

Discussion Paper

PPS Sampling with Panel Rotation for Estimating Price Indices on Services

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Sander Scholtus Arnout van Delden

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Summary

In the last decade, Statistics Netherlands (SN) has started publishing service producer price indices (SPPI) for a range of service types. To this end, quarterly price mutations are measured in a panel survey of enterprises. The original panel design involved stratified simple random sampling by size class. Over time, this panel has suffered severely from attrition. Also, the variable turnover is now available for all enterprises in the business register. In this paper, we present a redesign for the Dutch SPPI which is based on a probability proportional to size (PPS) sampling design, with turnover used as a size variable, involving 52 service types across 28 economic sectors.

In this redesign, we accounted for three 'non-standard' issues. Firstly, SN does not have a sampling frame of services. Instead, we need to estimate the service indices from a PPS sample of enterprises stratified by economic activity (NACE code). An enterprise may have activities in multiple services. We propose a ratio estimator for the SPPI based on a PPS sample of enterprises. Secondly, we addressed the allocation of the sample over different NACE codes, taking into account that each sector can have a different number of underlying services and a different relative importance. A Neyman allocation was used with a cost component for the number of underlying services. Finally, we wanted to design a rotating PPS panel, also accounting for births and deaths in the population, while still obtaining approximately unbiased estimators. In a simulation study, we compared different rotation strategies on the accuracy of their inclusion probabilities, and on the bias and variance of the estimators. We concluded that a Pareto sampling method gave the best results.

Keywords

PPS sampling, service indices, panel rotation

1. Introduction

Statistics Netherlands (SN) has been publishing quarterly producer price indices on services (SPPI) since 2002. To this end, a classification of services is linked to the NACE classification of enterprises by main economic activity. Separate indices are published for different types of services, as well as a total SPPI which is a weighted average of the separate SPPIs. To collect information about quarterly price mutations on services, SN conducts a panel survey for a sample of enterprises from the general business register (GBR). Initially, samples were drawn using a stratified simple random sampling design, with stratification by economic sector and size class (based on number of employees).

Two developments have prompted SN to reconsider the sampling and estimation procedure for the SPPI. Firstly, the annual turnover is now available for all enterprises in the Dutch GBR; this variable is derived from administrative data on tax declarations, combined with a monthly or quarterly census survey of the largest and most complex units (van Delden and de Wolf, 2013). This makes it possible to use probability proportional to size (PPS) sampling with turnover as a size variable. Secondly, the existing SPPI panels have suffered severely from attrition, due to a lack of structural panel maintenance. In fact, due to attrition, the existing panels are now treated de facto as a stratified simple random sample by sector; i.e., the additional stratification by size class is ignored during estimation. To prevent this attrition from occurring again in the future, SN wants to introduce annual panel rotation in the data collection process for the SPPI.

The switch to a PPS sampling design with panel rotation raises several methodological issues. First of all, SN has to work with a sampling frame of enterprises rather than services and it is not known at the population level in which types of services each enterprise is involved. Therefore, we need to develop an estimation strategy for price indices on service domains based on a stratified PPS sample of enterprises, where multiple service domains may belong to the same PPS stratum. Secondly, we would like to allocate the total sample size across the PPS strata in a way that optimises the accuracy of the total SPPI, while also ensuring that each service domain is covered sufficiently by the sample. Finally, no "perfect" fixedsize panel rotation method exists that exactly achieves the nominal inclusion probabilities of a general PPS sampling design for all units in all survey rounds (Grafström and Matei, 2015). Therefore, some approximate rotation procedure has to be used. We want to choose a rotation method for which the bias in the estimated price indices is negligible.

In the remainder of this paper, we explain these issues in more detail and describe the solutions we propose for the Dutch SPPI. The PPS estimator of the SPPI and its variance are given in Section 2. Sample allocation theory and its application to the Dutch SPPI are presented in Section 3. The panel rotation problem is discussed in Section 4. A conclusion follows in Section 5.

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2. SPPI estimation using a PPS panel from a fixed population

2.1 Estimation and inference at the sector level

As mentioned in the introduction, SPPI estimates are based on a panel survey of enterprises. The information in the GBR allows us to stratify the sample by economic sector, based on NACE codes. In some cases, the SPPI publication level coincides with a NACE sector. However, in many cases SN also publishes indices for service domains at a more detailed level. In this paper, we use the terms sector-level SPPI and domain-level SPPI to distinguish between these two situations. Throughout the paper, domains are defined in such a way that they belong to a single sector (i.e., a single PPS stratum).

