHEM ET AL. 2011). We extend the RAP formulated in Chapter 3 to include an additional quality indicator, namely the response representativity indicator (R-indicator). The R-indicator measures the variation in response rates between population groups that form the final respondent sample. The lower this variation is, the higher the degree of resemblance is achieved between the respondents and the entire sample to survey, leading thus to a higher quality in the survey results. However, this indicator is a nonlinear function of the group response rates and the overall response rate and it does not posses the additivity property. Thus, the application of the RAP algorithm is impeded.

We propose a two-step algorithm that solves the problem in an iterative manner. We reformulate the extended RAP such that the groups become independent of each other (several group-level problems, the number of which equals the number of groups). A master problem assigns resources to the groups and also considers the R-indicator constraint. Thus, by removing the response representativity constraint from the group problem, we can apply the RAP algorithm per group. In the second step of the algorithm, the master problem checks whether the solutions from the group-level problems satisfy the response representativity constraint. When this is not the case, a new allocation of resources is selected and the algorithm goes back to step one for another iteration. The algorithm stops when all feasible allocations have been checked and the allocation that yields the maximum response rate while satisfying the response representativity constraint represents the optimal solution. By careful adjustment of the set of feasible resource allocations and implementation of parallelization techniques, the algorithm gains a significant increase in speed. The master problem does not exploit in any way the structure of the response representativity constraint, therefore, other quality indicators can be addressed by simple addition of a corresponding constraint to the master problem. Chapter 5 provides a problem setup in this sense.

There are however few details that require additional consideration. The choice of discretization step for the domain of values of λ_g and β_g (i.e, the budget and capacity proportions available to each group problem) may "hide" the global optimum. However, a tradeoff must be made between refining the discretization step and the resulting increase in computational time. From our numerical experiments, it seems likely that the increase in computational time outweighs the additional gain in objective value. Another aspect is the size of the state space. A notable reduction was obtained when adjusting the domain of values from $[0, 1]$ to $[\lambda_g^{min}, \lambda_g^{max}]$, where λ_g^{min} represents the minimum fraction of budget required to obtain a strictly positive group response rate and λ_g^{max} the budget fraction that allows achievement of maximum group response rate. We have not explored additional state space reduction methods. A combination of such methods could potentially eliminate a larger number of suboptimal resource allocations, offering thus the possibility of refining the discretization step while keeping the computational time within reasonable limits (e.g., few hours).

5

The survey resource allocation problem and measurement errors

The emergence of web as a candidate survey mode has offered an increased potential to better resource planning through implementation of mixed-mode survey designs. Additional advantages may come from increased probability of contact and/or cooperation and more detailed recording of process information. As a consequence, survey organizations are considering switching from unimode to mixed-mode survey designs. However, there are some disadvantages to implementing mixed-mode designs, namely the impact of mode effects on survey estimates. Different modes lead to different responde levels and thus to different compositions of the respondent pools within the mode (see Bethlehem et al. 2011). In addition, due to concerns on security of sending information over the Internet, people may choose to alter their answers in web surveys. Thus, mode effects may dominate accuracy and comparability of statistics, see e.g., Jäckle et al. (2010) and Buelens et al. (2012). As a consequence, the discussion about addressing measurement errors in the context of mixed-mode designs has been amplified. Nonetheless, the interaction between nonresponse and measurement error in the context of multi-mode survey designs has not yet been studied extensively. Literature has shown recent interest in this area, see Olson (2007), Olson (2012) and Fricker and Tourangeau (2010), but additional research effort is imperative.

This chapter discusses the optimal resource allocation for an adaptive survey design in order to account for both measurement and nonresponse errors, which constitutes a novelty in the field. The main research question is how to include measurement errors in adaptive survey design optimization. The approach we adopt is based on what we termed *measurement profiles*, i.e., characteristics of respondents that may lead to undesirable response styles during the interview. Response styles are answering behaviors that are persistent through a considerable part of the interview, see, e.g., Tourangeau and Rasinski (1988), Krosnick (1991) and Baumgartner and Steenkamp (2001), and they influence negatively the quality of survey estimates. For example, the response style of social desirability is manifested when respondents offer a modified version of their true answer in order to be viewed favorably by others such as the interviewer. Thus, bias is introduced in the survey estimates.

We develop two models to handle measurement profiles within the RAP formulation. The first one combines the probability of response and the probability of a response style in a single quality objective function that replaces the response rate objective in the RAP formulation and the second one extends the RAP formulation with a constraint on the maximum acceptable proportion of respondents that show undesirable response styles. We compare the two models on real data from the Dutch Labor Force Survey.

5.1 Measurement errors in surveys: an introduction

While nonresponse is usually viewed as a type of error that affects the survey estimates as a whole, measurement errors are typically viewed as acting on single estimates. As a consequence, incorporating measurement errors in adaptive survey designs is not straightforward. A design may lead to an increase in measurement errors on some survey estimates but to a decrease on others. The conceptual difference between nonresponse and measurement errors is, however, not so big. Nonresponse also affects different estimates to different extents and both errors are conjectured to have common causes (see e.g., Groves 2006, Olson 2007 and Olson 2012). For this reason, we attempt to summarize measurement errors through response styles, and instead of defining measurement error on single survey estimates, we define a latent concept, the *measurement profile*.

Tourangeau and Rasinski (1988) introduced four phases followed by respondents in answering survey questions: *interpretation and comprehension*, *information retrieval*, *judgment*, and *reporting*. A measurement profile is a predisposition or mood of a respondent, defined by the person's characteristics such as age and ethnicity, that affects one or more of these four phases. A profile is persistent throughout the survey and may lead to certain response styles during the interview (for an overview of response styles see Baumgartner and Steenkamp 2001).

Measurement profiles may be derived from a mix of registry data and paradata observations, where paradata may be observations about the design of the survey, about the sampled persons or households outside the interview, e.g., about the area or the house of residence of the sampled person or household and about the answering behavior. Response styles can be derived from paradata and, in rare cases, from validation data available in registries. Figure 5.1 shows the subtle differences between measurement profiles and response styles. Measurement profiles consist of different types of information available at the outset of the survey such as the sampled person's characteristics (available from registry data) and interviewer's experience about the residential area. Response styles, on the other hand, become visible only during the survey fieldwork.

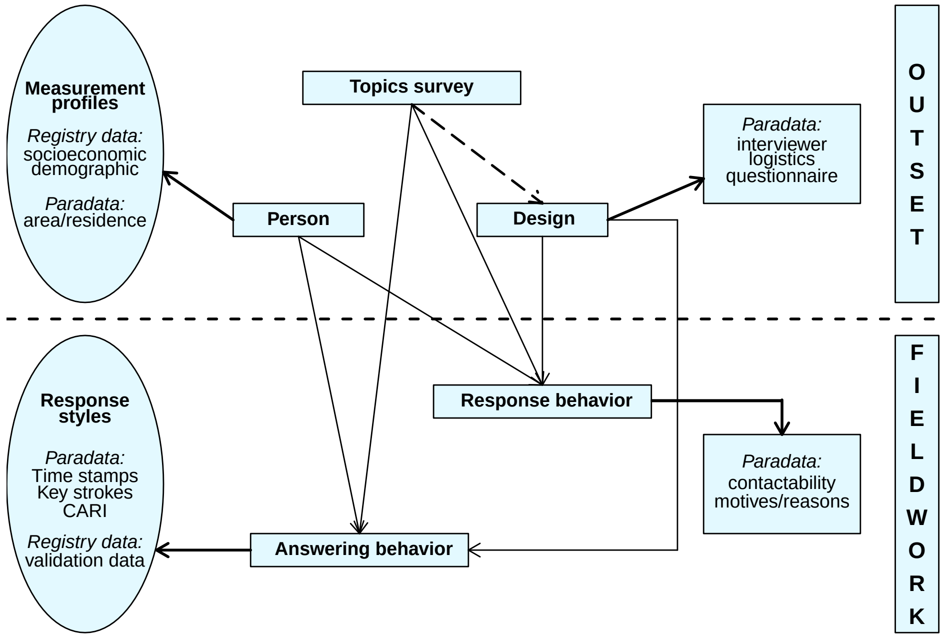


FIGURE 5.1: Measurement profiles vs. response styles.

The survey design features and the survey topics are also known at the outset of the survey fieldwork and most often the survey topics determine implicitly part of the design features, e.g., the presence of an interviewer or the length of the questionnaire. The various interactions between person's characteristics, survey design features and topics lead to certain response and answering behaviors that are observed during the survey fieldwork. Thus, given a set of design features and a set of characteristics, a person has a probability to show a certain response style. Measurement profiles are then characteristics that associate with a higher probability to show a response style. From fieldwork observations, additional knowledge can be gained for future surveys such as contact and cooperation patterns and validation of the registry data. Adaptive survey designs attempt to exploit the interactions between design features and person characteristics that mostly influence the response behavior.

In the following, we focus on manifestation of measurement errors for different types of reporting. A survey is by default run as self-report, i.e., every household member provides answers for themselves. However, when a household member is not available or incapable of participating, proxy-reporting can be used, i.e., other members of the household can provide answers for the unavailable member. On one hand, using proxy-reporting helps decreasing survey costs and increasing contact and participation rates. On the other hand, a proxy respondent may often be unaware or unfamiliar with the requested information about the other household members, which leads to decreased accuracy of the collected information and thus bias in the survey estimates. The obvious tradeoff is between allocating scarce resources and collecting inaccurate information. We analyze this tradeoff in the context of the Labor Force Survey which provides an estimate for the population unemployment rate.

5.2 Problem formulation

The RAP formulation in the previous chapters considers two survey design features, i.e., the survey mode and the contact protocol given by timing and number of contact attempts. Other design features such as interviewers with different skills, different advance letters and different types of reporting can be viewed as a survey mode with a specified contact and cooperation probability. For example, if contact and participation probabilities are provided for self- and proxy- reporting for all sample groups and time slots, the two types of reporting can simply be added to the set of survey modes. In this chapter, we extend the RAP formulation to address the quality of response obtained through different types of reporting. We use response styles to signal the potential presence of error in response. We view measurement profiles as characteristics that associate with a higher probability to show a response style.

As already suggested by the definition of the concept, occurrence of the response style is conditional on participation which is conditional on successful contact. Let RS_i be the 0-1 indicator for the response style for sample unit i. The response style probability, i.e., the probability that sample unit i shows the response style, is denoted by θ_i. Analogous to nonresponse, we can model the probability θ_i as a function of survey design features and person characteristics. Let $\theta_g(t,m)$ be the probability for finding a response style given cooperation at time slot t, survey mode m and population group g. We adopt two approaches to integrate the response style probability within the RAP formulation:

- Approach 1 - change objective function to include response style probability in the response rate. The new function expresses the proportion of sample units that respond without showing the specified response style. In other words, if the response style is observed, the response thus obtained is treated as a nonresponse. Therefore, the objective in this case is to maximize the response in the absence of the response style. Then, (3.4) becomes

$$\bar{\rho}^{ME} = \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)\,\big(1-\theta_g(t,m)\big). \tag{5.1}$$

- Approach 2 - an additional constraint on the proportion of respondents showing an undesirable response style. This proportion must be smaller than a specified threshold Θ. For example, if $\Theta = 4\%$, then on average the proportion of respondents showing an undesirable response style must be smaller than 4%. The constraint (further referred to as the *risk constraint*) is given by

$$\frac{\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)\, \theta_g(t,m)}{\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)} \leq \Theta. \tag{5.2}$$

Additionally, we transform the cost constraint (3.9) to account for the number of visits, i.e., the expected total number of visits is not allowed to be larger than a pre-specified value. Every survey approach "costs" one visit regardless of the outcome of that approach, which translates to $b^s(m) = b^{f_c}(m) = b^{f_r}(m) = 1$. For the sake of simplicity we assume that capacity is sufficient and thus discard the corresponding constraint.

In order to understand the factors that drive the optimal solution we conduct a gradual analysis. We start with a deterministic setting (Setting 0), where all sample units are allowed only one type of reporting and the average number of visits is fixed. In this case, no optimization is necessary. In Setting 1 we maximize the response rate subject to the choice of reporting type and number of visits. Setting 2 addresses measurement errors through Approach 1. Setting 3 merges Setting 1 and Approach 2 to address measurement errors. Setting 4 studies the impact of the response representativity constraint (4.2) on previous settings. Thus, the broadest problem formulation is as follows

$$\begin{aligned}
\max \quad & \bar{\rho}^{ME} = \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)\,\big(1-\theta_g(t,m)\big) \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} N_g b_g(t,m) \le B \\
& \sum_{m\in M} x_g(t,m) \le 1,\ \forall g\in\mathcal{G},\ \forall t\in\mathcal{T} \\
& 1 - 2\sqrt{\sum_{g\in\mathcal{G}} w_g(\rho_g^{ME} - \bar{\rho}^{ME})^2} \ge \alpha \\
& x_g(t,m)\in\{0,1\},\ \forall g\in\mathcal{G},\ \forall t\in\mathcal{T},\ \forall m\in\mathcal{M},
\end{aligned} \tag{5.3}$$

under Approach 1 and

$$\begin{aligned}
\max \quad & \bar{\rho} = \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m) \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} N_g b_g(t,m) \le B \\
& \sum_{m\in M} x_g(t,m) \le 1,\ \forall g\in\mathcal{G},\ \forall t\in\mathcal{T} \\
& 1 - 2\sqrt{\sum_{g\in\mathcal{G}} w_g(\rho_g - \bar{\rho})^2} \ge \alpha \\
& \frac{\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)\, \theta_g(t,m)}{\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)} \le \Theta \\
& x_g(t,m)\in\{0,1\},\ \forall g\in\mathcal{G},\ \forall t\in\mathcal{T},\ \forall m\in\mathcal{M}.
\end{aligned} \tag{5.4}$$

under Approach 2.

5.3 Problem solving technique

The problem formulations in (5.3) and (5.4) can be addressed using the two-step algorithm presented in Chapter 4. The application of the algorithm requires some preparatory steps.

Note that under Approach 1, the group-level problem (4.4) changes objective function in order to incorporate the response style probability. That is, by dropping the capacity constraint (given the assumed unlimited availability) and replacing the objective function, (4.4) becomes

$$\begin{aligned}
\max\ & \rho_g^{ME} = \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)\, \big(1-\theta_g(t,m)\big),\ \forall\, g \in \mathcal{G} \\
\text{s.t.}\ & \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} N_g b_g(t,m) \le B \\
& \sum_{m\in M} x_g(t,m) \le 1,\ \forall g \in \mathcal{G},\ \forall t \in \mathcal{T} \\
& x_g(t,m) \in \{0,1\},\ \forall g \in \mathcal{G},\ \forall t \in \mathcal{T},\ \forall m \in \mathcal{M},
\end{aligned} \tag{5.5}$$

Note that this formulation satisfies the additivity property that is a prerequisite for the application of the RAP algorithm. The master problem formulation addresses now the allocation of available visits to groups such that the response representativity constraint is met, i.e., (4.5) is reformulated as

$$\begin{aligned}
\max\ & \bar{\rho}^{ME} = \sum_{g\in\mathcal{G}} w_g\, \rho_g^{ME}(\lambda_g) \\
\text{s.t.}\ & \sum_{g\in\mathcal{G}} \lambda_g \le 1 \\
& 1 - 2\sqrt{\sum_{g\in\mathcal{G}} w_g \big[\rho_g^{ME}(\lambda_g) - \bar{\rho}^{ME}\big]^2} \ge \alpha \\
& \lambda_g \in [0,1],\ \forall g \in \mathcal{G} \\
& \rho_g^{ME}(\lambda_g) = \text{solution of the group-level problem (5.5)}, \forall g \in \mathcal{G}.
\end{aligned} \tag{5.6}$$

With this modification, the two-step algorithm can be applied right away.

Extending the RAP under Approach 2 to include the risk constraint requires more steps. First, we rewrite the risk constraint in (5.2) as follows

$$\sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)\, [\Theta - \theta_g(t,m)] \ge 0. \tag{5.7}$$

Note that in the master problem (5.6) the decision variables $x_g(t,m)$ are no longer present. The objective function and constraints are expressed in terms of ρ_g and λ_g for all $g \in \mathcal{G}$. That prevents adding the risk constraint (5.7) to the master problem. The alternative is to incorporate it in the group-level problem (4.4). By dropping the summation over g, we obtain

$$\rho_g^{RS} = \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)\, [\Theta - \theta_g(t,m)].$$

Note that ρ_g^{RS} possesses the additivity property which is the necessary condition for application of the RAP algorithm. However, by removing the summation over g, we cannot impose nonnegativity on ρ_g^{RS} anymore. To circumvent this issue, we first replace the objective function, the group response rate ρ_g, with ρ_g^{RS}. With this modification and dropping the capacity constraint given the assumed unlimited availability, (4.4) becomes

$$\begin{aligned} \max\ \ & \rho_g^{RS} = \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} w_g\, f_g(t-1)\, x_g(t,m)\, p_g(t,m)\, r_g(t,m)\, [\Theta - \theta_g(t,m)] \\ \text{s.t.}\ \ & \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}} N_g b_g(t,m) \le \lambda_g B \\ & \sum_{m\in M} x_g(t,m) \le 1,\ \forall t \in \mathcal{T} \\ & x_g(t,m) \in \{0,1\},\ \forall t\in\mathcal{T},\ \forall m \in \mathcal{M}. \end{aligned} \tag{5.8}$$

Second, we add the following constraint to the master problem,

$$\sum_{g\in\mathcal{G}} \rho_g^{RS} \ge 0.$$

Thus, the master problem becomes

$$\begin{aligned} \max\ \ & \bar{\rho} = \sum_{g\in\mathcal{G}} w_g\, \rho_g(\lambda_g) \\ \text{s.t.}\ \ & \sum_{g\in\mathcal{G}} \lambda_g \le 1 \\ & 1 - 2\sqrt{\sum_{g\in\mathcal{G}} w_g \left[\rho_g(\lambda_g) - \bar{\rho}\right]^2} \ge \alpha \\ & \sum_{g\in\mathcal{G}} \rho_g^{RS} \ge 0 \\ & \lambda_g \in [0,1],\ \forall g \in \mathcal{G} \\ & \rho_g(\lambda_g) = \rho_g^*\ \forall g \in \mathcal{G}, \end{aligned} \tag{5.9}$$

where ρ_g^* is obtained by applying (3.3) to the optimal solution $\left(x_g^*(t,m)\right)_{t,m}$ of the group-level problem (5.8). Thus, the risk constraint is addressed successfully and the problem can be solved by applying the two-step algorithm.

5.4 Case study: the Dutch Labor Force Survey

In this section, we describe the results of the experimental setup for implementing an ASD on data from the Dutch LFS from 2008 that accounts for both nonresponse and measurement errors. We define measurement profiles and model response style probabilities for different types of reporting using characteristics such as age, gender, ethnicity that are available in external registries. Hence, we implement a static ASD. Literature suggests that registry data may provide only partial explanation of different response styles. Paradata may be useful in supplementing the existing information. However, that will turn the design into a dynamic design, which requires additional theoretical considerations (see Chapter 7). For simplicity, we opted for implementation of a static design.

The response styles are defined based on observed differences between LFS response and registry data on employment. We expect at least two response styles, i.e., social desirability and satisficing. Social desirable answering behavior is manifested for respondents that feel that they should have a job and when they do not, they should be actively looking for a job. Satisficing behavior appears when respondents lack the motivation to answer follow-up questions, for example when they have multiple jobs they could shortcut the answering process by denying having additional jobs. The key issue is how to detect the two response styles.

In the Netherlands, there are two government registries that can provide information about the labor force. The POLIS register contains information about income from employment and social benefits; it does not however contain information on income from self-employment. From this register we can determine whether a person is employed and the number of jobs they have. The UWV werkbedrijf (UWV) assists people in finding a job. Unemployment benefits can be received only by those registered at UWV. Three main differences can be observed between survey answers and data available in POLIS and UWV, namely

- persons that are not employed according to the POLIS registration but respond in the LFS that they are;
- persons that are nor employed (POLIS) nor subscribed to an employment office (UWV) but respond in the LFS that they are registered at UWV;
- persons that are employed according to the POLIS registration, but respond that they are not employed.

We believe the first two differences indicate the tendency to provide socially desirable answers. The third difference is caused by satisficing because it occurs mostly when a respondent has multiple jobs and does not mention one of the jobs that is registered as employment in order to avoid follow-up questions. The measurement errors that occur in this manner considerably affect the estimation of unemployment rate for the population. Note that the two government registry data sets are essential to various ministries and they are frequently subject to quality checks. For this reason, we believe that there will be relatively few errors in the registry data themselves. For the sake of simplicity, we assume in our model that there is only one response style. The model can be easily extended to include additional response styles.

5.4.1 Numerical setup

The sample units are clustered into homogeneous group given age and we distinguish three groups, namely $\mathcal{G} = \{15-25, 26-55, 56-65\}$. The target population for LFS consists of persons aged 15 years and older, which is the potential labor force population. A great part of the 65+ population is retired, therefore we disregard this group in our

Mode	Probability	Group		
		15-25	26-55	56-65
$m=1$	$p_g(1)$	0.261	0.303	0.434
	$r_g(1)$	0.594	0.651	0.690
	$\theta_g(1)$	0.060	0.028	0.041
$m=2$	$p_g(2)$	0.392	0.367	0.461
	$r_g(2)$	0.594	0.651	0.69
	$\theta_g(2)$	0.078	0.035	0.047

Table 5.1: Estimated probabilities for contact, cooperation and response style probabilities (LFS 2008).

analysis. Traditionally, the LFS is conducted as a face-to-face survey with a maximum of six visits. In 2008, the LFS response rate was 57.9%. The proportion of groups $g \in \mathcal{G}$ in the population is

$$w_g = \{0.196, 0.624, 0.18\}.$$

Let $\mathcal{M} = \{0, 1, 2\}$ where 1 denotes self-reporting, 2 proxy-reporting and 0 no visit. Let $\mathcal{T} = \{1, 2, \ldots, 10\}$ be the time slots at which visits can be made to the sample units. In case of self-reporting, contact has to be established with the sampled person. In case of proxy-reporting, contact is defined as contact with the household the sampled person belongs to. Obviously, the contact probabilities are larger when proxy-reporting is allowed.

From the 2008 LFS data we estimated contact probabilities, cooperation probabilities and the response style probabilities for satisficing behavior for all groups $g \in \mathcal{G}$ and reporting types $m \in \mathcal{M}$ (see Table 5.1). The probability values for $m = 0$ are omitted since that involves no interaction with the group. As previously mentioned, the contact probabilities are higher in proxy-reporting. Also, the probabilities for the response style are higher in proxy-reporting which creates the tradeoff between allocation of scarce resources and collecting error-prone responses.

Remark 5.1. For the sake of simplicity, we assume that all the input probabilities are independent of the time slot and of the history of failed contact attempts.

For a sample of size $N = 10{,}000$ we study the RAP defined by the 4 settings described in Section 5.2. Table 5.2 revisits the setup of each analysis setting. We consider the following values for the constraint thresholds:

$$\Theta = \{3\%, 3.5\%, 4\%\},$$
$$\alpha = \{0.8, 0.85, 0.9\}.$$

Let b denote the average number of visits per address. We let $b \in \{2, 2.5, 3\}$. The maximum number of visits available for the entire survey will be given by $B = bN$, i.e., $B \in \{20{,}000, 25{,}000, 30{,}000\}$. Tables 5.13 and 5.11 at the end of the chapter show an overview of results for Settings 0-4, where for each setting it is indicated the number of

Setting	Reporting type	Number of visits	Risk constraint	Risk-free objective	Representativity constraint
Setting 0	Self-report Proxy-report	Fixed			
Setting 1	Optimize	Optimize			
Setting 2	Optimize	Optimize		Yes	
Setting 3	Optimize	Optimize	Yes		
	Optimize	Optimize			Yes
Setting 4	Optimize	Optimize		Yes	Yes
	Optimize	Optimize	Yes		Yes

Table 5.2: Overview data analysis setup.

visits needed and the resulting response rate, representativity indicator and proportion of respondents with response styles (here denoted as average risk from risk for manifestation of the response style). Few instances such as Setting 4, $\Theta = 3\%$ and $\alpha \in \{0.8, 0.85, 0.9\}$ yield only the trivial solution, i.e., $\left(x_g(t,m)\right)_{g,t,m} = 0$, therefore the risk for response styles cannot be computed in this case since no visits are carried out.

5.4.2 Optimization results

For a clear understanding, we group the results to answer the following questions:

- what setting yields the maximal response rate? (Section 5.4.2.1)
- what is the impact of allowing for proxy-reporting? (Section 5.4.2.2)
- what are the changes in the group optimal solutions across various model instances? (Section 5.4.2.3)
- what are the differences in addressing measurement error via the two approaches? (Section 5.4.2.4)
- what is the impact of the response representativity constraint? (Section 5.4.2.5)

In Tables 5.3 - 5.9, 1 denotes self-reporting, 2 proxy-reporting and 0 no visit. Given that some of the optimal solutions may not be feasible for implementation in practice due to many switches between self- and proxy-reporting for the same group, we add a constraint of the maximum number of such switches. The impact of this constraint is investigated in Section 5.4.2.6. A visual summary of the optimization results is provided in Section 5.4.2.7.

5.4.2.1 Maximal response rate

Higher response rates are obtained when proxy-reporting is allowed, due to higher contact probabilities in proxy-reporting, and for large values of b, i.e., more visits are allowed. From Setting 0 with proxy-report and Setting 1 we get that the maximal response rate given the input data is 64.2%, achieved for $b = 3$, i.e., a total of 30,000 visits were available to approach the entire sample.

Max number of visits	Group	Time slot										Response (%) rate (%)
		1	2	3	4	5	6	7	8	9	10	
20,000	15-25	2	0	0	0	0	0	0	0	0	0	23.3
	26-55	2	2	2	2	0	0	0	0	0	0	54.7
	56-65	2	2	2	2	2	0	0	0	0	0	65.9
25,000	15-25	2	2	2	2	2	2	0	0	0	0	56.4
	26-55	2	2	2	2	2	2	2	0	0	0	62.5
	56-65	2	2	2	2	2	2	2	2	2	2	68.9
30,000	15-25	2	2	2	2	2	2	2	2	2	2	59.0
	26-55	2	2	2	2	2	2	2	2	2	2	64.4
	56-65	2	2	2	2	2	2	2	2	2	2	68.9

TABLE 5.3: Optimal allocation of visits Setting 0 with proxy-reporting and Setting 1 (0 = no visit, 2 = proxy-reporting).

Although resulted from different models, the group strategies in these two settings are identical. There are two reasons that explain why the optimal strategy allocation in Setting 1 (see Table 5.3) chooses proxy-reporting at all visits. First, higher contact probabilities lead to higher response rates in the case of proxy-reporting. Second, reporting types cost the same, i.e., 1 visit, since we treat the constraint on the number of visits as a cost constraint regardless of the visit outcome. Thus, the optimal choice for the reporting type is proxy since it yields higher response.

Note that in Setting 1 for $b = 3$ there are sufficiently many visits available to completely handle all the groups. Hence, we can also interpret this case as an unconstrained problem, i.e., find the highest response given an unlimited number of visits. Note that also the representativity of this response is maximal, 0.939. This comes as no surprise, given the fact that all groups are approached at all time slots via the same reporting type.

5.4.2.2 Impact of proxy-reporting

We analyze the impact of proxy-reporting by comparing the two versions of Setting 0 (see Table 5.4 for the optimal solution for Setting 0 self-reporting). Proxy-reporting yields a higher response than self-reporting for any value of b. The structure of the strategies is, however, different. This is a consequence of the lower probability for contact in the case of self-reporting. Thus, more visits are required, which leads to higher costs. This is an important aspect in the case of group 15-25 that receives no visits at all for $b = 2$. In other words, it is preferable to “sacrifice” the response from group 15-25 in order to obtain higher response from the other groups. However, a zero visit-strategy for a group, i.e., the trivial solution, will not be acceptable for the response representativity constraint.

The average risk for the response style under self-reporting is never higher than 3.6%, while when proxy-reporting is allowed, it can reach 4.5% and higher. It is thus clear that the problem instances where the fraction of respondents showing the response style is constrained create a tradeoff between using self-reporting with lower probability for response styles but higher costs and proxy-reporting with lower costs, higher response

Max number of visits	Group	Time slot										Response (%) rate (%)
		1	2	3	4	5	6	7	8	9	10	
	15-25	0	0	0	0	0	0	0	0	0	0	0.0
20,000	26-55	1	1	1	1	0	0	0	0	0	0	49.7
	56-65	1	1	1	1	1	1	1	1	1	1	68.8
	15-25	1	0	0	0	0	0	0	0	0	0	15.5
25,000	26-55	1	1	1	1	1	1	1	0	0	0	59.9
	56-65	1	1	1	1	1	1	1	0	0	0	67.7
	15-25	1	1	1	1	1	0	0	0	0	0	46.3
30,000	26-55	1	1	1	1	1	1	1	1	1	1	63.3
	56-65	1	1	1	1	1	1	1	1	0	0	68.3

TABLE 5.4: Optimal allocation of visits Setting 0 with self-reporting (0 = no visit, 1 = self-reporting).

rates but also higher probability for response styles.

5.4.2.3 Group-strategy structure

As seen from Table 5.3, the strategies per group differ significantly when we vary the value of b. For example, for age group 15-25 only one visit is allowed for $b = 2$, whereas for $b = 2.5$ the strategy advises six visits. This is a notable design change. On the other hand, the strategy for age group 56-65 does not suffer from any changes between $b = 2.5$ and $b = 3$. Similar patterns can be observed over all settings.

The reason for such strategy structures lies in the input parameters, namely the group size, contact and cooperation probabilities. Age group 56-65 is a rather small group, only 1,800 sample units. Therefore, it is cheaper (i.e., small number of visits) to handle this group completely than any other (see again (3.9) for the cost computation). This age group is also the group with the highest contact and cooperation probabilities. Since our objective is to maximize the overall response rate, this group is preferred.

To explain the drastic change in strategy structure for age group 15-25, we need to take a comparative look with group 26-55. This is the largest group in our sample, 6,240 sample units, and therefore the costs for this group are the largest. However, it has a higher cooperation probability relatively to that of age group 15-25. As a consequence, in the optimal strategy more visits are spent on group 26-55 than on group 15-25 since this strategy yields a higher response. Therefore, when a small budget is available, age group 15-25 receives fewer visits than the other groups.

5.4.2.4 Addressing measurement errors

Under Approach 1, maximizing the adjusted response rate does not affect the structure of the original RAP formulation, only the objective function (see again (3.4) and (5.1) for comparison). The optimal solutions are presented in Table 5.5. The group strategies in this setting lead to a mix of reporting types. Upon a first sight, this is a counterintuitive results. Since proxy-reporting presents a high risk for erroneous response, an intuitive optimal survey approach would prescribe self-reporting for all visits. On the other hand,

Max number of visits	Group	Time slot 1	2	3	4	5	6	7	8	9	10	Response (%) rate (%)
	15-25	0	0	0	0	0	0	0	0	0	0	0.0
20,000	26-55	2	2	2	2	2	2	0	0	0	0	58.8
	56-65	1	1	2	2	1	1	1	1	1	1	65.9
	15-25	2	2	2	2	0	0	0	0	0	0	47.3
25,000	26-55	2	2	2	2	2	2	2	2	2	0	61.8
	56-65	2	2	1	1	1	2	2	2	2	2	65.6
	15-25	1	1	1	2	2	2	2	2	2	2	54.7
30,000	26-55	1	1	2	2	2	2	2	2	2	2	62.3
	56-65	1	1	1	1	1	1	2	2	2	2	66.0

Table 5.5: Optimal allocation of visits Setting 2
(0 = no visit, 1 = self-reporting, 2 = proxy-reporting).

such a strategy leads to high costs, as seen from Setting 0. Hence, a mix of reporting types is preferred. This is however not always affordable. For $b < 3$ only age group 56-65 can still afford the reporting type mix, while the other groups are approached only via proxy-reporting.

Under Approach 2, the survey researcher gains greater control over the resource allocation by tweaking the threshold Θ on the risk constraint. The corresponding optimal solutions are presented in Table 5.6. High values for the threshold Θ on average risk for response styles might not have any impact on the optimal allocation of visits. For example, if $\Theta = 5\%$, the group solutions would be just as in setting 2, where the average risk amounted to 4.51% for $b = 3$. Low values on the other hand, e.g., $\Theta = 3\%$, impose a decreased number of visits. Furthermore, for $\Theta < 2.8\%$ the problem becomes infeasible because the risk constraint can no longer be satisfied.

Note that age group 26-55 acts as a "risk-balancing" group in the sense that negative terms $\Theta - \theta_g(t, m)$ produced by visits to groups with higher risk for response styles than Θ must be accompanied by positive terms, in order to satisfy the nonnegativity constraint (5.7). In this sense, a large number of visits for group 26-55 would be necessary to balance the risk for one visit for either of the other groups. For this reason, for low values of b, no visits can be made to the other groups. In other words, there are not enough visits available to cover all the visits to group 26-55 necessary to balance the risk for response styles at an additional visit to one of the other two groups. Moreover, the structure of the optimal solution changes to favoring self-reporting, contrary to the remarks above stating that self-reporting is not used for low values of b, simply because self-reporting has a lower risk for response styles than proxy.

5.4.2.5 Adding the response representativity constraint

Setting 4 creates the most interesting problem instances since it investigates the allocation of visits to sample groups given the interaction of three opposing quality indictors. As previously mentioned, taking into account the response representativity is crucial for surveys of high-quality. Adding the response representativity constraint will have a

Θ	Max number of visits	Group	Time slot										Response rate (%)
			1	2	3	4	5	6	7	8	9	10	
		15-25	0	0	0	0	0	0	0	0	0	0	0.0
	20,000	26-55	1	1	1	2	2	2	2	2	2	2	64.1
		56-65	0	0	0	0	0	0	0	0	0	0	0.0
		15-25	0	0	0	0	0	0	0	0	0	0	0.0
3%	25,000	26-55	1	1	1	1	1	1	1	2	2	2	63.8
		56-65	1	0	0	0	0	0	0	0	0	0	30.0
		15-25	0	0	0	0	0	0	0	0	0	0	0.0
	30,000	26-55	1	1	1	1	1	1	1	2	2	2	63.8
		56-65	1	0	0	0	0	0	0	0	0	0	30.0
		15-25	0	0	0	0	0	0	0	0	0	0	0.0
	20,000	26-55	1	2	2	2	2	0	0	0	0	0	57.8
		56-65	1	1	2	2	0	0	0	0	0	0	62.6
		15-25	1	0	0	0	0	0	0	0	0	0	15.5
3.5%	25,000	26-55	1	1	1	2	2	2	2	2	2	0	63.7
		56-65	1	1	2	2	2	2	2	0	0	0	68.0
		15-25	1	1	1	1	0	0	0	0	0	0	41.7
	30,000	26-55	1	1	1	1	1	1	1	1	1	1	63.3
		56-65	1	1	1	1	1	1	0	0	0	0	66.7
		15-25	0	0	0	0	0	0	0	0	0	0	0.0
	20,000	26-55	2	2	2	2	2	2	0	0	0	0	60.9
		56-65	2	2	2	2	2	2	2	2	2	2	68.9
		15-25	2	2	0	0	0	0	0	0	0	0	37.4
4%	25,000	26-55	1	1	1	2	2	0	0	0	0	0	59.5
		56-65	2	2	2	2	2	2	2	2	2	2	68.9
		15-25	2	2	2	2	2	2	1	0	0	0	57.2
	30,000	26-55	1	1	1	1	1	1	2	2	2	2	63.9
		56-65	1	1	1	2	2	2	2	2	2	2	68.8

Table 5.6: Optimal allocation of visits Setting 3
(0 = no visit, 1 = self-reporting, 2 = proxy-reporting).

strong impact on the structure of the group optimal solutions. This holds particularly in the case of adding the constraint to setting 3. Here, the feasible region is significantly reduced, due to the conflicting situation created by combining the response representativity constraint with the risk constraint. Thus, a general decrease in the response rates is observed, especially for high values of α, the minimum required response representativity. Moreover, infeasibility is often encountered, due to either low values of Θ or high values of α. For the sake of clarity, we discuss only the optimal solutions for the problem instances where the response representativity constraint is applied to Setting 1 (see Table 5.7) and Setting 3 (see Table 5.8) for $b = 2$. Tables 5.12 – 5.15 at the end of the chapter present the remaining results. However, extending from Setting 3 for $\Theta = 3\%$ no results are obtained due to infeasibility.

A general pattern that can be observed in Setting 4 is that age group 15-25 is no longer omitted, as in the optimal solutions for Setting 2 and Setting 3. In order to reach the given thresholds for the R-indicator, this group has to be approached, which leads to a lower number of visits available to approach the other age groups. Thus, relative to previous settings, lower response rates are obtained.

α	Group	Time slot										Response (%)
		1	2	3	4	5	6	7	8	9	10	rate (%)
	15-25	2	2	0	0	0	0	0	0	0	0	37.4
0.8	26-55	2	2	2	0	0	0	0	0	0	0	48.6
	56-65	2	2	2	2	2	2	2	2	0	0	68.5
	15-25	2	2	0	0	0	0	0	0	0	0	37.4
0.85	26-55	2	2	2	0	0	0	0	0	0	0	48.6
	56-65	2	2	2	0	0	0	0	0	0	0	58.2
	15-25	2	2	0	0	0	0	0	0	0	0	37.4
0.9	26-55	2	2	2	0	0	0	0	0	0	0	48.6
	56-65	2	2	0	0	0	0	0	0	0	0	50.0

TABLE 5.7: Optimal allocation of visits Setting 4 (R-indicator threshold α) extended from Setting 1, $b = 2$ (0 = no visit, 1 = self-reporting, 2 = proxy-reporting).

α	Group	Time slot										Response (%)
		1	2	3	4	5	6	7	8	9	10	rate (%)
	15-25	2	2	0	0	0	0	0	0	0	0	34.5
0.8	26-55	2	2	2	0	0	0	0	0	0	0	46.9
	56-65	1	1	1	1	1	1	2	2	2	2	66.0
	15-25	2	2	2	0	0	0	0	0	0	0	42.5
0.85	26-55	2	2	2	0	0	0	0	0	0	0	46.9
	56-65	2	2	2	0	0	0	0	0	0	0	55.3
	15-25	2	2	2	0	0	0	0	0	0	0	42.5
0.9	26-55	2	2	2	0	0	0	0	0	0	0	46.9
	56-65	2	2	2	0	0	0	0	0	0	0	55.3

TABLE 5.8: Optimal allocation of visits Setting 4 (R-indicator threshold α) extended from Setting 3, $b = 2$ (0 = no visit, 1 = self-reporting, 2 = proxy-reporting).

5.4.2.5 Limit the number of reporting type switches

Optimal solutions such as those presented in Table 5.6 are less attractive from a practical point of view. Suppose for example that such a sequence of survey visits would be implemented. Following these recommendations the interviewer would have to permanently check whether at the given visit the household is allowed to use proxy-reporting or only self-reporting. This is both inconvenient and prone to error, therefore, in order to make such solutions more practical, we impose an additional constraint on the number of switches between reporting types. In other words, once there is a switch in the reporting type, continue with the chosen type or no approach. We thus assume that at most one switch is allowed. There are only two cases where this constraint affects the optimal sequence of visits. That is setting 2 for $b = 2$ and $b = 2.5$. Table 5.9 shows the new optimal solutions. A slight decrease in the response rates is observed (see Table 5.5 for comparison). The corresponding R-indicator and risk for response styles are presented in Table 5.10.

5.4.2.7 Visual summary of results

Figure 5.2 provides a graphical overview of response rates across the analyzed problem instances. The evolution of response rates when measurement errors are addressed is

Max number of visits	Group	Time slot										Response (%) rate (%)
		1	2	3	4	5	6	7	8	9	10	
	15-25	0	0	0	0	0	0	0	0	0	0	0.0
20,000	26-55	2	2	2	2	2	2	0	0	0	0	58.8
	56-65	1	1	2	2	2	2	2	2	2	2	65.9
	15-25	2	2	2	2	0	0	0	0	0	0	47.3
25,000	26-55	2	2	2	2	2	2	2	2	2	0	61.8
	56-65	2	2	2	2	2	2	2	2	2	2	65.5

Table 5.9: Optimal allocation of visits Setting 2 with an additional constraint on the number of switches between reporting types (0 = no visit, 1 = self-reporting, 2 = proxy-reporting).

Max number of visits	Response rate (%)	Number of visits	R-indicator	Average risk (%)
20,000	48.5	19,969	0.518	3.69
25,000	59.6	24,938	0.876	4.41

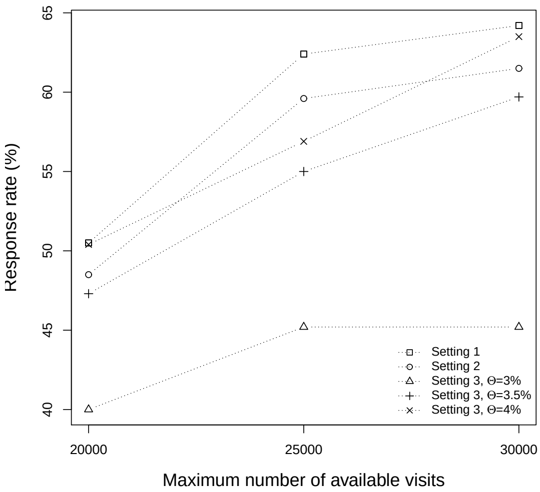
Table 5.10: Overview results Setting 2 with an additional constraint on the number of switches between reporting types (0 = no visit, 1 = self-reporting, 2 = proxy-reporting).

visualized in Figure 5.2a by comparing results from Settings 1-3. The impact of the response representativity constraint is visualized in Figure 5.2b by comparing results from Setting 4. Note that in Figures 5.2a and 5.2b the response rate is plotted as a function of "budget", i.e., the available number of visits. Dotted lines connect response rate levels resulted from models that differ only through the value of this parameter. Similarity in graphical symbols across figures does not indicate any relation between the plotted values.

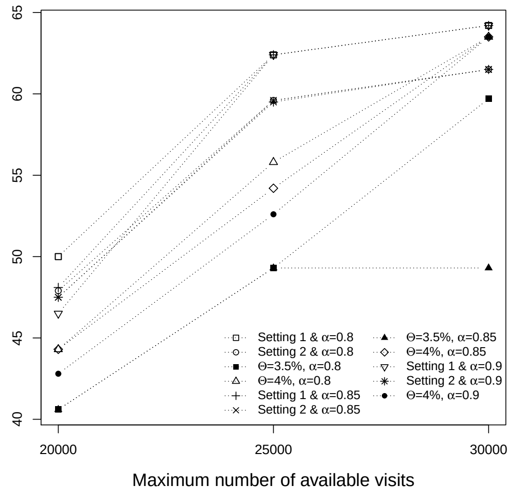
In Figure 5.2a, Setting 3 brings great differences in the response rate levels. For $\Theta = 4\%$ the highest difference occurs for $b = 2.5$, i.e., 25,000 visits available, although for both $b = 2.5$ and $b = 3$ the average risk in Setting 1 was around 4.5% (see Table 5.13). The relatively small drop in the response rate for $b = 3$ is explained by the higher number of visits that is available in this case, i.e., 30,000. Thus, a larger feasible region is created, which lead to a solution that simultaneously restrains the level of allowed risk for response styles to the given threshold Θ and yields a relatively high response rate. Decreasing Θ from 3.5% to 3% brings a more significant decrease in response rates. This decrease is amplified for larger values of b. Such behavior is a consequence of the contraction of the feasible region caused by stricter thresholds on the risk constraint.

Figure 5.2b depicts the decrease in the response rate levels due to additional restrictions imposed by the response representativity constraint. In some cases, e.g., when $\Theta = 3\%$, the feasible region is reduced to the empty set, therefore such instances are not displayed in the plot.