As a starting point, we take a sector-level SPPI that is not differentiated further into domains. Until Section 4, we consider the populations of enterprises and services to be fixed over time. For an enterprise b that belongs to NACE sector h, let $P_{hb}(t,q;0)$ denote its individual price index of services within that sector in quarter q of year twith respect to some base period 0, and let $X_{hb}(ref)$ denote its (annual) turnover in a reference year. Let N_h denote the number of enterprises in the population of sector h. A direct quarterly price index of services in sector h can be defined as:

$$I_h(t,q;0) = \frac{\sum_{b=1}^{N_h} X_{hb}(ref) P_{hb}(t,q;0)}{\sum_{b=1}^{N_h} X_{hb}(ref)} = \sum_{b=1}^{N_h} W_{hb}(ref) P_{hb}(t,q;0), \tag{1}$$

where $W_{hb}(ref) = X_{hb}(ref)/X_h(ref)$ and $X_h(ref) = \sum_{b=1}^{N_h} X_{hb}(ref)$ denotes the total turnover in sector h in the reference year. The choice ref = 0 in Formula (1) yields a standard Laspeyres price index. In practice, SN uses weights from an earlier reference period (ref < 0) for the SPPI, which makes (1) a so-called Young index (van der Grient and de Haan, 2011; IMF, 2004). For the remainder of Sections 2 and 3, we simplify the notation by suppressing the time indices, e.g. $P_{hb} \equiv P_{hb}(t,q;0)$ and $W_{hh} \equiv W_{hh}(ref)$.

We first review some results on how to estimate I_h from a PPS sample. Suppose a sample of n_h enterprises is taken from sector h, where the inclusion probability of enterprise b is proportional to its turnover in the reference period: $\pi_{hb} = n_h W_{hb}$. We assume here that $X_{hb} < X_h/n_h$ (or $W_{hb} < 1/n_h$) for all $b=1,\ldots,N_h$. Enterprises with $X_{hh} \ge X_h/n_h$ should be placed in a separate stratum and selected with certainty. In practice, this concerns a limited number of enterprises. We defer a discussion of this topic until Section 2.3.

For a PPS sample with turnover used as a size variable, the standard Horvitz-Thompson (HT) estimator of I_h has a very simple form:

$$\hat{I}_h = \sum_{i=1}^{n_h} \frac{w_{hi} p_{hi}}{\pi_{hi}} = \frac{1}{n_h} \sum_{i=1}^{n_h} p_{hi} = \bar{p}_h, \tag{2}$$

where the sampled units are indicated by $i=1,\ldots,n_h$ and lowercase characters denote sample observations. That is to say, under PPS sampling at the sector level, the unweighted sample mean of individual price indices is an unbiased estimator for the weighted price index in Formula (1); see also, e.g., Knottnerus (2011a, 2011b).

At SN, PPS samples are usually drawn by systematic sampling from a randomly ordered list (Banning et al., 2012, pp. 53-54). This corresponds to Procedure 2 of Brewer and Hanif (1983), who also listed 49 other procedures for selecting a PPS sample. Under the assumption that all $W_{hb} \ll 1$, an approximate variance formula for \hat{I}_h under this sampling procedure is (Hartley and Rao, 1962; Knottnerus, 2011b):

$$\operatorname{var}(\hat{I}_h) \approx \frac{1}{n_h} \sum_{b=1}^{N_h} W_{hb} \{ 1 - (n_h - 1) W_{hb} \} (P_{hb} - I_h)^2.$$
 (3)

To simplify this expression further, it is helpful to suppose that the deviations $\varepsilon_{hb} = P_{hb} - I_h$ are independently distributed with $E(\varepsilon_{hb}) = 0$ and $E(\varepsilon_{hb}^2) \propto W_{hb}^{\beta}$ for some value of β . In previous studies at SN with (S)PPI data, it was found that this model often holds with $\beta \approx 0$, which implies that the variance of the deviations does not depend on turnover. For $\beta = 0$, Knottnerus (2011b) derived the following approximation to the variance of \hat{I}_h :

$$\operatorname{var}(\hat{I}_{h}) \approx \sigma_{ph}^{2} \left\{ \frac{1}{n_{h}} - \frac{1}{N_{h}} (1 + CV_{wh}^{2}) \right\},$$

$$\sigma_{ph}^{2} = \sum_{b=1}^{N_{h}} W_{hb} (P_{hb} - I_{h})^{2},$$

$$CV_{wh}^{2} = \left\{ \frac{1}{N_{h} - 1} \sum_{b=1}^{N_{h}} (W_{hb} - \overline{W}_{h})^{2} \right\} / \overline{W}_{h}^{2} \approx \left(N_{h} \sum_{b=1}^{N_{h}} W_{hb}^{2} \right) - 1.$$

$$(4)$$

In these expressions, it is also assumed that the sample and population sizes are large enough so that n_h-1 and N_h-1 may be replaced by n_h and N_h . Furthermore, σ_{ph}^2 is a measure of the variability of the individual price indices and $CV_{wh}^2 = CV_{xh}^2$ denotes the squared coefficient of variation of turnover in sector h; note that $\overline{W}_h = 1/N_h$.