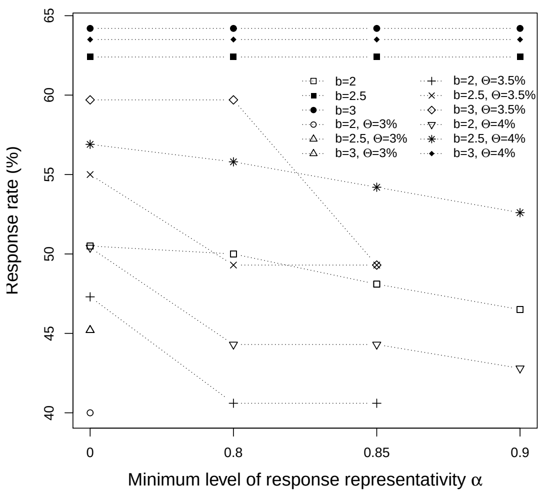
Figure 5.2c differs from the previous ones by visualizing the response rate as a function of α, the minimum level of response representativity. Settings 1, i.e., $\alpha = 0$, and 4 are



(A) Response rate evolution when addressing measurement error - Settings 1-3.



(B) Response rate evolution and the response representativity constraint - Setting 4.



(C) Response rate as a function of minimum level of response representativity α.

FIGURE 5.2: Response rate evolution across models - impact of various constraints on the response rate.

compared. Here, the dotted lines connect response rate levels resulted from models that differ only through the value of parameter α, i.e., Θ and b do not change along the lines. As noted before, similarity in graphical symbols across figures does not indicate any relation between the plotted values. The trend of most solutions in this graph is decreasing for increasing values of α, reaching infeasibility in the more extreme cases, e.g., for $\Theta = 3\%$. Moreover, high values of α, e.g., $\alpha = 0.9$ pose stricter conditions than Θ. Thus, the feasible region can be reduced to the empty set even in the case of $\Theta = 3.5\%$. Problem instances that lead to infeasibility are not present in the plot.

5.5 Concluding remarks

In this chapter, we discuss an optimization model that extends the framework of adaptive survey designs to specifically address nonresponse and measurement errors. Such research is motivated by the recently observed increase in the presence of measurement errors in survey practice. To our best knowledge, the work presented in this chapter constitutes the first attempt to addresses the interaction between nonresponse and measurement errors in the context of adaptive survey designs. Additionally, we investigate the resource allocation for such designs.

We evaluate whether the probability for undesirable response styles should be modeled as a correction in the objective function or addressed through a constraint. After carrying out the necessary adjustments, both problem formulations were successfully addressed through the two-step algorithm described in Chapter 4. Further research questions arise in both cases. For example, maximization of the error-free response rates assumes that, when the response style is observed, the corresponding response is in fact treated as nonresponse. In other words, by following this approach, we may throw away information. Therefore, the second approach, i.e., adding a constraint, may be favored. Nonetheless, setting an upper bound to the probability for undesirable response styles may be difficult in the sense that quantifying the impact of response styles on the survey estimates is still not fully addressed in the literature.

A few additional remarks are in place. To identify measurement profiles we used response styles defined as differences between survey answers and registry data. However, additional information such as paradata may be necessary for better identification of measurement profiles. Since paradata become available at the moment or after the interview, the question is how to refine the definition of response styles and measurement profiles during the survey fieldwork? Furthermore, from an implementation standpoint, the choice of survey design features and the number of subgroups that enter the adaptive design should be modest and backed up by historical survey data. Certain combinations might not have enough support in the historical survey data to accurately estimate the input parameters. The accurate estimation of the input parameters is paramount to obtaining effective survey designs. Therefore, sensitivity analysis should be conducted in order to assess the robustness of the optimal solutions.

Appendix: additional optimization results

Setting	Max number of visits	Response rate (%)	Number of visits	R-indicator	Average risk (%)
Setting 1 & $\alpha = 0.8$	20,000	50.0	19,738	0.806	4.43
	25,000	62.4	24,954	0.924	4.50
	30,000	64.2	25,689	0.939	4.50
Setting 1 & $\alpha = 0.85$	20,000	48.1	19,135	0.872	4.42
	25,000	62.4	24,954	0.924	4.50
	30,000	64.2	25,689	0.939	4.50
Setting 1 & $\alpha = 0.9$	20,000	46.5	18,612	0.911	4.41
	25,000	62.4	24,954	0.924	4.50
	30,000	64.2	25,689	0.939	4.50
Setting 2 & $\alpha = 0.8$	20,000	47.9	19,970	0.806	4.26
	25,000	59.6	24,995	0.875	4.38
	30,000	61.5	29,199	0.928	3.95
Setting 2 & $\alpha = 0.85$	20,000	47.5	19,859	0.919	4.51
	25,000	59.6	24,995	0.875	4.38
	30,000	61.5	29,199	0.928	3.95
Setting 2 & $\alpha = 0.9$	20,000	47.5	19,859	0.919	4.51
	25,000	59.5	24,957	0.905	4.44
	30,000	61.5	29199	0.928	3.95
$\Theta = 3.5\%$ $\alpha = 0.8$	20,000	40.6	19,847	0.862	3.49
	25,000	49.3	24,503	0.852	3.47
	30,000	59.7	29,317	0.820	3.49
$\Theta = 3.5\%$ $\alpha = 0.85$	20,000	40.6	19,847	0.862	3.49
	25,000	49.3	24,503	0.852	3.47
	30,000	49.3	24,503	0.852	3.47
$\Theta = 4\%$ $\alpha = 0.8$	20,000	44.3	19,521	0.885	3.99
	25,000	55.8	24,644	0.816	3.97
	30,000	63.5	28,809	0.928	3.99
$\Theta = 4\%$ $\alpha = 0.85$	20,000	44.3	19,521	0.885	3.99
	25,000	54.2	24,879	0.893	3.97
	30,000	63.5	28,809	0.928	3.99
$\Theta = 4\%$ $\alpha = 0.9$	20,000	42.8	19,892	0.935	3.96
	25,000	52.6	23,853	0.905	3.98
	30,000	63.5	28,809	0.928	3.99

TABLE 5.11: Overview results extended RAP for measurement errors Setting 4.

α	Group	Time slot										Response (%)
		1	2	3	4	5	6	7	8	9	10	rate (%)
0.8	15-25	2	2	0	0	0	0	0	0	0	0	34.5
	26-55	2	2	2	0	0	0	0	0	0	0	46.9
	56-65	1	1	1	1	1	1	2	2	2	2	66.0
0.85	15-25	2	2	2	0	0	0	0	0	0	0	42.5
	26-55	2	2	2	0	0	0	0	0	0	0	46.9
	56-65	2	2	2	0	0	0	0	0	0	0	55.3
0.9	15-25	2	2	2	2	2	0	0	0	0	0	50.3
	26-55	2	2	2	2	2	2	2	2	0	0	61.2
	56-65	2	2	2	2	2	2	0	0	0	0	64.0

TABLE 5.12: Optimal allocation of visits Setting 4 (R-indicator threshold α) extended from Setting 2, $b = 2.5$ (0=no visit, 1=self-reporting, 2=proxy-reporting).

Setting	Max number of visits	Response rate (%)	Number of visits	R-indicator	Average risk (%)
Setting 0 self-report	20,000	43.4	19,868	0.548	3.17
	25,000	52.6	24,979	0.629	3.29
	30,000	60.9	29,995	0.851	3.54
Setting 0 proxy-report	20,000	50.5	19,960	0.718	4.17
	25,000	62.4	24,954	0.924	4.50
	30,000	64.2	25,689	0.939	4.51
Setting 1	20,000	50.5	19,960	0.718	4.17
	25,000	62.4	24,954	0.924	4.50
	30,000	64.2	25,689	0.939	4.51
Setting 2	20,000	48.5	19,989	0.518	3.68
	25,000	59.6	24,995	0.875	4.38
	30,000	61.5	29,199	0.928	3.95
Setting 3 $\Theta = 3\%$	20,000	40.0	19,488	0.379	2.96
	25,000	45.2	21,762	0.487	2.99
	30,000	45.2	21,762	0.487	2.99
Setting 3 $\Theta = 3.5\%$	20,000	47.3	19,894	0.531	3.49
	25,000	55.0	24,980	0.609	3.47
	30,000	59.7	29,317	0.820	3.49
Setting 3 $\Theta = 4\%$	20,000	50.4	19,805	0.499	3.80
	25,000	56.9	24,990	0.796	3.99
	30,000	63.5	28,809	0.928	3.99

TABLE 5.13: Overview results extended RAP for measurement errors Settings 0 - 3.

α	Max number of visits	Group	Time slot 1	2	3	4	5	6	7	8	9	10	Response (%) rate (%)
0.8	20,000	15-25	1	1	0	0	0	0	0	0	0	0	27.0
		26-55	1	1	1	0	0	0	0	0	0	0	43.1
		56-65	1	1	0	0	0	0	0	0	0	0	46.9
	25,000	15-25	1	1	1	0	0	0	0	0	0	0	35.4
		26-55	1	1	1	1	1	0	0	0	0	0	54.4
		56-65	1	1	0	0	0	0	0	0	0	0	46.9
	30,000	15-25	1	1	1	1	0	0	0	0	0	0	41.7
		26-55	1	1	1	1	1	1	1	1	1	1	63.3
		56-65	1	1	1	1	1	1	0	0	0	0	66.7
0.85	20,000	15-25	1	1	0	0	0	0	0	0	0	0	27.0
		26-55	1	1	1	0	0	0	0	0	0	0	43.1
		56-65	1	1	0	0	0	0	0	0	0	0	46.9
	25,000	15-25	1	1	1	0	0	0	0	0	0	0	35.4
		26-55	1	1	1	1	1	0	0	0	0	0	54.4
		56-65	1	1	0	0	0	0	0	0	0	0	46.9
	30,000	15-25	1	1	1	0	0	0	0	0	0	0	35.4
		26-55	1	1	1	1	1	0	0	0	0	0	54.4
		56-65	1	1	0	0	0	0	0	0	0	0	46.9
0.9	20,000	15-25	0	0	0	0	0	0	0	0	0	0	0.0
		26-55	0	0	0	0	0	0	0	0	0	0	0.0
		56-65	0	0	0	0	0	0	0	0	0	0	0.0
	25,000	15-25	0	0	0	0	0	0	0	0	0	0	0.0
		26-55	0	0	0	0	0	0	0	0	0	0	0.0
		56-65	0	0	0	0	0	0	0	0	0	0	0.0
	30,000	15-25	0	0	0	0	0	0	0	0	0	0	0.0
		26-55	0	0	0	0	0	0	0	0	0	0	0.0
		56-65	0	0	0	0	0	0	0	0	0	0	0.0

TABLE 5.14: Optimal allocation of visits Setting 4 (R-indicator threshold α) extended from Setting 3, $\Theta = 3.5\%$ (0=no visit, 1=self-reporting, 2=proxy-reporting).

α	Max number of visits	Group	Time slot										Response (%) rate (%)
			1	2	3	4	5	6	7	8	9	10	
		15-25	1	2	0	0	0	0	0	0	0	0	32.7
	20,000	26-55	1	2	2	0	0	0	0	0	0	0	46.9
		56-65	2	1	0	0	0	0	0	0	0	0	48.0
		15-25	2	2	0	0	0	0	0	0	0	0	37.4
0.8	25,000	26-55	1	1	1	2	2	2	0	0	0	0	59.5
		56-65	2	2	2	2	0	0	0	0	0	0	63.2
		15-25	2	2	2	2	2	2	1	0	0	0	57.2
	30,000	26-55	1	1	1	1	1	1	2	2	2	2	63.9
		56-65	1	1	1	2	2	2	2	2	2	2	68.8
		15-25	1	2	0	0	0	0	0	0	0	0	32.7
	20,000	26-55	1	2	2	0	0	0	0	0	0	0	46.9
		56-65	2	1	0	0	0	0	0	0	0	0	48.0
		15-25	2	2	2	0	0	0	0	0	0	0	46.1
0.85	25,000	26-55	1	1	1	1	1	2	0	0	0	0	58.3
		56-65	2	2	0	0	0	0	0	0	0	0	49.0
		15-25	2	2	2	2	2	2	1	0	0	0	57.2
	30,000	26-55	1	1	1	1	1	1	2	2	2	2	63.9
		56-65	1	1	1	2	2	2	2	2	2	2	68.8
		15-25	2	2	0	0	0	0	0	0	0	0	37.4
	20,000	26-55	1	1	1	0	0	0	0	0	0	0	43.1
		56-65	1	2	0	0	0	0	0	0	0	0	48.0
		15-25	2	2	2	0	0	0	0	0	0	0	46.1
0.9	25,000	26-55	1	1	1	2	2	0	0	0	0	0	56.3
		56-65	2	2	0	0	0	0	0	0	0	0	46.9
		15-25	2	2	2	2	2	2	1	0	0	0	57.2
	30,000	26-55	1	1	1	1	1	1	2	2	2	2	63.9
		56-65	1	1	1	2	2	2	2	2	2	2	68.8

TABLE 5.15: Optimal allocation of visits Setting 4 (R-indicator threshold α) extended from Setting 3, $\Theta = 4\%$ (0=no visit, 1=self-reporting, 2=proxy-reporting).

6

Adaptive survey designs to minimize survey mode effects

Previous chapters dealt with modelling adaptive survey designs that maximize some notion of survey quality given limited resources, where the survey quality definition is given by a covariate-based quality function such as the response rate or the representativity indicator. This simplified the analysis since we did not have to employ the answers to the survey variables in order to define our objective function. In some cases, however, and increasingly more in the recent years, the focus has shifted towards item-based quality functions such as the estimated nonresponse bias, that are applied to a single survey variable. This is a very daunting task due to various reasons. First, it employs answers to the specified survey variable, which are missing in the case of nonrespondents, and thus additional assumptions are required to carry out the analysis. Second, in surveys with many survey variables, it may be difficult to interpret the quality function since it may lead to conflicting conclusions for different variables. Nonetheless, for major economic indicators such as the unemployment rate, estimated through the Labor Force Survey (LFS), it is worthwhile to investigate what factors decrease the accuracy of such estimators and whether the effect can be addressed through better survey designs.

In this chapter, we focus on addressing survey mode effects in the context of adaptive survey designs (ASDs). Survey mode effects appear when differences are noticed between estimates obtained from surveys using different survey modes, i.e., when the same question asked in different modes receives different answers. Assessing the impact of mode effects on survey estimates has become a crucial question due to the increasing appeal of mixed-mode designs. There are multiple reasons why survey practitioners prefer less and less unimode designs such as increased costs in carrying out face-to-face surveys, decreasing coverage in telephone surveys and low participation in online surveys (Fan and Yan 2010). As a consequence, survey organizations have been steadily restructuring their unimode designs into mixed-mode designs. However, there is significant theoretical and practical evidence (see Jäckle et al. 2010 and Dillman et al. 2009) that mode effects may sometimes be large relative to the precision. They may lead to incomparable statistics in time or over population subgroups and they may increase bias. Assessment of mode effects does not however follow a generally accepted technique. Literature and experimental studies note various viewpoints on what survey components suffer more from mode effects and how to test for manifestation on mode effects. The common ground among these perspectives is determining whether mode effects are item-specific or systematic phenomena (Klausch et al. 2013b). Subsequently, when item-specific, changes in the survey design can be made in order to address the corresponding mode effects. If however, systematic errors are observed, then modes are incomparable and carrying out a mixed-mode design could result into misleading conclusions.

We proceed with the analysis of item-specific mode effects. Survey mode effects stream from mode selection effect, i.e., different people have access to different modes, and mode measurement effects. However, as reported in Schouten et al. (2012), for the Dutch LFS, the mode selection effect for target variables can be adjusted for, due to strong auxiliary information available from registers. As a consequence, in the LFS case, the analysis of mode effects may focus only on mode measurement effects. Moreover, it enables analysis of the adjusted mode effects, i.e., analysis of differences between modes after nonresponse adjustments.

Given this result, we develop a model that minimizes mode measurement effects that may impact the unemployment rate estimate, one of the key statistics produced in the LFS. In our analysis, we consider three survey modes, namely, web (CAWI), phone (CATI) and face-to-face (CAPI). As observed from historical data, web surveys are cheap to run but possibly more prone to measurement effects than CATI or CAPI. At the same time, CAPI surveys produce more reliable survey estimates but they are very expensive to run. A mixed-mode design balances costs but the mode measurement effects are harder to quantify and control. Therefore, we investigate what mode combinations, from a specified list of combinations, should form the adaptive design such that mode measurement effects are minimized. To our best knowledge, this is the first research attempt of its kind and due to its flexibility, our methodology can be used as a basis for more complex settings that aim at addressing mode effects.

6.1 Survey mode effects: an introduction

Running mixed-mode surveys offers many advantages compared to unimode surveys such as lower costs and increased coverage. However, their implementation comes with a series of potential difficulties of which most troublesome is data comparability across modes. de Leeuw (2005) and Jäckle et al. (2010) note that before designing and implementing mixed-mode surveys, survey practitioners should be able to understand and quantify the impact of mode effects on data quality. Therefore, extensive research by means of field studies and testing of mode differences is strongly recommended.

The most common framework to assess mode effects is given by the cognitive models of survey response process (see Tourangeau et al. 2000) which analyze the phases of the response process, i.e., interpretation and comprehension of the survey question, information retrieval, judgment and reporting of the answer, that are influenced by mode. As a result, manifestation of an answering behavior, e.g., social desirability, satistificing, is perceived as a mode effect that can lead to response bias. Another method for assessing mode effects is to test for differences in various quality indicators and response distributions (see Link and Mokdad 2005, Greenfield et al. 2000). If significant differences are displayed across modes then the analyzed survey items or indicators are subject to mode effects. A third approach is a model-based approach that analyzes the impact of various modes on the probability of providing the same answer under the different modes for two persons that possess the same true state on the question topic (see Millsap 2011).

Although sufficient tools are available to identify response differences across modes, the main challenge is how to decide that such differences translate into data quality difference between surveys (see Biemer 1988). Differences in mode coverage, sampling frames and nonresponse bias could easily perturb responses, making it hard for the researchers to disentangle the mode effect. Additionally, differences in questionnaire design across modes are an easy trap for overstatement of the mode effects existence (see Dillman 2000). The reason is that even if a questionnaire is designed specifically for a given survey mode, mode effects may still occur. Furthermore, mode effects may impact only certain survey estimates (see de Leeuw 1992). Dillman et al. (2009) have shown that different people are attracted by different modes which results in an inhomogeneous sample creating thus a selection effect next to a measurement effect.

As suggested by Biemer (1988), evaluating the mode effect impact on data quality could be done by comparing responses to a "gold standard" such as external records or prior knowledge on the direction of error. Following this recommendation we develop a mode effect indicator that aims at quantifying the deviation caused by mode on a survey item against a selected benchmark. Subsequently, we develop an optimization model that assigns optimally survey resources in order to minimize the mode effect impact on data quality given the mode effect indicator. We apply this evaluation method on the Dutch LFS in order to assess the mode effect on one of the survey items, i.e., the unemployment rate estimate. In the following we present the optimization model to develop an adaptive design that minimizes the mode effects given the selected benchmark and the mode effect indicator.

6.2 Problem formulation

The most influential survey design features are the mode and the number of visits/calls, which is due to their significant influence on survey costs and quality. Therefore, for our adaptive design framework, we focus on various combinations survey mode - number of attempts, further denoted as *survey strategies*, and we let the decision variables denote the allocation probability of a survey strategy to the survey units. For example, the set of survey strategies in the case study in Section 6.4 is given by

$$\begin{aligned}\mathcal{S} = \{&\text{CAWI, CATI2, CATI2+, CAPI3, CAPI3+,}\\ &\text{CAWI-CATI2, CAWI-CATI2+, CAWI-CAPI3, CAWI-CAPI3+}, \emptyset\},\end{aligned}$$

where CAPI denotes face-to-face interviews, CATI telephone interviews and CAWI web survey. Note that the strategies alternate between no restriction of calls, i.e., CATI2+ and CAPI3+, and limitation to two calls for CATI, i.e., CATI2, and three for CAPI, i.e., CAPI3. Note that also the sampling design is addressed in the model by including the nonsampling strategy $\emptyset$ in the survey strategy set. The specified maximum number of attempts has been selected given the available historical data, where the greatest proportion of the overall response is obtained within the specified number of attempts. The selected mode combinations are a result of current practice, where web is part of most mixed-mode designs due to its reduced costs. Additionally, we are interested in discerning the mode measurement effect between unimode and mixed-mode strategies.

Population units are clustered into $\mathcal{G} = \{1, \ldots, G\}$ groups given a set of characteristics X such as age, ethnicity, that can be extracted from external sources of data. Let $p\,(s, g)$ be the allocation probability of strategy s to group g. Note that in the current chapter, we implicity model the sampling probabilities. In other words, if $p\,(s, g) > 0$, then a proportion $p\,(s, g)$ from group g is sampled and approached through strategy s. Denote by $p\,(\emptyset, g)$ the nonsampling probability. We then have that

$$\sum_{s\in\mathcal{S}} p\,(s, g) + p\,(\emptyset, g) = 1,\ \forall\, g \in \mathcal{G}. \tag{6.1}$$

We define the mode effect measure as the nonresponse adjusted difference between the survey estimate $\bar{y}_{s,g}$ and a benchmark estimate $\bar{y}_{BM}$ of the population mean $\bar{Y}$, where the survey estimate $\bar{y}_{s,g}$ is obtained by allocating strategy $s \in \mathcal{S}$ to group $g \in \mathcal{G}$. Let $D(s, g)$ denote this difference. The mode effect measure is expressed as

$$D(s, g) = \bar{y}_{s,g} - \bar{y}_{BM},\ \forall\, s \in \mathcal{S}, g \in \mathcal{G}, \tag{6.2}$$

with values in the same domain as $\bar{y}_{s,g}$ and $\bar{y}_{BM}$. We further refer to $D(s, g)$ as the *mode effect measure* or the *mode difference*. The mode effect measure considers the adjusted mode measurement effects, i.e., the survey estimates have been adjusted for nonresponse. For convenience, we refer in the following to mode effects but they are always adjusted for selection effects. The population average mode difference with respect to the indicated benchmark BM is given by

$$\bar{D}_{BM} = \left| \sum_{s,g} \frac{w_g p\,(s,g)\,\rho\,(s,g)\,D(s,g)}{\sum_{s'\in\mathcal{S}} p\,(s',g)\,\rho\,(s',g)} \right|,$$

where N_g is the population size of group g, $w_g = N_g/N$ represents the proportion of group g in the entire population N and $\rho\,(s,g)$ the response probability for group g if strategy s is assigned. In other words, the impact of any mode differences that arise when applying strategy s to group g are moderated by the contribution of the obtained group response to the total response, weighted by the group size. Since only response could trigger mode effects, the contribution of each group g to the overall mode effect measure needs to be proportional to the group's contribution to the overall response. $\bar{D}_{BM}$ takes values in the same domain as $D(s,g)$. Our goal is to minimize the overall mode effect $\bar{D}_{BM}$ by optimally assigning strategies $s \in \mathcal{S}$ to groups $g \in \mathcal{G}$, i.e.,

$$\underset{p(s,g)}{\text{minimize}}\ \bar{D}_{BM} = \left| \sum_{s,g} \frac{w_g p\,(s,g)\,\rho\,(s,g)\,D(s,g)}{\sum_{s'\in\mathcal{S}} p\,(s',g)\,\rho\,(s',g)} \right| \tag{6.3}$$

Note that $\bar{y}_{s,g}$ is a nonresponse adjusted estimate of $\bar{Y}$, while $\rho\,(s,g)$ is an unweighted estimate of the group g response probability in strategy s (see for details Section 6.4.2). We assume that the nonresponse adjustment does not influence the contribution of each group and strategy to the overall response. This allows us to write the objective function as in (6.3), while performing nonresponse adjustment within the optimization would be a very complex perhaps even irrealizable technique.

Scarcity in resources and other practical aspects impose a number of constraints in our model. A limited budget B is available to setup and run the survey. Let $c\,(s,g)$ be the unit cost of applying strategy s to one unit in group g (for estimation details, see Section 6.4.4). The cost constraint is formulated as follows

$$\sum_{s,g} N_g\, p\,(s,g)\, c\,(s,g) \leq B. \tag{6.4}$$

To ensure a minimal precision for the survey estimate of $\bar{Y}$, a minimum number R_g of respondents per group is required. This translates to the following constraint

$$\sum_{s} N_g\, p\,(s,g)\, \rho\,(s,g) \geq R_g,\ \ \forall g \in \mathcal{G}. \tag{6.5}$$

In addition to the objective function we address the mode effect also through a constraint. The structure of the objective function could lead to an unbalanced solution. For example, let a group g_i be assigned a strategy s such that the corresponding $D(s,g_i)$ is a large negative value and the other groups $g \in \mathcal{G}\backslash g_i$ receive strategies that yield positive $D(s,g)$ values. Thus, the large negative $D(s,g_i)$ is canceled out but group g_i will have a very different behavior compared to the other groups, which renders mutual comparison among groups impossible. To prevent the occurrence of such solutions, we limit the absolute difference in the mode effect measured for any two groups by the following constraint

$$\left| \frac{\sum_{s\in\mathcal{S}} p\,(s,g_i)\rho\,(s,g_i)D(s,g_i)}{\sum_{s\in\mathcal{S}} p\,(s,g_i)\rho\,(s,g_i)} - \frac{\sum_{s\in\mathcal{S}} p\,(s,g_j)\rho\,(s,g_j)D(s,g_j)}{\sum_{s\in\mathcal{S}} p\,(s,g_j)\rho\,(s,g_j)} \right| \leq M,\ \forall\, g_i, g_j \in \mathcal{G}. \tag{6.6}$$

For practical reasons we also introduce a constraint on the maximum sample size, i.e.,

$$\sum_{s,g} N_g p\,(s,g) \leq S_{max}. \tag{6.7}$$

The constraints on the decision variables $p\,(s,g)$ concern their definition as allocation probabilities, i.e.,

$$\begin{aligned} &0 \leq p\,(s,g) \leq 1,\ \forall\, s \in \mathcal{S},\ g \in \mathcal{G} \\ &\sum_{s\in\mathcal{S}} p\,(s,g) \leq 1,\ \forall\, g \in \mathcal{G}, \end{aligned} \tag{6.8}$$

where inequality is sufficient since every $p\,(s,g)$ implies assignment of strategy s after sampling from group g. Equality is necessary when taking into account the nonsampling probability $p\,(\emptyset,g)$ as in (6.1). However, since mode effects cannot be defined in case of nonsampling, we have excluded this variable from the model and adjusted the constraints accordingly. Additionally, we require that at least one $p\,(s,g)$ be strictly positive,

$$\sum_{s\in\mathcal{S}} p\,(s,g) > 0,\ \forall g \in \mathcal{G}, \tag{6.9}$$

to avoid computational errors such as division by zero.

Objective function (6.3) together with constraints (6.4) – (6.9) form the optimization model to minimize overall mode effects in the context of adaptive survey designs, which leads to a nonconvex nonlinear problem (NNLP).

6.3 Algorithm

The previous section dealt with defining an adaptive design that optimally allocates survey resources in order to minimize mode effects. The model formulation however poses difficulties in terms of finding a suitable algorithm to solve the problem to optimality. The constraints on the maximum difference between group mode effects make the problem nonconvex and hard to solve. Therefore, most general-purpose nonlinear solvers cannot do better than a local optimum. In such cases, the choice for a starting point of search for an optimum plays an important role in trying to achieve the best local optimum. Given these considerations, we opt for a two-step approach where, in the first step, we solve a linear programming problem (LP) that addresses the linear constraints (6.4), (6.5) and (6.7) – (6.9) and use the optimal solution thus obtained as a starting point for a local search algorithm to solve the NNLP.

In the following we present a reformulation of the mode effect problem such that the absolute value signs are discarded.

$$
\begin{aligned}
\text{minimize}\quad & t \\
\text{subject to}\quad & \sum_{s,g} \frac{w_g\, p\,(s,g)\,\rho\,(s,g)\,D(s,g)}{\sum_{s'\in\mathcal{S}} p\,(s',g)\,\rho\,(s',g)} \le t \\
& -\sum_{s,g} \frac{w_g\, p\,(s,g)\,\rho\,(s,g)\,D(s,g)}{\sum_{s'\in\mathcal{S}} p\,(s',g)\,\rho\,(s',g)} \le t \\
& \sum_{s,g} N_g\, p\,(s,g)\, c\,(s,g) \le B \\
& \sum_{s} N_g\, p\,(s,g)\,\rho\,(s,g) \ge R_g,\ \ \forall g \in \mathcal{G} \\
& \frac{\sum\limits_{s\in\mathcal{S}} p\,(s,g_i)\,\rho\,(s,g_i)\,D(s,g_i)}{\sum\limits_{s\in\mathcal{S}} p\,(s,g_i)\,\rho\,(s,g_i)} - \frac{\sum\limits_{s\in\mathcal{S}} p\,(s,g_j)\,\rho\,(s,g_j)\,D(s,g_j)}{\sum\limits_{s\in\mathcal{S}} p\,(s,g_j)\,\rho\,(s,g_j)} \le M,\ \ \forall\, g_i, g_j \in \mathcal{G} \\
& \frac{\sum\limits_{s\in\mathcal{S}} p\,(s,g_j)\,\rho\,(s,g_j)\,D(s,g_j)}{\sum\limits_{s\in\mathcal{S}} p\,(s,g_j)\,\rho\,(s,g_j)} - \frac{\sum\limits_{s\in\mathcal{S}} p\,(s,g_i)\,\rho\,(s,g_i)\,D(s,g_i)}{\sum\limits_{s\in\mathcal{S}} p\,(s,g_i)\,\rho\,(s,g_i)} \le M,\ \ \forall\, g_i, g_j \in \mathcal{G} \\
& \sum_{s,g} N_g\, p\,(s,g) \le S_{max} \\
& 0 \le p\,(s,g) \le 1,\ \ \forall\, s \in \mathcal{S},\ g \in \mathcal{G} \\
& \sum_{s\in\mathcal{S}} p\,(s,g) \le 1,\ \ \forall\, g \in \mathcal{G} \\
& 0 \le t.
\end{aligned}
\tag{6.10}
$$

Since $|f(x)| = \max\{f(x), -f(x)\}$, we can rewrite the objective function via an additional variable t and impose that $f(x) \le t$ and $-f(x) \le t$. Moreover, t has to be nonnegative. The reformulation of constraint (6.6) follows from the definition of the absolute value, i.e., $|f(x)| \le M \Leftrightarrow -M \le f(x) \le M$. The remaining constraints do not change from their initial formulation.

Before we sketch the LP, note that the dummy variable t in (6.10) appears only in two nonlinear constraints that would not be part of the LP. Hence, formulating the LP with the same objective function as (6.10) is senseless. Consequently, one of the linear constraints should be reformulated as the objective function. We choose for minimization of costs as the LP objective. The resulting problem formulation is given in (6.11).

To solve the linear problem, we use the simplex method available in `R` in package *boot*. Our proposed two-step algorithm thus handles (6.11) in the first step. Denote by x^*_{LP} the optimal solution obtained in the LP. In the second step, x^*_{LP} is submitted to a nonlinear optimization algorithm as a starting point in order to solve (6.10). For this step, we use nonlinear algorithms available in NLOPT (see Johnson 2013), an open-source library for nonlinear optimization that can be called from `R` through the *nloptr* package.

$$
\begin{aligned}
\min \quad & \sum_{s,g} N_g \, p\,(s,g)\, c\,(s,g) \\
\text{s.t.} \quad & \sum_{s,g} N_g \, p\,(s,g)\, \rho\,(s,g) \geq R_g, \; \forall g \in \mathcal{G} \\
& \sum_{s} N_g \, p\,(s,g) \leq S_{max} \\
& 0 \leq p\,(s,g) \leq 1, \; \forall\, s \in \mathcal{S}, \, g \in \mathcal{G} \\
& \sum_{s \in \mathcal{S}} p\,(s,g) \leq 1, \; \forall\, g \in \mathcal{G}.
\end{aligned}
\tag{6.11}
$$

Note that the choice for the LP objective function is also motivated by the intention to shorten the runtime in case of infeasibility due to limited budget. The algorithm does *not* perform the second optimization step if the LP objective value, i.e., minimum necessary budget to satisfy the survey design constraints, is larger than the available budget B.

Given that the performance of these algorithms is problem-dependent, we choose to combine two local search algorithms in order to increase the convergence speed. Global optimization algorithms are available in the NLOPT library but their performance for our problem was significantly worse than the selected local optimization algorithms. The two selected local search algorithms are COBYLA (Constrained Optimization by Linear Approximations), introduced by Powell (1998) (see Roy 2007 for an implementation in `C`) and the Augmented Lagrangian Algorithm (AUGLAG), described in Conn et al. (1991) and Birgin and Martinez (2008). The COBYLA method builds successive linear approximations of the objective function and constraints via a simplex of $n+1$ points (in n dimensions), and optimizes these approximations in a trust region at each step. The AUGLAG method combines the objective function and the nonlinear constraints into a single function, i.e., the objective plus a penalty for any violated constraint. The resulting function is then passed to another optimization algorithm as an unconstrained problem. If the constraints are violated by the solution of this sub-problem, then the size of the penalties is increased and the process is repeated. Eventually, the process must converge to the desired solution, if that exists.

As local optimizer for the AUGLAG method we choose MMA (Method of Moving Asymptotes, introduced in Svanberg 2002), based on its performance for our numerical experiments. The concept behind MMA is as follows. At each point $\mathbf{x}$, MMA forms a local approximation, that is both convex and separable, using the gradient of $f(\mathbf{x})$ and the constraint functions, plus a quadratic penalty term to make the approximations conservative, e.g., upper bounds for the exact functions. Optimizing the approximation leads to a new candidate point $\mathbf{x}$. If the constraints are met, then the process continues from the new point $\mathbf{x}$, otherwise, the penalty term is increased and the process is repeated.

The reason for using two local search algorithms is that AUGLAG performs better in finding the neighborhood of the global optimum but COBYLA provides a greater accuracy in locating the optimum. Therefore, the LP optimal solution is first submitted to AUGLAG and after a number of iterations, when the improvement in the objective value is below a specified threshold, the current solution of AUGLAG is submitted to COBYLA for increased accuracy. However, for large feasible regions, i.e., large values for budget and sample size, the computational time can increase up to tens of minutes if the accuracy tolerance for COBYLA is set very low. From a practical perspective, it is considered accurate enough if the obtained objective value is within 10^{-4} away from the global optimum. Any further accuracy gains are completely blurred by the sampling variation and accuracy of the input parameters themselves.

6.4 Case study: the Dutch Labor Force Survey

The Dutch LFS is a monthly household survey using a rotating panel with five waves at quarterly intervals. The first wave was conducted using face-to-face interviews up to 2009. Over the years 2010-2012, the first wave was gradually redesigned to a mixed-mode survey employing web, telephone and face-to-face. In the four subsequent waves, data are collected by telephone. During these re-interviews, a condensed questionnaire is applied to establish changes in the labor market position of the respondents. The face-to-face contact strategy for the LFS consists of a maximum of six visits to the address. If no contact was made at the sixth visit, then the address is processed as a noncontact.

The key statistics produced based on the LFS data are estimates of the percentage of persons employed, unemployed and not in the labor force in the Netherlands and in various regional and socio-demographic subpopulations. The target population consists of persons aged 15 years and older (i.e., the potential labor force population). For all members of participating households, demographic variables are observed. For the target variables, only persons aged 15 years and older are interviewed. When a household member cannot be contacted, proxy interviewing is allowed by members of the same household. Households in which one or more members do not respond are treated as nonresponding households.

In order to keep the exposition simple, we restrict ourselves to the first wave. We use 2010–2012 LFS data to estimate various input parameters for the optimization model. Although the LFS sample is based on addresses, it is possible to zoom in on the individual level using the municipal registration of population data. Erroneous records such as in-existent addresses or empty house addresses are removed from the sample.

In order to investigate mode effects, we augmented the LFS with data from the POLIS

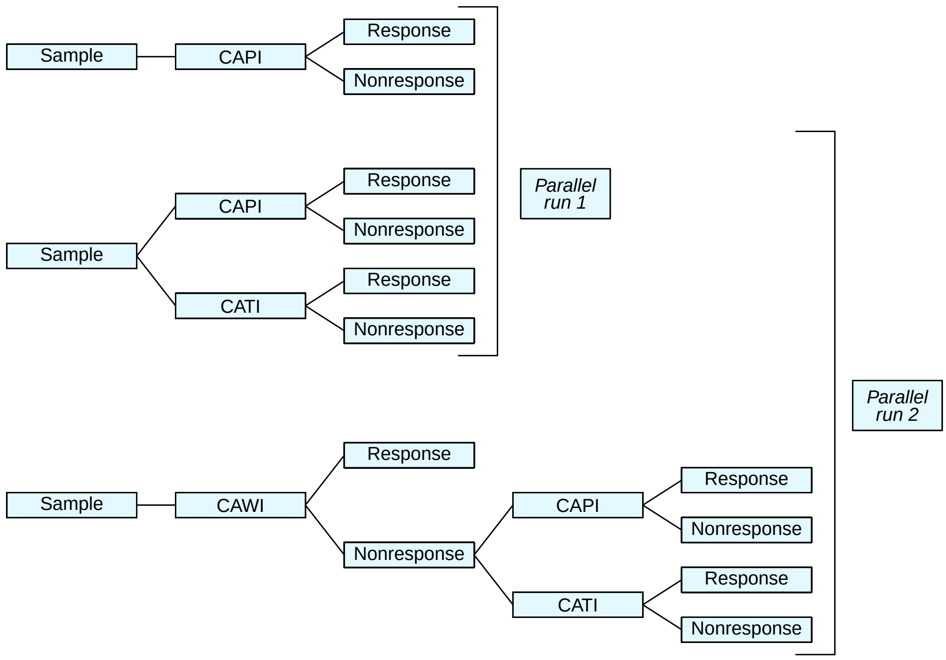


FIGURE 6.1: Parallel runs LFS redesign from unimode (CAPI) to mixed-mode.

and UWV werkbedrijf registers. The POLIS register contains information about income from employment and social benefits; it does not however contain information on income from self-employment. From this register we can determine whether a person is employed and the number of jobs they have. The UWV assists unemployed people in finding a job. Unemployment benefits can be received only by those registered at UWV.

The redesign of the LFS from a unimode (CAPI) to a mixed-mode design involved two parallel runs of different designs (see Figure 6.1). As a consequence, adjustment across the parallel runs is necessary for certain input parameters (see Section 6.4.3). The first LFS mixed-mode run involved a parallel CATI-CAPI design where sample units were approached in CAPI only if no registered phone number was available or if the household size exceeded 3. The current mixed-mode design (second run) offers all sample units a web questionnaire. CAWI nonrespondents are subsequently approached in CATI or CAPI given availability of registered phone and the household size, following the same rules as in the first mixed-mode design. Thus, households with more than 3 members or without a publicly available phone number are approached in CAPI. Due to this structure, a CAWI-CAPI design, where all CAWI nonrespondents would be approached in a CAPI follow-up, is not observable. Sections 6.4.2 and 6.4.3 discuss an approximation method for producing suitable estimates of the optimization input parameters in the absence of such historical information.

We note here that accuracy in the estimators of the optimization input parameters (i.e., response probabilities $\rho(s,g)$, unit costs $c(s,g)$ and mode differences $D(s,g)$) is crucial for a successful implementation of the optimal design in practice. However, in this paper, we have not performed sensitivity analyses, which would be necessary to assess

Characteristic	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
65+		✓							
No 65+			✓	✓	✓	✓			
$15-26$			✓			✓			
No $15-26$				✓	✓		✓	✓	
Household size ≤ 3		✓	✓	✓	✓	✓	✓	✓	
Household size > 3									✓
Ethnicity non-western				✓			✓		
Other ethnicities					✓			✓	
UWV registration	✓								
No UWV registration		✓	✓	✓	✓	✓	✓	✓	✓
POLIS employed						✓	✓	✓	
POLIS not employed		✓	✓	✓	✓				
w_g (%)	7.46	19.77	2.38	1.53	10.97	15.59	3.91	33.50	4.89

TABLE 6.1: Population clustering given set of characteristics X

the robustness of the optimal design.

6.4.1 Population groups

The population units are clustered into $\mathcal{G} = \{g_1, g_2, \ldots, g_9\}$ homogeneous groups (see Table 6.1) given the following characteristics, $X =$ (age, household size, UWV registration, POLIS registration of employment, ethnicity). The proportion w_g of each group in the total population is also provided. The characteristics enumerated in X were selected based on their close relationship to the survey target variables and the sampling frame variables. The list of characteristics may be extended, but the resulting groups should be big enough to ensure satisfactory precision of the optimization parameters.

Population clustering, as presented in Table 6.1, should be interpreted as follows. The tick sign marks what characteristic from X is active for the given group. For example, group 1 clusters population units that are registered at UWV. For this group, none of the other characteristics are relevant. For group 4, a number of characteristics are used in defining the cluster, namely, age (it does *not* contain units older than 65 or younger than 26 years of age), household size (population units in this group belong to households with less than 3 members), ethnicity (population units in this group have non-western origins), UWV registration (units in this group do *not* have a UWV registration) and POLIS registration of employment (according the POLIS records, units in this group are *not* employed).

The LFS targets people with age between 15 and 64 years, therefore, undersampling occurs for addresses with households consisting only of persons of 65 years of age and higher. Moreover, households with persons of age between 15 and 26 or from non-western countries present more interest therefore such households are oversampled. Let $\mathcal{G}' = \{g'_1, g'_2, \ldots, g_5\}$ be the clustering of population units according to the set of characteristics $X' =$ (age, UWV registration, ethnicity) (see Table 6.2) that have unequal sampling

Characteristic	g'_1	g'_2	g'_3	g'_4	g'_5
65+	✓				
No 65+		✓	✓	✓	✓
15 − 26			✓		
No 15 − 26				✓	✓
UWV registration		✓			
No UWV registration			✓	✓	✓
Ethnicity non-western				✓	
Other ethnicities					✓

TABLE 6.2: Sampling strata definition.

probabilities. This clustering occurs before the random sampling procedure has been carried out and we further refer to $\mathcal{G}'$ as the *sampling strata*. Information in Table 6.2 should be interpreted analogously to Table 6.1.

Similarly to constraint (6.5), we impose that a minimum number of households in each $g' \in \mathcal{G}'$ should be a respondent by the end of the survey. To formulate such a constraint, the response probabilities have to estimated at (g, g') level (see Section 6.4.2). The corresponding constraint is given by

$$\sum_{s,g} N_{g,g'}\, p\,(s,g)\, \rho\,(s,g,g') \geq R_{g'},\ \forall g' \in \mathcal{G}',$$

where $N_{g,g'}$ represents the population size for group (g, g').

6.4.2 Estimation of response probabilities

In every survey each population unit is assigned a non-zero probability of being sampled through a random selection procedure. Let d_i^D be the inverse of this probability for a population unit i which is most commonly known as the *design weight*. The sample estimate of a population mean $\bar{Y}$ is then computed as

$$\bar{y} = \frac{1}{N} \sum_{i \,\in\, sample} d_i^D Y_i,$$

where Y_i is the value of parameter Y for unit i. This yields an unbiased estimate of the population mean and it is also known as the *Horvitz-Thompson estimator*. SÄRNDAL ET AL. (1992) modify the Horvitz-Thompson estimator to account for nonresponse, i.e.,

$$\bar{y}^r = \frac{1}{N} \sum_{i \,\in\, resp} \frac{d_i^D Y_i}{\rho_i},$$

where ρ_i represents the unknown response probability of unit i and $resp$ the respondent sample. The unknown response probabilities can be replaced by their corresponding estimates based on auxiliary information (see SÄRNDAL 1981) or by the Horvitz-Thompson estimator for the mean response probability (see BETHLEHEM 1988) that also uses auxiliary information. Take d_i^A be the inverse of the Horvitz-Thompson estimator for the mean response probability. We then have

$$\tilde{y}^r = \frac{1}{N} \sum_{i \in resp} d_i^D d_i^A Y_i.$$

In order to prevent large capacity variations between subsequent modes in mixed-mode surveys, a subsampling of the remaining nonrespondents is carried out before the follow-up mode. In this case, the Horvitz-Thompson estimator is given by

$$\tilde{y}^r_{\mathrm{MM}} = \frac{1}{N} \sum_{i \in resp} d_i Y_i,$$

with

$$d_i = d_i^D d_i^A d_i^S,$$

the total adjusted weight and d_i^S is the subsampling rate. Note that $\tilde{y}^r_{\mathrm{MM}}$ can be used for unimode surveys if d_i^S is set to 1 for all units i that did not respond in the first mode but were respondents in the follow-up mode.

Aggregating individual response probabilities to group g level yields the following

$$\rho\,(s,g) = \frac{\sum_{i \in g} d_i \frac{1}{d_i^A} R_i^s}{\sum_{i \in g} d_i R_i^s},$$

where $R_i^s \in \{0,1\}$ indicates whether unit i is a respondent through strategy s.

A final step in the estimation of the response probabilities is the sub-/over-sampling of rate of the sampling strata $\mathcal{G}'$. Let $z(g')$ be the sub-/over-sampling rate for stratum $g' \in \mathcal{G}'$, relative to a base stratum g'_{base}. Then the unadjusted design weight for group g', i.e., the design weight in the absence of sub-/over-sampling, is given by $d^D_{g',\mathrm{UN}} = z(g')\, d^D_{g'_{base}}$. The sample size is computed as

$$n = \sum_{g' \in \mathcal{G}'} d^D_{g',\mathrm{UN}}\, N_{g'},$$

with $N_{g'}$ the population size of stratum g'. Assuming all $N_{g'}$ are known, we can now derive $d^D_{g'_{base}}$ by replacing $d^D_{g',\mathrm{UN}}$ accordingly. With this adjustment, the response probabilities estimates are given by

$$\rho\,(s,g,g') = \frac{\sum_{i \in (g,g')} d^D_{i,\mathrm{UN}} d_i^S R_i^s}{\sum_{i \in (g,g')} d_i R_i^s}, \tag{6.12}$$

where the summation is taken only over units in (g, g'). Aggregating over all sampling strata we obtain the response probability estimates for group g, i.e.,

$$\rho\,(s,g) = \sum_{g' \in \mathcal{G}'} \frac{N_{g'}}{N_g}\, \rho\,(s,g,g'),\ \forall\, s \in \mathcal{S},\ g \in \mathcal{G}. \tag{6.13}$$

Table 6.3 presents the estimated response probabilities $\rho\,(s,g)$ from available data. Note that given the complex definition of these estimates, it is not possible to compute directly their standard deviations. We perform a bootstrap analysis in order to assess the

$\rho(s,g)$	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
CAWI	23.2% (0.3)	23.6% (0.6)	15.5% (0.6)	10.8% (0.6)	27.9% (0.4)	27.7% (0.2)	17.5% (0.5)	36.7% (0.2)	22.4% (0.5)
CATI2	12.2% (0.5)	31.4% (1.1)	8.5% (0.8)	4.7% (0.8)	19.7% (0.6)	13.3% (0.4)	7.2% (0.5)	18.1% (0.4)	21.2% (0.8)
CATI2+	20.8% (0.6)	41.3% (1.1)	15.2% (1.0)	8.6% (1.0)	31.1% (0.7)	23.8% (0.5)	14.3% (0.7)	33.3% (0.5)	37.5% (0.9)
CAPI3	43.5% (1.5)	53.5% (1.7)	42.2% (2.4)	34.1% (2.4)	45.1% (1.1)	45.3% (0.9)	35.9% (1.5)	46.7% (0.7)	54.6% (1.4)
CAPI3+	52.4% (1.3)	58.3% (1.6)	51.0% (2.5)	41.2% (2.2)	51.2% (1.1)	54.9% (0.8)	46.0% (1.4)	56.8% (0.7)	61.4% (1.3)
CAWI-CATI2	28.3% (0.4)	41.0% (0.8)	20.2% (0.7)	13.9% (0.8)	36.3% (0.4)	34.0% (0.3)	20.8% (0.5)	44.5% (0.3)	23.1% (0.5)
CAWI-CATI2+	32.8% (0.4)	48.4% (0.7)	23.8% (0.8)	17.5% (0.9)	42.1% (0.5)	41.1% (0.3)	25.8% (0.6)	52.1% (0.3)	24.4% (0.5)
CAWI-CAPI3	46.3% (0.5)	57.7% (1.0)	38.6% (1.0)	32.7% (1.0)	50.0% (0.6)	51.0% (0.4)	39.3% (0.7)	58.9% (0.4)	50.0% (0.5)
CAWI-CAPI3+	49.8% (0.5)	58.3% (0.9)	43.4% (0.9)	36.6% (0.9)	52.6% (0.5)	54.7% (0.4)	44.3% (0.6)	62.0% (0.4)	54.2% (0.5)

TABLE 6.3: Estimated response probabilities per strategy s and group g.

standard errors that are provided in brackets. Additionally, given the weighting technique necessary to estimate the response propensities, the weights d_i need to be adjusted in the bootstrap analysis in order to correctly scale up bootstrap sample estimates to the same population composition.

As expected, restricted strategies, i.e., strategies with a cap on the number of attempts, yield lower response probabilities than full strategies. The only reason why restricted strategies may be present in the optimal solution would be the incurred lower costs (see Section 6.4.4). Additionally, note that all mixed-mode strategies yield higher response probabilities than the CAWI-only strategy and the mixed-mode involving CAPI is more appealing in terms of response than mixed-mode involving CATI. However, in terms of costs, the situation is opposite, i.e., mixed-mode with CATI being significantly less expensive than mixed-mode with CAPI.

However, when the selected strategy s does not have historical support, i.e., there is no survey design in historical data that matches the strategy s specific combination of survey mode - number of attempts, additional modeling is necessary. The structure of the current LFS mixed-mode design assigns CAWI nonrespondents to CATI or CAPI given availability of phone numbers. Thus, CAWI nonrespondents with a publicly available phone number are approached in CATI in the follow-up. Correspondingly, CAWI nonrespondents without a publicly available phone number are approached in CAPI in the follow-up. For our analysis this translates to lack of historical information for strategies CAWI-CAPI3 and CAWI-CAPI3+ that assign all CAWI nonrespondents to CAPI. As a consequence, we must build approximations for response probabilities $\rho(s_8, g, g')$ and

$\mathcal{G}$	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
$\lambda(g)$	38.1% (0.9)	76.4% (1.6)	30.2% (2.0)	22.4% (2.2)	60.0% (1.1)	38.9% (0.7)	32.0% (1.3)	53.4% (0.6)	62.4% (1.2)

TABLE 6.4: Estimated probabilities for registered phone for group $g \in \mathcal{G}$.

$\rho\,(s_9, g, g')$. This is done as follows

$$\rho\,(s_9, g, g') = \frac{\sum_{i\in(g,g')} d_i^D d_i^S R_i^{s_9}}{\sum_{i\in(g,g')} d_i R_i^{s_9}} + \frac{\rho\,(s_5, g, g')\lambda(g, g')}{\rho\,(s_3, g, g')}\left[\frac{\sum_{i\in(g,g')} d_i^D d_i^S R_i^{s_7}}{\sum_{i\in(g,g')} d_i R_i^{s_7}} - \frac{\sum_{i\in(g,g')} d_i^D d_i^S R_i^{s_1}}{\sum_{i\in(g,g')} d_i R_i^{s_1}}\right], \tag{6.14}$$

where $\lambda(g, g')$ represents the probability for registered phone number in group (g, g'). More specifically, the response probability for strategy CAWI-CAPI3+, $\rho\,(s_9, g, g')$, sums up two terms. The first term computes the response probability for strategy s_9 according to formula (6.12). The second term adjusts this result for the available historical data, where s_9 is not a stand-alone design. Thus, the second term represents an approximation of the response probability CATI respondents would have if approached in CAPI in the mixed-mode design. This probability is obtained by subtracting $\rho\,(s_1, g, g')$ from $\rho\,(s_7, g, g')$, i.e., removing CAWI respondents from the pool of respondents to strategy CAWI-CATI2+, and adjusting the result by the probability for registered phone for respondents in CAPI3+ and the response ratio between strategies CAPI3+ and CATI2+. Table 6.4 shows the estimated probability for registered phone $\lambda(g)$, where we aggregate from the (g, g') level similarly to the response probabilities, i.e.,

$$\lambda(g) = \sum_{g'\in\mathcal{G}'} \frac{N_{g'}}{N_g}\,\lambda(g, g'),\ \forall\, g. \tag{6.15}$$

The response probability for s_8 is computed analogously to $\rho\,(s_9, g, g')$,

$$\rho\,(s_8, g, g') = \rho\,(s_9, g, g')\frac{\sum_{i\in(g,g')} d_i^D d_i^S R_i^{s_8}}{\sum_{i\in(g,g')} d_i^D d_i^S R_i^{s_9}},$$

with the distinction that only units that respond within three CAPI visits in the follow-up are considered.

6.4.3 Estimation of the mode effect measure $D(s, g)$

For the mode effect measure $D(s, g)$, two benchmarks were selected after consultation with practitioners, i.e., $BM_1 = \bar{y}_{CAPI}$ and $BM_2 = 1/3 * (\bar{y}_{CAWI} + \bar{y}_{CATI} + \bar{y}_{CAPI})$, where $\bar{y}_{mode}$ represents the average unemployment rate estimated via the indicated survey mode. The first benchmark assumes that the average unemployment rate estimated via CAPI only, i.e., when the LFS was carried out only as a face-to-face interview, represents the true average unemployment rate. The second benchmark assumes there is no preferred mode, hence, it assigns an equal weight to each of the three modes. Given a benchmark, the mode effect measure for group g under strategy s is computed as

$$D(s,g) = \frac{\sum_{i\in g} d_i R_i^s \, y_i^{unemployed}}{\sum_{i\in g} d_i \, R_i^s \, (y_i^{unemployed} + \, y_i^{employed} + \, y_i^{nonlabor})} - \bar{y}_{BM}, \tag{6.16}$$

where $\bar{y}_{BM}$ represents the benchmark estimate of the unemployment rate, $y_i^{unemployed}$ the number of unemployed household members, $y_i^{employed}$ the number of employed household members and $y_i^{nonlabor}$ the number of household members aged younger than 15. Note that the unemployment rate estimate is a quantity in $[0,1]$, therefore $D(s,g) \in [-1,1]$ which implies that $\bar{D}_{BM} \in [0,1]$.

As remarked in Section 6.4.2, due to the structure of the mixed-mode design, estimation of $D(s,g)$ for $s \in$ {CAWI-CAPI3,CAWI-CAPI3+} cannot be carried out directly. Moreover, adjustments are necessary to account for estimation of quantities across the parallel runs of the LFS. Let $D(s,s',g) = \bar{y}_{s,g} - \bar{y}_{s',g}$ be such an adjustment step between estimates of the unemployment rate obtained in strategies s and s'. Then,

$$D(s,g) = D(s',g) + D(s,s',g), \text{ for } s' \neq s. \tag{6.17}$$

Using (6.17), the adjusted mode differences are computed as

$$\begin{aligned}
D(s_1,g) &= D(s_3,g) + D(s_1,s_3,g) \\
D(s_6,g) &= D(s_6,s_2,g) + D(s_2,g) \\
D(s_7,g) &= D(s_7,s_3,g) + D(s_3,g) \\
D(s_8,g) &= p_{CAWI} D(s_1,g) + (1 - p_{CAWI}) D(s_4,g) \\
D(s_9,g) &= p_{CAWI} D(s_1,g),
\end{aligned} \tag{6.18}$$

for all $g \in \mathcal{G}$, with p_{CAWI} the proportion of CAWI respondents in the total respondent sample in the mixed-mode design. Tables 6.5 and 6.6 present the estimated mode differences against the two benchmarks.

Generally, $D_{BM_1}(s,g) > D_{BM_2}(s,g)$ for strategies involving CAWI. This is understandable since BM_2, as a mode mix, is "closer" to CAWI than BM_1. Furthermore, for BM_1, the mode differences $D(s_1,g)$ are higher than $D(s,g)$ for $s \neq s_1$. Looking at differences across groups, group 4 produces the highest $D(s,g)$ relative to the other groups for both benchmarks. Although units in this group do not have a UWV registration, i.e,. they are not looking for a job, they are registered as unemployed. Moreover, it has been observed in the past that unemployment rates for non-western ethnicities are generally higher than for other ethnicities. Additionally, group 3, that includes unemployed young people (15-26), displays slightly higher mode differences than the other groups. Population group 2 yields usually very low values, which can be explained by the fact that most population units aged 65 and higher fall into this group. Most often, such persons are either retired or employed, therefore producing a group unemployment rate very close to 0. Note that the standard deviations of $D_{BM_1}(\text{CAPI3+}, g)$ will always be 0 since its value is constant

$D_{BM_1}(s,g)$	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
CAWI	1.5%	0.0%	−2.3%	−4.5%	0.9%	−0.4%	−2.2%	0.6%	−0.4%
	(1.0)	(0.5)	(1.5)	(3.1)	(0.7)	(0.4)	(1.5)	(0.5)	(0.6)
CATI2	−0.2%	−0.1%	−2.6%	−6.8%	−1.0%	−0.9%	−1.1%	0.2%	−1.3%
	(0.7)	(0.1)	(0.9)	(1.8)	(0.4)	(0.3)	(1.1)	(0.4)	(0.4)
CATI2+	−0.1%	−0.1%	−2.3%	−4.9%	−0.6%	−1.0%	−0.8%	−0.2%	−1.2%
	(0.7)	(0.1)	(0.8)	(1.7)	(0.4)	(0.3)	(1.0)	(0.3)	(0.4)
CAPI3	−0.5%	−0.1%	0.0%	0.7%	−0.1%	0.0%	0.5%	0.3%	0.1%
	(0.3)	(0.1)	(0.4)	(0.6)	(0.1)	(0.1)	(0.3)	(0.1)	(0.1)
CAPI3+	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
CAWI-CATI2	0.9%	0.0%	−2.4%	−3.4%	−0.1%	−0.7%	−4.4%	0.9%	−0.7%
	(1.0)	(0.4)	(1.5)	(3.7)	(0.6)	(0.5)	(1.9)	(0.5)	(0.6)
CAWI-CATI2+	0.9%	−0.1%	−3.7%	−1.7%	0.5%	−0.7%	−3.0%	0.6%	−0.4%
	(0.9)	(0.3)	(1.4)	(3.2)	(0.7)	(0.4)	(1.4)	(0.5)	(0.6)
CAWI-CAPI3	0.7%	0.0%	−1.2%	−1.6%	0.6%	−0.3%	−1.0%	0.5%	−0.2%
	(0.6)	(0.3)	(0.8)	(1.4)	(0.5)	(0.3)	(0.8)	(0.3)	(0.3)
CAWI-CAPI3+	0.9%	0.0%	−1.2%	−2.0%	0.6%	−0.3%	−1.2%	0.4%	−0.2%
	(0.6)	(0.3)	(0.8)	(1.4)	(0.5)	(0.3)	(0.8)	(0.3)	(0.3)

TABLE 6.5: Estimated mode differences against benchmark $BM_1 = \bar{y}_{CAPI}$.

across bootstrap runs, i.e., it is always equal to zero given the definitions of the survey strategy and the benchmark, respectively.

6.4.4 Estimation of unit costs

The cost estimation process follows closely the actual cost computations from practice. This means that all major cost-incurring activities are taken into consideration such as average number of attempts until contact, interview time and travel time. Other costs such as questionnaire design or interviewer training, are considered one-time costs that occur before the start of the data collection, hence, they do not depend on the selected strategy or group. Consequently, it is not necessary to include overhead costs in the analysis. Furthermore, we assume that LFS workload for CAPI and CATI interviewers resulting from the optimization model gets subsumed in regular interviewer workloads, i.e., small allocated samples can be treated as larger ones since they are part of larger workloads. With these assumptions we do not have to account for clustering of addresses. Essentially, the $c(s,g)$ estimate represents the expected costs to address one population unit from group g using strategy s, i.e., it includes the corresponding response probability $\rho(s,g)$ such that the outcome of the survey attempt is considered.

Note that in the case of $c(s_1,g)$ the standard deviation will always be 0. The costs for the CAWI-only strategy do not depend on the response rate but only on the sample size, i.e., sending a web questionnaire to all sample units, which is constant across the bootstrap runs.

$D_{BM_2}(s,g)$	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
CAWI	1.0% (0.5)	0.1% (0.3)	−0.8% (0.9)	−1.4% (1.8)	0.8% (0.4)	0.1% (0.2)	−1.2% (0.8)	0.5% (0.2)	0.1% (0.3)
CATI2	−0.6% (0.3)	−0.1% (0.2)	−1.0% (0.6)	−3.7% (1.4)	−1.2% (0.2)	−0.5% (0.2)	−0.1% (0.8)	0.1% (0.2)	−0.8% (0.2)
CATI2+	−0.6% (0.2)	−0.1% (0.2)	−0.8% (0.5)	−1.7% (1.0)	−0.7% (0.2)	−0.5% (0.1)	0.2% (0.5)	−0.3% (0.1)	−0.6% (0.2)
CAPI3	−1.0% (0.7)	−0.1% (0.2)	1.6% (0.8)	3.8% (1.6)	−0.2% (0.4)	0.5% (0.2)	1.5% (0.8)	0.2% (0.3)	0.6% (0.3)
CAPI3+	−0.5% (0.5)	0.0% (0.2)	1.6% (0.7)	3.1% (1.4)	−0.1% (0.4)	0.5% (0.2)	1.0% (0.7)	−0.1% (0.3)	0.5% (0.3)
CAWI-CATI2	0.4% (0.5)	0.0% (0.3)	−0.9% (1.0)	−0.3% (2.9)	−0.2% (0.4)	−0.2% (0.3)	−3.4% (1.5)	0.7% (0.3)	−0.1% (0.4)
CAWI-CATI2+	0.5% (0.4)	0.0% (0.2)	−2.1% (0.8)	1.5% (2.0)	0.4% (0.4)	−0.2% (0.2)	−2.0% (0.8)	0.5% (0.2)	0.1% (0.3)
CAWI-CAPI3	0.3% (0.2)	0.0% (0.1)	0.4% (0.3)	1.5% (0.6)	0.5% (0.2)	0.2% (0.1)	0.0% (0.3)	0.4% (0.1)	0.3% (0.1)
CAWI-CAPI3+	0.4% (0.1)	0.0% (0.1)	0.4% (0.3)	1.1% (0.5)	0.5% (0.2)	0.2% (0.1)	−0.2% (0.3)	0.3% (0.1)	0.3% (0.1)

TABLE 6.6: Estimated mode differences against benchmark $BM_2 = 1/3 * (\bar{y}_{CAWI} + \bar{y}_{CATI} + \bar{y}_{CAPI})$.

6.4.5 Optimization results

In our numerical experiments, we explore the solution structure for various values of the constraint thresholds, namely we let

$$B \in \{160{,}000; 170{,}000; 180{,}000\}$$
$$M \in \{1\%; 0.5\%; 0.25\%\}$$
$$S_{max} \in \{9{,}500; 12{,}000; 15{,}000\}.$$

For the minimal precision constraints, we keep the constraints' right-hand side terms unchanged, with the following values

$$R_{g'} = (165.35, 533.50, 1359.34, 303.03, 2135.15)$$
$$R_g = (533.69, 162.07, 142.47, 84.77, 529.79, 933.66, 215.83, 1603.43, 290.63).$$

These values have been computed such that a 95% confidence interval is built for the population unemployment rate given the survey estimate. Since $\bar{D}_{BM} \in [0,1]$, it follows that the left hand side of (6.6) also takes values in the $[0,1]$ interval. The low values chosen for the threshold M are determined by the maximum absolute differences in mode effects observed among the groups when x^*_{LP}, the LP optimal solution, is applied (see Table 6.8). If for example, $M \geq 2.06\%$, the optimal solution for $S_{max} = 9{,}500$ and BM_1 would simply be the LP solution. Table 6.9 provides an overview of the optimization results for the original nonlinear problem in (6.10).

Two conclusions can be drawn. First, increasing the sample size and/or the budget brings the objective value down, reaching 0 for $S_{max} = 15{,}000$ and $B = 180{,}000$ for

$c(s,g)$	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
CAWI	1.6 (0.0)	1.6 (0.0)	1.6 (0.0)	1.5 (0.0)	1.6 (0.0)	1.6 (0.0)	1.6 (0.0)	1.6 (0.0)	1.5 (0.0)
CATI2	4.9 (0.1)	5.8 (0.1)	4.3 (0.1)	3.9 (0.1)	5.3 (0.1)	5.3 (0.1)	4.4 (0.1)	5.6 (0.0)	7.2 (0.1)
CATI2+	6.0 (0.1)	6.6 (0.1)	5.0 (0.1)	4.3 (0.1)	6.3 (0.1)	6.8 (0.1)	5.3 (0.1)	7.4 (0.1)	10.0 (0.2)
CAPI3	38.8 (0.4)	34.5 (0.5)	36.4 (0.5)	35.7 (0.8)	35.9 (0.3)	39.9 (0.2)	38.7 (0.5)	39.7 (0.2)	45.4 (0.5)
CAPI3+	46.2 (0.6)	38.7 (0.6)	43.9 (0.7)	43.3 (1.1)	41.6 (0.4)	47.6 (0.3)	47.7 (0.6)	47.5 (0.2)	51.2 (0.5)
CAWI-CATI2	3.7 (0.0)	4.2 (0.1)	3.9 (0.1)	3.9 (0.1)	3.6 (0.0)	3.7 (0.0)	3.8 (0.0)	3.3 (0.0)	3.4 (0.0)
CAWI-CATI2+	4.1 (0.1)	4.6 (0.1)	4.2 (0.1)	4.2 (0.1)	4.0 (0.1)	4.3 (0.0)	4.3 (0.1)	3.9 (0.0)	3.6 (0.0)
CAWI-CAPI3	27.5 (0.3)	25.6 (0.7)	28.1 (0.6)	30.4 (0.8)	24.7 (0.4)	26.7 (0.3)	31.1 (0.5)	24.0 (0.2)	31.3 (0.4)
CAWI-CAPI3+	33.0 (0.4)	27.3 (0.7)	35.2 (0.9)	36.5 (1.2)	30.2 (0.6)	32.5 (0.4)	38.5 (0.8)	29.5 (0.3)	36.2 (0.6)

TABLE 6.7: Estimated unit costs (in euros) per strategy s and group g.

Sample size (S_{max})	Objective value (min costs)	Benchmark	Mode effect ($\bar{D}_{BM}$)	Max difference in mode effects (M)	Response rate
9,500	123,748.50	BM_1	0.16%	2.06%	48.0%
		BM_2	0.29%	3.31%	
11,000	88,408.95	BM_1	0.05%	5.97%	39.9%
		BM_2	0.19%	2.98%	
12,500	82,270.72	BM_1	0.08%	5.97%	36.9%
		BM_2	0.21%	2.98%	
15,000	74,350.44	BM_1	0.12%	5.97%	29.4%
		BM_2	0.25%	2.39%	

TABLE 6.8: Overview optimization results linear programming formulation - minimize costs.

all levels of M. Second, using BM_2 as benchmark, yields lower objective values than BM_1 (except the case of S_{max} = 9,500), which is mainly due to the smaller values of $D(s,g)$. Additionally, there is an increased similarity among groups with respect to the deviation from the benchmark, i.e., $D_{BM_2}(s,g)$'s are close in absolute value. This allows feasibility even for $M = 0.01\%$ and for B = 180,000 and S_{max} the algorithm still yields an objective value very close to 0 (0.00001%).

A more counterintuitive effect is shown by the invariance of the objective value given decreasing values of M for BM_2. The reason is that the yielded solution is a point contained in all three feasible regions, i.e., a stricter bound on the maximum difference in group mode effects does not remove this point from the larger feasible region. The same holds for the invariance of $\bar{D}_{BM_2}$ with respect to increasing budget for S_{max} = 9,500, where larger feasible regions do not add points that could improve the objective value. For $\bar{D}_{BM_1}$ on the other hand, any change in the budget level or a lower value of M causes a change in the objective value. Note that the improvement step in the objective value

S_{max}	B	BM	M	$\bar{D}_{BM}$	M	$\bar{D}_{BM}$	M	$\bar{D}_{BM}$
9,500	160,000	BM_1 BM_2	1%	0.155% 0.170%	0.5%	Infeasible	0.25%	Infeasible
	170,000	BM_1 BM_2	1%	0.131% 0.170%	0.5%	Infeasible	0.25%	Infeasible
	180,000	BM_1 BM_2	1%	0.100% 0.170%	0.5%	Infeasible	0.25%	Infeasible
12,000	160,000	BM_1 BM_2	1%	0.097% 0.046%	0.5%	0.119% 0.046%	0.25%	0.123% 0.046%
	170,000	BM_1 BM_2	1%	0.076% 0.036%	0.5%	0.093% 0.036%	0.25%	0.101% 0.036%
	180,000	BM_1 BM_2	1%	0.009% 0.014%	0.5%	0.058% 0.014%	0.25%	0.095% 0.014%
15,000	160,000	BM_1 BM_2	1%	0.051% 0.006%	0.5%	0.094% 0.006%	0.25%	0.112% 0.006%
	170,000	BM_1 BM_2	1%	0.020% 0.004%	0.5%	0.080% 0.004%	0.25%	0.097% 0.004%
	180,000	BM_1 BM_2	1%	0.005% 0.000%	0.5%	0.058% 0.000%	0.25%	0.095% 0.000%

TABLE 6.9: Overview optimization results nonlinear problem - minimize overall mode effects in LFS.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
CAWI	0%	0%	0%	1%	0%	0%	0%	0%	0%
CATI2	0%	0%	14%	22%	31%	0%	0%	0%	0%
CATI2+	0%	0%	6%	2%	12%	0%	0%	0%	0%
CAPI3	39%	0%	3%	4%	2%	0%	0%	0%	0%
CAPI3+	0%	0%	70%	71%	5%	0%	65%	0%	0%
CAWI-CATI2	0%	4%	0%	0%	0%	0%	0%	0%	0%
CAWI-CATI2+	0%	96%	2%	0%	45%	43%	0%	100%	0%
CAWI-CAPI3	0%	0%	0%	0%	2%	29%	0%	0%	100%
CAWI-CAPI3+	61%	0%	5%	0%	2%	29%	35%	0%	0%

TABLE 6.10: Strategy assignment given optimal solution for $S_{max} = 9{,}500$, $B = 170{,}000$, $M = 1\%$, BM_1.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
CAWI	2%	10%	0%	0%	0%	99%	0%	7%	0%
CATI2	1%	13%	0%	20%	0%	0%	41%	3%	0%
CATI2+	5%	4%	1%	2%	0%	0%	0%	5%	0%
CAPI3	22%	0%	0%	77%	0%	0%	3%	1%	6%
CAPI3+	8%	7%	81%	0%	0%	0%	45%	56%	85%
CAWI-CATI2	0%	1%	1%	0%	98%	0%	0%	14%	0%
CAWI-CATI2+	60%	39%	0%	0%	0%	0%	10%	6%	0%
CAWI-CAPI3	2%	5%	17%	0%	0%	1%	0%	8%	7%
CAWI-CAPI3+	0%	20%	1%	1%	1%	0%	0%	0%	2%

TABLE 6.11: Strategy assignment given optimal solution for $S_{max} = 15{,}000$, $B = 170{,}000$, $M = 1\%$, BM_1.

decreases when tighter bounds are imposed on the group difference in mode effects.

We can analyze the impact of the sample size by comparing the optimal solutions for $S_{max} = 9{,}500$ and $S_{max} = 15{,}000$. Consider $B = 170{,}000$, $M = 1\%$ and BM_1 with the corresponding optimal solutions given in Tables 6.10 and 6.11. For a clearer exposition

S_{max}	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
9500	0.06%	0.07%	0.00%	0.00%	0.00%	0.22%	0.23%	0.17%	0.09%
15000	0.08%	0.08%	0.21%	0.06%	0.55%	0.36%	0.36%	0.04%	0.01%

TABLE 6.12: Sampling probabilities for $S_{max} = 9{,}500$ and $S_{max} = 15{,}000$, when $B = 170{,}000$, $M = 1\%$, BM_1.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
CAWI	2%	0%	0%	0%	0%	2%	3%	0%	0%
CATI2	3%	1%	28%	0%	2%	1%	1%	2%	3%
CATI2+	45%	94%	12%	7%	62%	0%	80%	44%	21%
CAPI3	42%	0%	40%	1%	0%	0%	12%	2%	14%
CAPI3+	0%	0%	7%	88%	0%	1%	0%	3%	3%
CAWI-CATI2	1%	2%	2%	0%	0%	0%	0%	1%	59%
CAWI-CATI2+	0%	1%	1%	1%	36%	79%	0%	47%	0%
CAWI-CAPI3	6%	1%	7%	3%	0%	16%	3%	0%	0%
CAWI-CAPI3+	0%	0%	2%	1%	0%	1%	1%	1%	0%

TABLE 6.13: Strategy assignment given optimal solution for $S_{max} = 12{,}000$, $B = 160{,}000$, $M = 1\%$, BM_1.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
CAWI	42%	0%	0%	0%	0%	100%	0%	63%	20%
CATI2	0%	0%	0%	0%	10%	0%	01%	0%	21%
CATI2+	0%	0%	0%	0%	32%	0%	0%	0%	7%
CAPI3	0%	0%	5%	73%	57%	0%	0%	0%	4%
CAPI3+	58%	0%	67%	3%	1%	0%	55%	31%	20%
CAWI-CATI2	0%	0%	0%	0%	0%	0%	0%	0%	1%
CAWI-CATI2+	0%	100%	0%	0%	0%	0%	0%	6%	1%
CAWI-CAPI3	0%	0%	28%	1%	0%	0%	45%	0%	1%
CAWI-CAPI3+	0%	0%	0%	22%	0%	0%	0%	0%	25%

TABLE 6.14: Strategy assignment given optimal solution for $S_{max} = 12{,}000$, $B = 180{,}000$, $M = 1\%$, BM_1.

B	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
160000	0.09%	0.09%	0.02%	0.48%	0.53%	0.27%	0.17%	0.06%	0.03%
180000	0.07%	0.07%	0.00%	0.00%	0.00%	0.42%	0.24%	0.20%	0.00%

TABLE 6.15: Sampling probabilities for $B = 160{,}000$ and $B = 180{,}000$, when $S_{max} = 12{,}000$, $M = 1\%$, BM_1.

of the results, we do not provide the values of $p\,(s,g)$ directly, since they can be as small as 10^{-14}, but instead a derived quantity, namely the probability of being assigned a strategy after having been sampled, which is computed as

$$\mathbb{P}\{\text{assign}|\,\text{sample}\} = \frac{p\,(s,g)}{1 - p\,(\emptyset,g)}.$$

Additionally, Table 6.12 presents the corresponding sampling probabilities, i.e., $1 - p\,(\emptyset,g)$, which allows the reader to derive the individual values of $p\,(s,g)$. Note that in Table 6.12, the sampling probabilities for groups g_3, g_4 and g_5 are only rounded to 0%, otherwise constraint (6.9) would be violated, and their actual value is equal to

0.0000001%. The very low sampling rates for these groups can be explained through their large deviations in mode effect from the benchmark that cannot be balanced by other groups when the maximum sample size is small. For larger sample sizes, e.g., S_{max} = 15,000, we see that the sampled proportion of these groups increases significantly. Consequently, the proportion of other population groups may go down in order to lower the average mode difference $\bar{D}_{BM}$, e.g., groups g_8 and g_9.

The impact of available budget can be most clearly seen for S_{max} = 12,000 and BM_1, when the objective value drops from 0.097% for B = 160,000 to 0.009% for B = 180,000. The corresponding solutions are provided in Tables 6.13 and 6.14 and the sampling probabilities are given in Table 6.15. Strategy CAPI3+ is often chosen in large proportions when the available budget is sufficient, which leads to a low objective function. On the other hand, for smaller budget levels, the optimal solution presents a mix between telephone strategies and mixed-mode strategies. Although it may seem appropriate to assign CAWI strategies that are cheapest, the corresponding mode effect deviations from benchmark are significantly higher (see again the group mode differences in Table 6.5).

A more careful consideration of the optimal solution reveals that its implementation in practice may be difficult from a logistics point of view. Take for example, the solution from Table 6.10. Carrying out the survey design prescribed by this solution implies offering a CAWI-only survey to only 3 sample units due to the fact the group g_4 is small. Additionally, only 4% of group g_2, i.e., 73 sample units, would receive CAWI-CATI2, while the remaining 96% would receive CAWI-CATI2+. If we adjust this solution by sending the three units from CAWI to one of the interviewer-assisted modes and approach entire group g_2 in CAWI-CATI2+, then the costs will increase by 0.2% and the objective function will decrease by 0.22%. It follows that, if slightly more budget becomes available, then the optimization yields a better objective value and the corresponding optimal solution becomes more practical.

6.5 Discussion

This chapter discusses an optimization model that combines the mathematical framework of adaptive designs with mode effect assessment methods in the attempt to minimize mode effects for a given survey. The research presented here is motivated by the recent development of survey mixed-mode designs. Introducing mixed-mode surveys helps survey organizations get a better grasp on their expenditures. However, the added mode effects may have a strong impact on the accuracy of statistics. As Jäckle et al. (2010) point out, many mode effects are nonlinear in nature and appropriate adjustment methods are still not available. To our best knowledge, this is the first research attempt of its kind and due to its flexibility, our methodology can be used as a basis for more complex settings that aim at addressing mode effects. However, our method requires that candidate strategies have been implemented and accurate estimates exist of mode differences in response and survey outcomes.

We use the adjusted mode effect for the comparison between the survey estimate and a "gold standard" as suggested by Biemer (1988). We propose an optimization model that develops an adaptive survey design such that the overall population mode effect is minimized, subject to constraints on differences in mode effects between important population groups. If it is the designer's choice to focus on a different survey item, then the method is still applicable. Note that, in this case, an appropriate "gold standard" must be specified and the optimization input parameters must change accordingly. If it is the designer's choice to address multiple items simultaneously, then a composite indicator of the mode effects influencing these items must be developed. We find this a challenging task since the survey mode may have different effects on different survey items. In this case, an approach similar to the one suggested in Chapter 5 may be more suitable, where an indicator that summarizes measurement effects across survey items, namely the measurement profile, is employed.

We illustrate our methodology on the unemployment rate, one of the key statistics of the Labor Force Survey (LFS). In our case study on LFS data, we are able to focus on mode measurement effects since mode selection effect can largely be adjusted for given auxiliary information, as concluded by Schouten et al. (2012). We find that, for realistic values of the input parameters, the overall mode effect can be brought to zero. We also study the differences between applying two different "gold standards" in the definition of the mode effect measure. It follows that, if the population groups have similar behavior with respect to the benchmark, then it is easy to lower the overall mode effect even for low budget levels.

The accuracy of input parameters to the optimization model requires additional consideration. Section 6.4.2 dealt with the estimation of the optimization input parameters, i.e., response probabilities, unit costs and mode differences, for all considered population groups and survey strategies. This analysis step is of crucial importance for a successful implementation of the yielded optimal solution. Hence, sufficient historical data should be available to produce reliable optimization input parameters. However, since implementation of mixed-mode designs is rather new, it could happen that certain survey strategies are not backed up by historical data, as it was also the case for our numerical experiments. In such situations, approximation methods could be applied. As a consequence, sensitivity analysis should be performed to test the robustness of the optimal solution in case of small perturbations in the input parameters. Future research should develop a robust and effective model.

7

Dynamic learning in adaptive survey designs

Schouten et al. (2013) mention that adaptive survey designs arose in analogy to medical statistics literature on (adaptive) clinical trials. Here, treatments are varied over various patient groups before the start of the trial but the allocation of treatments can change during the trial according to the patient responses. A clinical trial may be adapted to patient groups according to various rules such as allocation to the treatment with more responses, adding or removing treatments (see Bretz et al. 2009). In the field of survey designs, an adaptive clinical trial design may translate to a *dynamic adaptive survey design* under the condition that no new treatments are added during data collection. Previous chapters described *static adaptive designs*, where a set of design features (survey mode and number of visits per mode) that influence survey costs and quality, is allocated to sample units at the start of the data collection and their respective allocation does not change throughout the survey. Dynamic adaptive designs on the other hand alter the survey strategy allocation during the survey when additional information, i.e., paradata (response process information or additional information on the sample unit) is observed. Thus, the strategy allocation has to adapt according to the newly observed data.

In general, the input parameters to the resource allocation problem (RAP) in adaptive surveys designs may be subject to uncertainty. Sampling variations or changes in people's opinions and beliefs could cause shifts in parameter values between historical data and current design implementation. For large surveys with a great economic impact (e.g., the Labor Force Survey that estimates the population's unemployment rate) such differences are of great concern. It would be thus desirable that the model updates its parameters on the go. For example, take $p(g, s)$, the probability of contact for population group g given strategy s. Historical data provide a mean estimate for $p(g, s)$. However, this value may differ from the true contact probability for group g when strategy s is applied. Hence, by including a learning method in the model, new observations from the true distribution would serve updating the belief on $p(g, s)$. Nonetheless, the goal cannot solely be learning the true parameter values, the model should still optimize survey quality given constraints on the limited resources. Hence, a tradeoff is created between learning the true parameter values and optimal allocation of survey strategies given the current value estimates. Such a tradeoff is known in the literature as the *exploration-exploitation tradeoff* and it is formally studied within reinforcement learning where an agent must learn what actions are best to take so as to maximize some notion of cumulative reward by interacting with its environment (see Sutton and Barto 1998). Reinforcement learning problems have been most thoroughly studied through Markov decision processes within the framework of *multi-armed bandit problems* (MABs).

We focus on budgeted MABs that describe the agent's decision-making process when costs are attached to each pull of an arm and only a limited budget is available. As such, the agent must consider how to learn and exploit the arms to gain maximum expected reward within the available budget. However, high-dimensionality of the problem and absence of independence between arms due to the cost constraint have hindered the development of exact algorithms. In the absence of constraints, the problem admits an elegant optimal policy, the Gittins index (see Gittins 1979), that computes an index for each arm separately, i.e., independently of the other arms, and plays the arm with the highest index. However, in the presence of constraints, the separability property of the index policy vanishes and the Gittins index is no longer optimal. For these reasons, the budgeted MAB has only recently been addressed in the literature. In the following, we introduce the notation for budgeted MABs in adaptive survey designs and we present an algorithm that solves the problem to optimality using dynamic programming. Additionally, we discuss some algorithmic features to improve tractability for bandits with more than two arms in the cost constrained context. To our best knowledge, this is the first research step towards developing dynamic learning techniques for survey designs.

7.1 Literature review multi-armed bandit problems

The name of the research field is an analogy to the decision process a gambler undergoes when choosing which slot machines he should play in order to maximize a notion of cummulative reward. Each slot machine, i.e., a bandit arm, has a probability for reward that is unknown to the gambler. Every arm pull results either in a success, which leads to receiving a reward, or a failure. Thus, the decision problem the gambler is faced with is which slot machines are best to play such that the expected reward over some period of time is maximized. As such, the gambler's dilemma is to choose between obtaining high immediate rewards, i.e., play arms that in his current knowledge have a high probability for reward, or explore other arms and acquire information for better decisions in the future.

Generally, an n-armed bandit problem arises when a decision maker is repeatedly faced with a choice among n actions. After each choice, in case of success, the decision maker receives a reward from a stationary probability distribution that depends on the selected action. The objective is to maximize the long-term discounted rewards. If the decision maker knew the expected reward of every action over the considered period of time, then the problem would be trivial, i.e., he would have to always choose the action with the highest expected reward. However, in practical settings, the decision maker may have no knowledge or only some estimates on the expected rewards. Two extreme situations mark the range for policies the decision maker can follow, namely, the *greedy policy*, that always chooses the action with highest immediate reward, i.e., only exploiting current knowledge about the action's expected rewards, and the *exploring policy* where the goal is to improve the estimate on the action's expected reward while the immediate reward may be lower. However, since it is not possible to both explore and exploit within the same action selection, the decision maker is confronted with the celebrated *exploitation-exploration tradeoff*. Sutton and Barto (1998) stress that whether it is better to explore or exploit depends in a complex way on the precise values of the estimates, uncertainties, and the number of remaining plays.

First formulations of multi-armed bandit problems date back to Wald (1950) and Robbins (1952) that introduce the concept of sequential design of experiments in statistical theory. Prior to the development of this concept, statistical experiments were based on fixed sample size designs which altered the randomness of the studied variables. In sequential designs, the size and composition of the samples are not fixed in advance but they are functions of the observations themselves. However, the complex probability problems that occur in this setting are difficult to solve. Robbins (1952) introduced a simplification of the problem that describes the dilemma of tossing either of two coins of unknown bias (i.e., probability of heads is unknown) when only n tosses are available, each head brings a reward of one unit while the tail brings no reward and the objective is to maximize the total reward. The coins problem, generalized to n coins, has become the basis of n-armed bandit problem formulations.

The sequence of arm pulls clearly depends on the observed sequence of successes and failures, i.e., we learn from the result of previous pulls which arm might have a higher probability for success and we may continue pulling that respective arm. Intuitively, the more prior information exists on the success probability the easier it becomes to choose the "better" arm. In particular, Jones and Gittins (1972) introduce the model-based stochastic multi-armed bandit problem where a prior distribution is available for the arm's success probability. This problem formulation has been extensively studied in decision theory (see for example Whittle 1988 and Bertsekas 2001). For the infinite horizon setting with discounted rewards, a unique greedy optimal solution exists, offered by Gittins (1979), that is also known as the *Gittins index*. The optimal policy in this case describes that, at every decision stage, the arm with the largest Gittins index should be pulled. The associated Gittins index can be interpreted as the reward

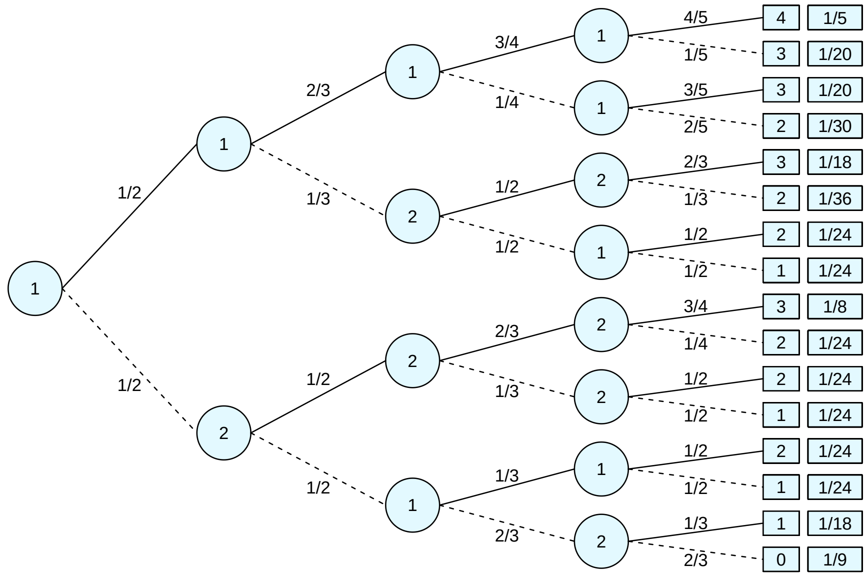


FIGURE 7.1: Decision tree for a classic undiscounted MAB with 2 arms, maximum 4 decision epochs and uniform priors for each arm. End rectangles: final reward (no discounting) and probability of achieving that reward. Nodes: the arm pulled. Full lines: arm transition in case of a success. Dotted lines: arm transition in case of a failure. Line label: probability of outcome.

that can be achieved by pulling the arm from the current state onwards until a stopping time. The optimality of the index policy implies that the problem exhibits a separability property whereby the optimal decision at each step is obtained by computations performed separately for each bandit-arm. Exploiting this property translates into an efficient decision making algorithm. A visualization of the MAB decision tree with two arms and no prior knowledge, i.e., uniform prior distributions for both arms' success probabilities, is provided in Figure 7.1 (adjusted from STOUT AND HARDWICK 2006). The expected reward for this MAB is 2.28.

Most often, the performance of a proposed policy is measured in terms of *regret*, defined as the difference between the expected return of the optimal strategy, i.e., consistently performing the best action, and the proposed policy's expected return. Using statistical assumptions, LAI AND ROBBINS (1985) proved that the regret over T pulls can be asymptotically as small as $O(\ln T)$. In other words, for $T \to \infty$, the regret converges to $O(\ln T)$. Furthermore, they proved that this bound is optimal by showing that there is no other policy with a better asymptotic performance. However, it may be difficult or impossible to determine the right statistical assumptions for a given domain. AGRAWAL (1995) introduced a family of policies that are easier to compute and AUER ET AL. (2002) developed policies that achieve logarithmic regrets uniformly over time, e.g., ϵ_n-*greedy* and *upper confidence bound* (UCB). In the ϵ_n-greedy algorithm, at each step n, the arm with the best current estimate of the average reward value is pulled with probability $(1-\epsilon_n)$ while a different arm is randomly pulled with probability ϵ_n. In other words, the algorithm exploits with probability $(1-\epsilon_n)$ and explores with probability ϵ_n.

The value of ϵ_n decreases as n grows, that is, as time goes by, the algorithm focuses more on exploitation while it decreases the probability of exploration. On the other hand, the UCB algorithm maintains an upper confidence bound estimate for each arm's real mean reward value.

However, many practical settings are more complex than the problems described above, in particular, pulling an arm is costly and only a limited budget is available. The MAB formulation in this case is known as the *budgeted MAB* . There are, however, various versions of formulating a budgeted MAB. For example, only the exploration phase is constrained by the budget, thus limiting the samples to estimate the arm rewards while the exploitation phase is cost-free (see, e.g., Guha and Munagala 2007 and Bubeck et al. 2009). Another version assigns costs to switching from one arm to another (see e.g., Agrawal et al. 1988 and Banks and Sundaram 1994). The more complex and realistic settings, however, bound both phases by a single budget and attach different cost levels to pulling different arms. Intuitively, the optimal solution for the budgeted MAB is not to pull the optimal arm repeatedly (as is the case of classic MAB) since the total number of pulls is finite given the limited budget, but rather to pull a combination of arms that maximizes the total reward within the available budget. As a consequence, it is not sufficient to learn the expected reward of only the highest-value arm but also the other arms' rewards since they may appear in the optimal combination. Thus, existing algorithms cannot apply since they concentrate on learning only the value of the highest expected reward arm (see Tran-Thanh et al. 2010). Moreover, by reducing a knapsack problem to a special coins problem where the coins have different costs, Madani et al. (2004) proved that the budgeted MAB is NP-HARD.

The source of intractability when applying standard dynamic programming to compute the optimal policy for an n-armed bandit problem is the size of the joint state space $O(m^n)$ where m represents the maximum number of states across all arms. However, Stout and Hardwick (2006) suggest that using some algebraic manipulation of indices, great state space reductions can be obtained. Additionally, implementation of parallelization techniques bring increased efficiency in memory access and therefore the problem becomes tractable. In the following, we formulate the budgeted MAB for survey designs and we introduce an adjusted dynamic programming algorithm to efficiently compute the optimal policy.

7.2 Problem formulation

The problem formulation for budgeted MABs in adaptive designs is similar to the RAP formulation from Chapter 3. We briefly revisit here the RAP setup.

Consider a survey sample consisting of N units that can be clustered into homogeneous groups based on characteristics, such as age, gender, and ethnicity (information that can be extracted from external sources of data). Let $\mathcal{G} = \{1, \ldots, G\}$ be the set of homogeneous groups with size N_g for group $g \in \mathcal{G}$ in the survey sample. The data collection is divided into time slots, denoted by the set $\mathcal{T} = \{1, \ldots, T\}$, at which units in a group can be approached for a survey. The survey itself can be conducted using certain interview modes, such as a face-to-face, phone, web/paper; the set of different modes is denoted by $\mathcal{M} = \{1, \ldots, M\}$. At each time slot $t \in \mathcal{T}$ one can decide to approach units in group $g \in \mathcal{G}$ for a survey using mode $m \in \mathcal{M}$. A survey request ends with success if two steps are achieved, i.e., successful contact and participation by answering the questionnaire.

The RAP formulation in (3.12) uses of historical data to estimate group-dependent contact probabilities $p_g(t,m)$ and participation probabilities $r_g(t,m)$. However, as practitioners note, even when extensive historical data is available, these estimates are subject to uncertainty due to sample variations, changes in people's opinions, beliefs etc. Given the strong influence such parameters exert on the optimal solution, updating these estimates when new information is acquired becomes of crucial importance. In the following, we extend the RAP formulation to incorporate a learning method and we map it to a budgeted MAB with discounted rewards.

Let each combination (g,m) define a bandit arm. This means that approaching group g via mode m translates to pulling arm (g,m). For notation simplification, let $\mathcal{I} = \{1, \ldots, G \cdot M\}$ be the set of bandit arms, where each $i \in \mathcal{I}$ corresponds to a unique (g,m) combination. Let $\rho(i)$ be the true response probability for arm i. Then, $\rho(i,t)$ defines the current estimate of the response probability for arm i if pulling arm at time t. Let $x(i,t)$ be a binary 0-1 decision variable that indicates whether arm i is pulled at time t. Each arm pull results in either response, i.e., success, or nonresponse, i.e., failure. Thus, the reward for pulling arm i at time t is $\rho(i,t)$. We are interested in maximizing the expected response, i.e.,

$$\max \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \rho(i,t)\, x(i,t). \tag{7.1}$$

The update of the response probability at $t+1$ given observation of success or failure at time t is as follows

$$\begin{aligned}\rho(i,t+1) = &\big(1 - x(i,t)\big)\, \beta\, \rho(i,t) \\ &+ x(i,t) \left[\rho(i,t)\, \beta\, \frac{\rho(i,t)\, n(i,t) + 1}{n(i,t) + 1} + \big(1 - \rho(i,t)\big)\, \beta\, \frac{\rho(i,t)\, n(i,t)}{n(i,t) + 1} \right],\end{aligned}$$

where

$$n(i,t) = \sum_{t'=1}^{t} x(i,t') \tag{7.2}$$

represents the total number of pulls for arm i until current time point t and β the discounting factor. In other words, if arm i is not pulled at time t, i.e., $x(i,t) = 0$, the expected response at $t+1$ is equal to the discounted value of the expected response from time t. On the other hand, if $x(i,t) = 1$, then the update method must take into consideration both the event of a success and of a failure. Note that the number of successes up until time t is given by $\rho(i,t)\, n(i,t)$, i.e., the current estimate of the success probability times the total number of arm pulls. Furthermore, at time t exactly one arm can be pulled, yielding the constraint

$$\sum_{i \in \mathcal{I}} x(i,t) = 1, \ \forall t \in \mathcal{T}. \tag{7.3}$$

Let $c\,(i)$ denote the cost incurred by pulling arm i. Given a limited budget B, the cost constraint is given by

$$\sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} c\,(i)\, x(i,t) \leq B,$$

under the assumption that the cost does not depend on the outcome of the arm pull. If, however, different costs are incurred for success or failure, then the cost constraint becomes

$$\sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} x(i,t) \big[\rho(i,t)\, c_s(i) + \big(1 - \rho(i,t)\big)\, c_f(i)\big] \leq B, \tag{7.4}$$

where $c_s(i)$ represents the costs in case of success and $c_f(i)$ the costs in case of failure. This means that the costs of pulling arm i are computed in expectation given the two possible outcomes of the pull, i.e., success or failure. In survey designs usually, there is a significant difference in the incurred costs between obtaining response or nonresponse. Therefore, we choose to employ the later formulation of the cost constraint. The budgeted MAB for survey designs with discounted rewards can now be formulated as

$$\begin{aligned}
\max \quad & \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \rho(i,t)\, x(i,t) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{I}} x(i,t) = 1, \ \forall\, t \in \mathcal{T} \\
& \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} x(i,t) \big[\rho(i,t)\, c_s(i) + \big(1 - \rho(i,t)\big)\, c_f(i)\big] \leq B \\
& n(i,t) = \sum_{t'=1}^{t} x(i,t'), \ \forall\, i \in \mathcal{I}, \ t \in \mathcal{T} \\
& \rho(i,t+1) = \big(1 - x(i,t)\big)\, \beta\, \rho(i,t) \\
& \qquad + x(i,t) \left[\rho(i,t)\, \beta\, \frac{\rho(i,t)\, n(i,t) + 1}{n(i,t) + 1} \right. \\
& \qquad \left. + \big(1 - \rho(i,t)\big)\, \beta\, \frac{\rho(i,t)\, n(i,t)}{n(i,t) + 1} \right], \ \forall\, i \in \mathcal{I}, \ t < T \\
& x(i,t) \in \{0,1\}, \ \forall\, i \in \mathcal{I}, t \in \mathcal{T} \\
& \rho(i,1) = p_i, \ \forall\, i \in \mathcal{I},
\end{aligned} \tag{7.5}$$

where p_i denotes the initial estimate for the success probability for arm i. Note that the objective function may result in a value larger than 1.

Finding the optimal policy for (7.5) is in fact a Markov decision problem, as we will show in the next section. We can solve the MDP by dynamic programming, i.e., iterating over the Bellman optimality equations (2.5), appropriately adjusted for the problem at hand. However, applying dynamic programming implies computing an action for each joint state of all I arms. The size of the joint state space is $O(m^I)$, where m is the maximum number of states an arm process can be in. This raises various computational challenges such as massive memory requirements, due to the necessity of storing the entire decision tree, and multiple nested loops with dynamic indices for accessing the tree. These issues translate usually in non-tractability of the problem. Nonetheless, an efficient algorithm can be developed by conveniently adjusting the size of the state space and consequently of the decision tree and by applying parallelization techniques to speed up the access to the tree. We begin the next section by presenting the MDP formulation and we follow with the details of the dynamic programming implementation.

Remark 7.1. Note that the discounting factor β acts as a moderator over how much time the policy spends on exploration. For a value of β close to 1, the policy will allow extended exploration, whereas lower values of β lead to more conservative policies.

Remark 7.2. For the sake of simplicity, we omit the constraint on capacity present in the RAP formulation in (3.12). However, the model can easily be extended to also address limited capacity.

7.3 Solving the budgeted MAB via dynamic programming

In this section, we present a tractable algorithm that employs dynamic programming to address the budgeted MAB. We first show that by assuming a suitable prior distribution on $\rho(i)$, the success probability for arm i, the learning method does not need to keep track of the entire probability distribution but only of the distribution parameters.

Note that, since pulling arm i can result in either success or failure, the sequence of successes and failures forms a Bernoulli process and its success probability is $\rho(i)$. By choosing a suitable prior distribution for $\rho(i)$, e.g., a member of a class of distributions conjugate to the Bernoulli family of distributions, one would only need to keep track of the distribution parameters instead of the entire probability distribution (see Bernardo and Smith 1994). Such a distribution is the Beta distribution. An additional benefit of choosing a Beta distribution is that the posterior distribution obtained when using a Bayesian approach to update the current estimate of $\rho(i)$ is again a Beta distribution (see also Theorem 7.1).

Theorem 7.1. *(Th. 5.8 in* Bhulai *2002) Let $x = (x_1, \ldots, x_n)$ be a sample size of size $n \in \mathbb{N}$ from a Bernoulli distribution with an unknown parameter θ. Suppose that the* *prior distribution of θ is given by a Beta distribution with parameters $\alpha > 0$ and $\beta > 0$. Then the posterior distribution is also given by a Beta distribution, with parameters $\alpha' = \alpha + \sum_{i=1}^{n} x_i$ and $\beta' = \beta + n - \sum_{i=1}^{n} x_i$.*

In our case, let α_i and β_i denote the parameters for the Beta distribution of possible values of $\rho(i)$. Then, the corresponding number of successes recorded at arm i is given by $\alpha_i - 1$ and the number of failures is $\beta_i - 1$. If $\alpha_i - 1 = 0$ and $\beta_i - 1 = 0$, then $\mathcal{B}(1,1)$ is the uniform distribution, which corresponds to having no prior information on $\rho(i)$. By the Bayes' rule, from a Beta prior distribution with parameters $\mathcal{B}(\alpha_i, \beta_i)$ we obtain a Beta posterior $\mathcal{B}(\alpha_i + 1, \beta_i)$ in case of a success or $\mathcal{B}(\alpha_i, \beta_i + 1)$ in case of a failure. Furthermore, at any time point t, the estimate of the response probability for arm i is

$$\rho(i,t) = \frac{\alpha_i}{\alpha_i + \beta_i}. \tag{7.6}$$

Let the state space of the MDP be denoted by

$$\mathcal{S} = \mathcal{T} \times [0, B] \times (\mathbb{N}_0 \times \mathbb{N}_0)^I,$$

where each $s \in \mathcal{S}$ is defined as $s = \big(t, b, (\gamma_i)_{i \in \mathcal{I}}\big)$, with t the current time point, b the remaining budget and $\gamma_i = (\alpha_i - 1, \beta_i - 1)$, the number of successes and failures for arm i observed up until time t. In order to apply dynamic programming we need to show that our problem satisfies the prerequisites of finite horizon MDPs. In other words, we need to show that in our case the decision-making horizon is finite and the state space is also finite. To this end, we discretize interval $[0, B]$ of the available budget. Given the discretized budget values, we can now state the following.

Proposition 7.2. *The decision-making horizon is finite.*

The proof is trivial. Let c_{min} be the minimum cost incurred by pulling an arm. The maximum number of pulls is then given by $\lfloor B/c_{min} \rfloor$, where $\lfloor a \rfloor$ denotes the largest integer smaller than a. Since exactly one arm pull is allowed at any time point, it follows that the decision-making horizon is limited by $T_{max} = \lfloor B/c_{min} \rfloor$.

Proposition 7.3. *The state space of the budgeted MAB is finite.*

To prove this result, note that given the budget restriction, the total number of pulls is finite. As a consequence, the number of pulls for any arm $i \in \mathcal{I}$ is finite which implies that the maximum number of successes or failures per arm is also finite. Also, by Proposition 7.2 the time horizon is finite, since exhaustion prevents taking further actions. Furthermore, if we consider that the available budget is expressed in euros, then at any state s, the available budget can be one of the values of a finite set. Hence, the state space is finite.

The set of arms available for pulling in state s depends on the available budget. Let $\mathcal{A}_s$

denote the action space with

$$\mathcal{A}_s = \left\{ a \in \mathcal{I} \,\middle|\, b - \sum_{a \in \mathcal{I}} \Big[\rho(a,t)\, c_s(a) + \big(1 - \rho(a,t)\big)\, c_f(a) \Big] \geq 0 \right\},$$

where taking action a, i.e., pulling arm a, decreases the available budget by the associated expected costs $\rho(a,t)\, c_s(a) + \big(1 - \rho(a,t)\big)\, c_f(a)$, with $\rho(a,t)$ given by (7.6). More specifically, given the state s the process is in, choosing an action translates to choosing to pull arm a provided that there is sufficient budget left to handle the outcome of the arm pull. Note that not pulling any arm at time t is not optimal, therefore action $a = 0$ is not an element of $\mathcal{A}_s$, $\forall s$.

Once an arm is pulled in state s, the state evolves according to the outcome of the arm pull. The transition probabilities are thus given by

$$p(s,a,s') = \begin{cases} \dfrac{\alpha_a}{\alpha_a + \beta_a}, & \text{for } s' = \Big(t+1, b - c_s(a), \big((\gamma_j)_{j \neq a}, \alpha_a, \beta_a - 1\big)\Big), \\ \dfrac{\beta_a}{\alpha_a + \beta_a}, & \text{for } s' = \Big(t+1, b - c_f(a), \big((\gamma_j)_{j \neq a}, \alpha_a - 1, \beta_a\big)\Big), \\ 0, & \text{otherwise}, \end{cases}$$

with $\gamma_j = (\alpha_j - 1, \beta_j - 1)$.

The expected direct reward in state s given action a is a realization from the corresponding Beta distribution of the success probability for the pulled arm, i.e.,

$$r(s,a) = \frac{\alpha_a}{\alpha_a + \beta_a}. \tag{7.7}$$

The tuple $(\mathcal{S}, \mathcal{A}, p, r)$ completely defines the MDP (see also definition in Section 2.2) for the budgeted MAB in (7.5). The Bellman optimality equations in this case are given by

$$V(s) = \max_{a \in \mathcal{A}_s} \left\{ \frac{\alpha_a}{\alpha_a + \beta_a} \big[1 + \beta V(s')\big] + \frac{\beta_a}{\alpha_a + \beta_a} \beta V(s'') \right\}$$

with

$$\begin{aligned} s' &= (t+1, b - c_s(a), \big((\gamma_j)_{j \neq a}, \alpha_a, \beta_a - 1\big) \\ s'' &= (t+1, b - c_f(a), \big((\gamma_j)_{j \neq a}, \alpha_a - 1, \beta_a\big) \\ \gamma_j &= (\alpha_j - 1, \beta_j - 1), \end{aligned} \tag{7.8}$$

where $V(s)$ denotes the optimal discounted reward starting from state $s \in \mathcal{S}$, with $s = (t, b, (\gamma_i)_{i \in \mathcal{I}})$ and $\gamma_i = (\alpha_i - 1, \beta_i - 1)$. By using the optimality equations, we balance both obtaining immediate response, i.e., exploitation of arm i, and learning about the response probability of other arms, i.e., conducting exploration.

Given the results in Proposition 7.2 and Proposition 7.3, the theoretical framework for finite-horizon finite state space MDPs becomes applicable. In particular, for all $s = (T_{max} + 1, b, (\gamma_i)_{i \in \mathcal{I}})$ we have $V(s) = 0$ since the decision-making horizon ends at T_{max}. Then, the optimal policy can be obtained through dynamic programming in a finite number of iterations. By iterating backwards from T_{max} over the value function in (7.8), the total expected discounted reward is found in $V(s_0)$, with s_0 the initial state given by $t = 1$, $b = B$, and $\big(\gamma_i = (0,0)\big)_{i\in\mathcal{I}}$, if no prior information is available on the success probability for all arms.

For a classic MAB, i.e., without a cost constraint, Bhulai (2002) shows that the state space can be significantly reduced since, after pulling an arm N times, sufficient information has been obtained about its success probability. In other words, basing future decisions on the information available after N pulls will not cause great differences in the obtained discounted rewards. Consequently, once $\alpha_i + \beta_i = N + 2$, arm i does not change state anymore. In case the maximum number of pulls given by the available budget is larger than $I \cdot N$ we can use this remark to impose a stricter upper bound on the total number of combinations for (α_i, β_i), $\forall i$. The corresponding value function is given by

$$\underline{V}(s) = \max_{a\in\mathcal{A}_s}\Bigg\{\mathbb{1}_{\alpha_a+\beta_a=N+2}\bigg(\frac{\alpha_a}{\alpha_a+\beta_a} + \beta\underline{V}(s)\bigg) + \mathbb{1}_{\alpha_a+\beta_a<N+2}\bigg(\frac{\alpha_a}{\alpha_a+\beta_a}\left[1+\beta\underline{V}(s')\right] + \frac{\beta_a}{\alpha_a+\beta_a}\,\beta\underline{V}(s'')\bigg)\Bigg]\Bigg\}$$

with

$$\begin{aligned} s' &= (t+1, b - c_s(a), \big((\gamma_j)_{j\neq a}, \alpha_a, \beta_a - 1) \\ s'' &= (t+1, b - c_f(a), \big((\gamma_j)_{j\neq a}, \alpha_a - 1, \beta_a\big) \\ \gamma_j &= (\alpha_j - 1, \beta_j - 1), \end{aligned}$$

where, if $\alpha_a + \beta_a = N + 2$, the corresponding state is frozen.

Remark 7.3. The derivation of the total number of combinations for (α_i, β_i) is as follows. For each α_i in $\{1, \ldots, N+1\}$, β_i must satisfy $\alpha + \beta \leq N + 2$. For example,

$$\begin{aligned} \alpha_i &= 1 &&\Rightarrow \beta_i \in \{1,\ldots,N+1\} &&\Leftrightarrow N+1 \text{ values} \\ \alpha_i &= 2 &&\Rightarrow \beta_i \in \{1,\ldots,N\} &&\Leftrightarrow N \text{ values} \\ &\vdots \\ \alpha_i &= N &&\Rightarrow \beta_i \in \{1,2\} &&\Leftrightarrow 2 \text{ values} \\ \alpha_i &= N+1 &&\Rightarrow \beta_i \in \{1\} &&\Leftrightarrow 1 \text{ value.} \end{aligned}$$

In total, this gives $\frac{(N+1)(N+2)}{2}$ possible combinations for (α_i, β_i).

Bhulai (2002) shows that in practice N need not be large (depending on the discount factor β) in order to obtain an ϵ-optimal solution, i.e., obtain a solution that is less than ϵ away from the optimal. Due to fast convergence of the discounted reward, N may take small values, for example for $\beta = 0.9$ for a two-armed bandit problem it is sufficient to take $N = 28$.

The next feature of the algorithm improves both memory access and computational

Algorithm 4 Tractable dynamic programming to solve a budgeted MAB

Step 1: Generate values between 1 and $N+2$ for pair (α_I, β_I), the Beta distribution parameters for the success probability at arm I, such that $\alpha_I + \beta_I \leq N+2$.

Step 2: Loop over generated values of (α_I, β_I) and build list of ordered calculations for all arms

Step 3: Start at the beginning of the list and compute $V(s)$ recursively for the corresponding $s \in \mathcal{S}$. Discard computations from previous recursion step.

time. Note that the value function V at a state s requires information only from $2 \cdot I$ previous states. More specifically, for a two-armed bandit in state $s = (t, b, \gamma_1, \gamma_2)$, with $\gamma_i = (\alpha_i - 1, \beta_i - 1)$, $V(s)$ is computed given the value function at states

$$\begin{aligned}
s_1 &= (t+1, b - c_s(1), \alpha_1, \beta_1 - 1, \alpha_2 - 1, \beta_2 - 1) \\
s_2 &= (t+1, b - c_f(1), \alpha_1 - 1, \beta_1, \alpha_2 - 1, \beta_2 - 1) \\
s_3 &= (t+1, b - c_s(2), \alpha_1 - 1, \beta_1 - 1, \alpha_2, \beta_2 - 1) \\
s_4 &= (t+1, b - c_f(2), \alpha_1 - 1, \beta_1 - 1, \alpha_2 - 1, \beta_2),
\end{aligned}$$

therefore, only the discounted rewards for these states need to be stored in the memory simultaneously. As a consequence, one can order the calculations starting at the final $2 \cdot I$ states and continue forth. Thus, nested loops that iterate over each possible value of the γ_i pairs for all $i \in \mathcal{I}$ are no longer necessary. Furthermore, the list of ordered calculations can be pre-computed, using appropriate algebraic manipulations that translate the definition of a state s into a member of the list, reducing thus the high-dimensionality of nodes in the decision tree to 1. Additionally, the optimal action for any given state is uniquely identified by its position in the ordered list. Consequently, once all computations for states at time point t are performed given the value function for states at time $t+1$, previous computations may be discarded from the memory. The steps of our tractable dynamic programming to solve a budgeted MAB are sketched in pseudocode in Algorithm 4.

7.4 Simulation results

This section presents an extensive numerical experiment to illustrate application of our algorithm to solve a budgeted MAB in the context of adaptive survey designs. Consider two groups, $\mathcal{G} = \{1, 2\}$, and two survey modes, $\mathcal{M} = \{1, 2\}$. This leads to the set of arms $\mathcal{I} = \{1, 2, 3, 4\}$. We let the time horizon of the problem be defined by the maximum number of pulls affordable given the available budget B and let $B \in \{500, 1{,}000, 1{,}500\}$. Two numerical setups are considered, where in Setup 1 pulling costs are close in value and in Setup 2 the more rewarding arms are more expensive. Table 7.1 provides the input data, where the pulling costs are given in pairs $(c_s(i), c_f(i))$, i.e., the costs in case

Parameter		Setup 1				Setup 2			
		Arm 1	Arm 2	Arm 3	Arm 4	Arm 1	Arm 2	Arm 3	Arm 4
γ_i^0		$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
$(c_s(i), c_f(i))$		$(50,30)$	$(50,20)$	$(40,20)$	$(40,30)$	$(20,5)$	$(60,40)$	$(30,10)$	$(50,30)$
$\rho(i)$	Set 1	0.7	0.8	0.7	0.8	0.5	0.8	0.5	0.8
	Set 2	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
	Set 3	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
	Set 4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	Set 5	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3

TABLE 7.1: Simulation input parameters.

of success and failure, respectively, and $\gamma_i^0 = (\alpha_i^0, \beta_i^0)$ the prior information for each arm, in terms of number of successes and failures. We assume no prior information available, therefore the Beta prior distribution for each arm is $\mathcal{B}(1,1)$, i.e., the uniform distribution. We compare the expected discounted rewards under three settings, namely the budgeted MAB, the classic MAB (where the Gittins index is the optimal policy) and the full information problem (where the true success probabilities are known for all arms) for two values of the discounting factor, namely $\beta = 0.8$ and $\beta = 0.9$. In the full information case, the optimal policy is to pull the arm with the highest success probability until the budget is exhausted. For correct comparability, we simulate the system dynamics and we limit the horizon of the classic MAB by keeping track of how expensive the arm pulls would be if the costs from the budgeted MAB were considered. Thus, the policy is halted when the expenses overrun the budget. The simulation setup includes five sets of true success probabilities, provided in Table 7.1, four of which do not differentiate success probabilities across arms for a clearer exposition of results.

The expected discounted rewards for the three problem settings are presented in Table 7.2. Note that differences between the classic and the budgeted MAB become more visible for values of β closer to 1 when the impact of the additional future reward that the classic MAB still receives does not vanish through discounting. Note also that the presented values are rounded up to the third digit after the decimal point. The results are visually summarized in Figure 7.3, where the full information for the classic MAB has been added for comparison purposes. This value is computed given a large number of pulls, i.e., $\lfloor B/c_{min} \rfloor = 300$, that is not reached in the budgeted setting. As expected, larger budget yields higher rewards. Note that in both Set 1 and Set 3 the highest response probability is 0.8 therefore calculating the discounted reward in full information yields identical values. The expected discounted reward for the classic MAB does not change across setups since it is not influenced by either costs or true response probabilities.

A perhaps less intuitive result is that under Setup 2 the budgeted MAB yields a higher expected discounted reward than in Setup 1. This aspect also appears in the full information setting, except for Set 1. The explanation lies in the total number of pulls that can be performed. Except for arm 2, the costs are generally lower in Setup 2 than in

Setup	B	β	Budgeted MAB	Classic MAB	Full information				
					Set 1	Set 2	Set 3	Set 4	Set 5
Setup 1	500	0.8	2.941	3.151	3.780	3.346	3.780	2.430	1.478
		0.9	4.848	6.717	5.967	5.399	5.967	4.073	2.595
	1000	0.8	3.137	3.151	3.988	3.495	3.990	2.498	1.499
		0.9	6.229	6.717	7.483	6.670	7.535	4.845	2.945
	1500	0.8	3.150	3.151	3.999	3.499	3.999	2.499	1.500
		0.9	6.607	6.717	7.869	6.932	7.894	4.974	2.993
Setup 2	500	0.8	3.061	3.151	3.702	3.497	3.994	2.499	1.499
		0.9	5.770	6.717	5.683	6.760	7.623	4.926	2.987
	1000	0.8	3.142	3.151	3.971	3.499	3.999	2.500	1.500
		0.9	6.478	6.717	7.229	6.992	7.982	4.999	2.999
	1500	0.8	3.150	3.151	3.998	3.500	4.000	2.500	1.500
		0.9	6.649	6.717	7.756	6.999	7.999	5.000	3.000

Table 7.2: Expected discounted rewards:
budgeted MAB, classic MAB and full information.

Budget	MAB	Set 1	Set 2	Set 3	Set 4	Set 5
500	Budgeted	3.499	3.326	3.787	2.389	1.452
	Classic	3.339	3.218	3.627	2.352	1.426
1000	Budgeted	3.691	3.493	3.987	2.501	1.499
	Classic	3.652	3.478	3.961	2.484	1.502
1500	Budgeted	3.699	3.499	4.001	2.491	1.500
	Classic	3.674	3.502	4.005	2.481	1.495

Table 7.3: Simulation results Setup 1: simulation
average discounted rewards, budgeted and classic MAB, $\beta = 0.8$.

Setup 1. Therefore, the budgeted MAB affords more pulls in Setup 2, yielding a higher expected total reward. On average, the budgeted MAB performs 36.59 pulls in Setup 2 ($\beta = 0.9$) and only 26.49 in Setup 1 ($\beta = 0.9$). Set 1 does not follow this argument due to its specific costs - true response probabilities combination. Note that the cheaper arms have lower response probabilities than in Setup 1. This implies that after performing the maximum number of pulls allowed by the budget on the best arm, i.e., arm 4, the cheapest of the most rewarding arms, any additional available pulls will bring 37.5% less reward per pull. The optimal policy under the full information setting is to pull arm 4 until budget is exhausted. In setup 1, given the expected costs, the budget remaining after arm 4 is no longer affordable does not cover additional pulls. In setup 2 however, pulls may be performed additionally on arm 1.

Intuitively, the expected discounted reward under full information is higher than in the MAB cases. The optimization results for Set 4 and Set 5 show that this is not always the case. In fact, given the prior distribution of the success probability, the expected discounted reward obtained in MAB may overestimate the true reward value. Assuming a uniform prior distribution on all arms leads to a first estimate of the success probability equal to 0.5. As a consequence, when the true success probabilities are lower, the MDP reward definition in (7.7) overestimates the success probability. Given the budget

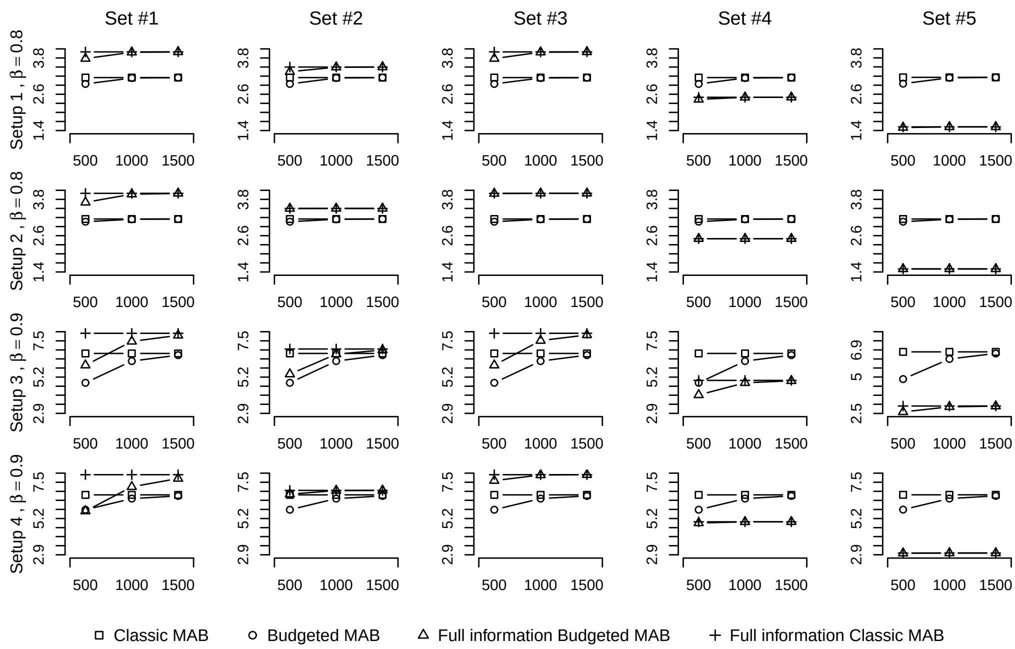


FIGURE 7.2: Optimization results: expected discounted rewards budgeted MAB, classic MAB, full information budgeted, full information classic.

constraint and the discounting factor, a great weight is assigned to the reward obtained in the first steps, thus the expected discounted reward is higher (see again Figure 7.3).

We discuss further the average system dynamics given 10,000 simulations. Figure 7.2 displays the differences between the discounted reward obtained by the budgeted MAB and the classic MAB where we keep track of the pull costs. The y-axis is synchronized across the five sets of success probabilities and centered around 0 to improve readability. In each plot, the black line marks the average difference in rewards and Table 7.3 shows the simulation average discounted rewards for the two settings. On average, when the budget is small, the classic MAB performs worse since running out of budget occurs in at most 12 pulls (if arm 4 is pulled from the start and only success is obtained at every pull, then the budget can cover only 12 pulls), with an average of 11.49. The average number of pulls for the budgeted MAB is 13.33. When the budget increases, the increase in the reward for the classic MAB is faster than in the budgeted case and for $B = 1{,}000$ the differences diminish considerably. An overview of these results is presented in Figure 7.4.

From such standpoint, the results for $B = 1{,}500$ appear counterintuitive. One expects that the average reward differences slowly converge to 0 when the budget increases, whereas in our case they become negative. The explanation can be found in the structure of the optimal policies for the two settings. Note that in the classic MAB the Gittins index focuses on finding the best arm and continues pulling this arm. The budgeted MAB is a finite-horizon problem where less rewarding arms may be present in the optimal policy if they are cheap. As a consequence, towards the end of the horizon, the policy

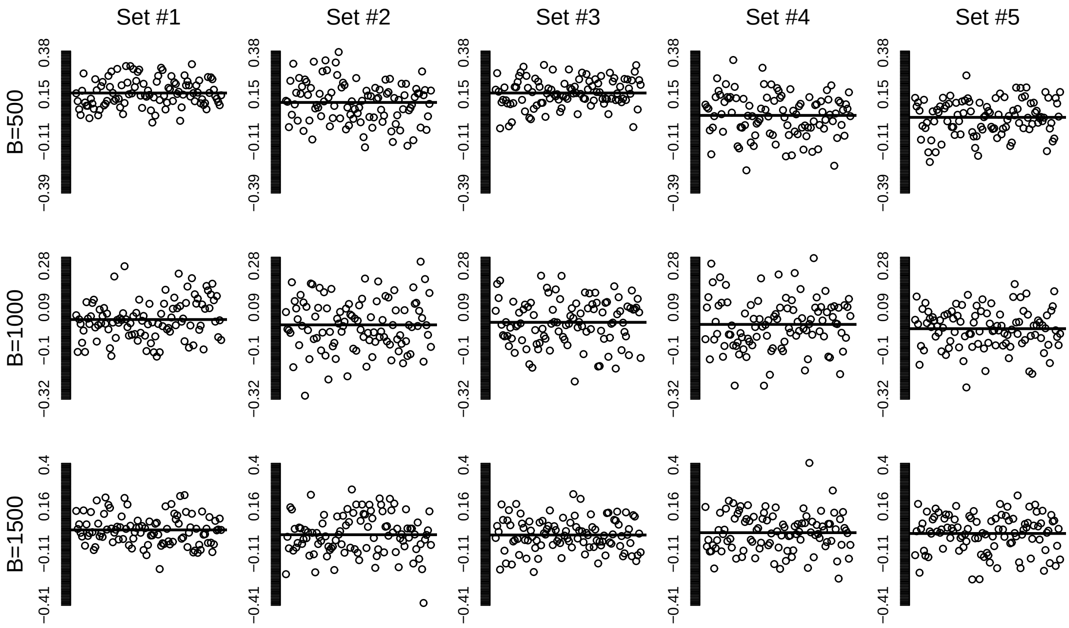


FIGURE 7.3: Simulation results Setup 1: differences in the discounted reward between the budgeted MAB and the classic MAB, $\beta = 0.8$. Black line: average difference.

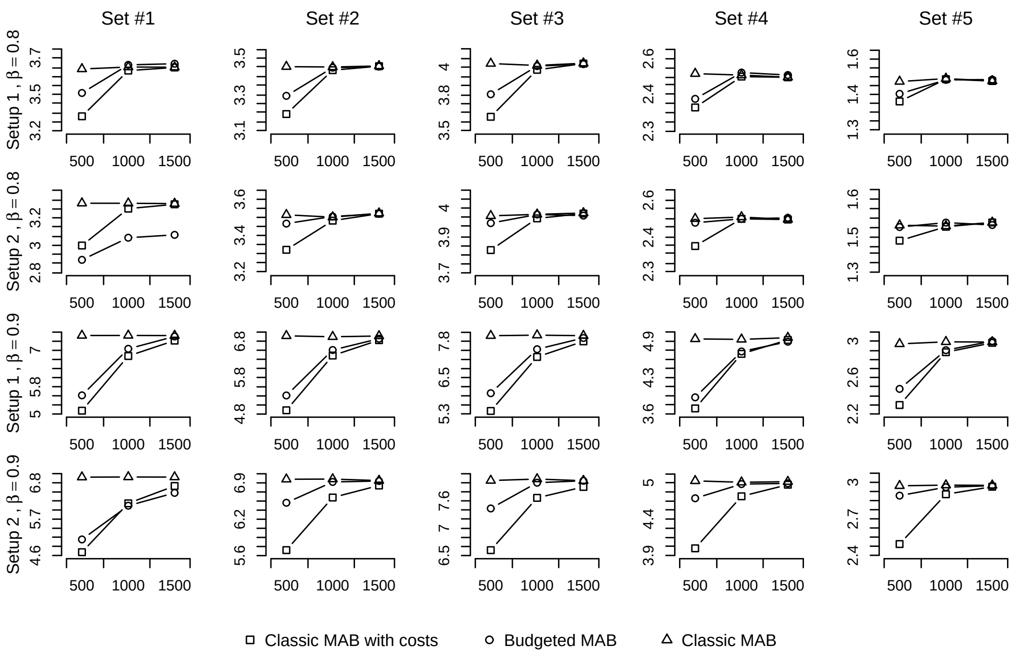


FIGURE 7.4: Simulation results: average discounted rewards budgeted MAB, classic MAB and classic MAB that accounts for costs.

is more "cautious" and alternates between pulling rewarding but expensive arms and pulling less rewarding but cheaper arms in order to obtain a higher discounted reward than spending the remaining budget only the more rewarding arms. A much larger budget is necessary to allow the budgeted MAB replicate the Gittins index. It appears thus that in certain settings it may be more useful to be less cautious than the policy prescribed by the budgeted MAB and employ instead the optimal policy given by the

Gittins index until the budget is exhausted.

The computations are performed on a 2650L cluster computer with 16 parallel processors and 32 GB of memory. Parallelization techniques are implemented in order to increase the speed of the simulation runs. The extensive RAM is required for storing all the final states for larger budget (2,750,517,000 states for $B = 1{,}000$). Smaller problems such as a 3-arm bandit problem can be handled by our algorithm on any machine with 1GB of memory. For our experiments the computational times were under 3,600 seconds.

7.5 Discussion

The work presented in this chapter is a first attempt to formulate a dynamic learning problem in the context of adaptive survey designs. We formulate a budgeted MAB by extending the RAP formulation for survey designs to incorporate a learning method for the RAP parameters that are subject to uncertainty. We make one assumption in order to keep the problem formulation simple. We collapse the two steps to a successful survey approach, i.e., contact and cooperation, into one, namely, response. Extending the presented framework to address contact and cooperation separately brings forth additional challenges. Note that the step of cooperation is triggered by successful contact. In other words, obtaining success at pulling arm i for contact, requires pulling arm j for cooperation. In no other situation does arm j get pulled. This problem could be modeled as a MAB with dependent arms, see for example Pandey et al. (2007), by pairing arm i for contact and the corresponding arm j for cooperation. However, in their problem formulation, it is assumed that once arm i from cluster $C_{[i]}$ is pulled, information about all arms in the cluster is received, thus all arms change state. This is not true in our case, since if contact fails, no information about cooperation probability can be gathered.

For the budgeted MAB, index policies stop being optimal since the arms are no longer separable in the context of budget constraints. We propose to solve the problem using dynamic programming, with some additional features that make the problem tractable. Based on Theorem 7.5 in Bhulai (2002), a significant decrease in the state space can be obtained. We show that at each step a small number of previous states is in fact necessary to compute the value function for the current state. By clever manipulations of the state description, memory requirement and computational speed can be improved significantly. We empirically show the performance of our algorithm by comparison with the classical MAB and the full information problem. It turns out that for relatively small budgets our algorithm outperforms the Gittins index. The differences in the expected discounted rewards decrease with the increase in budget and from a budget level on, the cautious behavior of the algorithm that takes costs into considerations may lead to underperformance compared to the Gitins index.

The problem formulation we consider may be easily generalized to other application areas such as online advertising and clinical trials. However, few questions remain open such as addressing large problem instances, e.g., bandits with more than 100 arms. We believe that by splitting the problem in several fewer-armed bandit problems and adding a master problem that manages the budget allocation among the small MABs may be the key to developing tractable algorithms.

8

Future research directions

The research presented in the thesis constitutes one of the first attempts to formalize adaptive survey designs as resource allocation problems. The motivation behind this effort comes from the increased difficulty survey organizations experience in producing high-quality survey estimates. Literature in the field notes various methods to increase the level of quality, however, these methods most often require use of additional (potentially unavailable) resources. Through our framework, the limited availability of resources is optimally exploited in order to maximize a provided notion of survey quality. Several research questions arise in this context. This chapter aims at highlighting those open questions that may constitute interesting venues for future research.

The basic formulation of adaptive designs as a resource allocation problem (RAP) is presented in Chapter 3. Here, we consider survey mode and timing of contact as tools to maximize response rate given their strong impact on the use of resources. A natural extension is to combine additional survey design features, such as offering incentives to increase cooperation or use interviewers with different skills to re-approach refusers. The RAP formulation utilizes the assumption that probability of contact is independent of the history of contact attempts which allows casting it as an MDP. However, this may be a strong assumption in practice. An interesting question is how to relax this assumption and keep the problem Markovian.

In Chapter 4 we investigate the modeling challenges of addressing two quality indicators, namely the response rate and the response representativity indicator. The reformulation of RAP as an MDP is facilitated by an endogenous additivity property, which is not satisfied by the representativity indicator. A two-step algorithm is developed to adequately address this problem. However, implementation of the algorithm becomes feasible for a discretized domain of values for the decision variables. Thus, a dilemma arises. On the one hand, choosing a small discretization step results in long computational times. On the other hand, a rough discretization step may "hide" the global optimum. An interesting question is at which point the additional gain in value obtained by a refined discretization step becomes unappealing given the corresponding increase in the computational time. Furthermore, one would be interested in evaluating beforehand what values lead to suboptimal policies. We prove a lower bound and an upper bound on the proportion of resources to be allocated to each group, outside which only suboptimal policies exist. However, we have not investigated how to discard suboptimal policies given the definition of the response representativity indicator.

Chapter 5 and Chapter 6 discuss how to design adaptive surveys in order to address both nonresponse and measurement errors. One approach, as presented in Chapter 5, uses external data to summarize measurement errors over the various survey items in one indicator, the measurement profile. An appealing extension would be to use other sources of data, that become available during the survey fieldwork (paradata), for a refined definition of a measurement profile. This question can further be extended to how to define a measurement profile before start of fieldwork in the absence of external data. The second approach, presented in Chapter 6, investigates an item-dependent indicator. A natural extension would be to address multiple item-dependent indicators or to develop a composite indicator, which may prove a daunting task.

For all models presented so far, it holds that accurate estimation of the optimization input parameters is crucial to obtaining effective survey designs. As a consequence, the choice of survey design features and the number of groups that enter the adaptive design should be backed up by historical survey data. However, when a new survey is launched, there is no historical support to produce the optimization parameters. Hence, the RAP is extended in Chapter 7 to incorporate a method that updates the parameter estimates when new observations become available. It is an open research question how to determine the optimal policy for budgeted multi-armed bandit problems where the Gittins index is no longer optimal. We show that ϵ-optimal policies may be obtained by dynamic programming for small instances. An interesting question is whether a similar approach to the two-step algorithm may help address intractability of larger problem instances.

The algorithms presented in Chapters 4–6 have been tested using survey historical data from the Dutch Labor Force Survey. Although the performance of the presented algorithms has not yet been tested in practice, Statistics Netherlands is currently running projects that aim at introducing adaptive survey designs and a more resource-efficient data collection. It would be interesting to include current research results in an automated design tool to be used by other survey organizations as well.

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