To apply Formula (4) in practice, we only need to estimate σ_{nh}^2 , since N_h and CV_{wh}^2 can be obtained directly from the sampling frame. Knottnerus (2011b) discussed how to estimate σ_{ph}^2 from a PPS sample. For a suitable estimator of σ_{ph}^2 in the case of a simple random sample, see Formula (17) below.

Estimation and inference at the domain level

Next, we consider a sector h with $D_h \ge 2$ underlying service domains for which separate SPPIs have to be estimated. An enterprise in sector h can be active within zero, one, or more than one of these service domains. Let X_{hdb} denote the turnover of enterprise b from services within domain d, with $X_{hdb} = 0$ for enterprises that do not provide these services. For an enterprise that is active within domain d, let P_{hdb} denote its associated price index. Analogously to (1), the direct price index for domain d is defined as

$$I_{hd} = \frac{\sum_{b=1}^{N_h} X_{hdb} P_{hdb}}{\sum_{b=1}^{N_h} X_{hdb}}, \quad d = 1, \dots, D_h,$$
 (5)

where the product $X_{hdb}P_{hdb}$ is taken to be zero when $X_{hdb}=0$.

As noted in the introduction, the information in the sampling frame does not allow SN to determine beforehand which enterprises are active within each domain. Therefore, we again select a PPS sample of size n_h at the sector level, with inclusion probabilities based on the sector turnover X_{hb} as before. The sampled units are then contacted by telephone to obtain their turnover specification by service domain. The panel for domain d effectively consists of all units in the total PPS sample with $x_{hdi} > 0$. Note that this implies that the effective sample size in domain d is stochastic.

To find an estimator for I_{hd} , it is convenient to rewrite Formula (5) as follows:

$$I_{hd} = \frac{\frac{1}{X_h} \sum_{b=1}^{N_h} X_{hdb} P_{hdb}}{\frac{1}{X_h} \sum_{b=1}^{N_h} X_{hdb}} = \frac{\sum_{b=1}^{N_h} \frac{X_{hb}}{X_h} \frac{X_{hdb}}{X_{hb}} P_{hdb}}{\sum_{b=1}^{N_h} \frac{X_{hb}}{X_h} \frac{X_{hdb}}{X_{hb}}} = \frac{\sum_{b=1}^{N_h} W_{hb} Y_{hdb}}{\sum_{b=1}^{N_h} W_{hb} G_{hdb}},$$
(6)

where $Y_{hdb}=G_{hdb}P_{hdb}$ and $G_{hdb}=X_{hdb}/X_{hb}$ is the fraction of turnover that enterprise b derives from domain d. The numerator and denominator of (6) are both expressions of the form (1). Hence, they can both be estimated from the PPS sample using a HT estimator of the form (2). In this manner, we obtain a ratio estimator for I_{hd} :

$$\hat{I}_{hd} = \frac{\bar{y}_{hd}}{\bar{g}_{hd}} = \frac{\sum_{i=1}^{n_h} y_{hdi}}{\sum_{i=1}^{n_h} g_{hdi}} = \frac{\sum_{i=1}^{n_h} g_{hdi} p_{hdi}}{\sum_{i=1}^{n_h} g_{hdi}} = \sum_{i=1}^{n_h} \frac{1}{n_h} \frac{g_{hdi}}{\bar{g}_{hd}} p_{hdi}.$$
(7)

Recall that, to estimate a price index at the sector level, each sampled unit receives the same weight $(1/n_h)$. To estimate a domain-level SPPI, as the last expression in (7) shows, these basic weights are adjusted by a factor g_{hdi}/\bar{g}_{hd} that accounts for the fraction of turnover that a unit derives from the domain. In particular, units that are not active within the domain ($g_{hdi} = 0$) receive a zero weight.

By the standard properties of a ratio estimator, \hat{I}_{hd} in (7) is asymptotically unbiased for I_{hd} (Cochran, 1977). It should be noted that the denominator of (6) equals

 $G_{hd} = \sum_{h=1}^{N_h} X_{hdb} / X_h$, the overall share of domain d in the total turnover of sector h. To obtain \hat{I}_{hd} , we have estimated this quantity by the sample mean \bar{g}_{hd} . In fact, a reasonable proxy for G_{hd} is usually available from the Structural Business Statistics or the National Accounts. Thus, it is not strictly necessary to estimate the denominator of (6) from the PPS panel, and we could estimate I_{hd} instead by $\hat{I}_{hd,alt} = \bar{y}_{hd}/G_{hd}$, which is unbiased. However, it is well known that, in practice, the ratio estimator \hat{I}_{hd} typically leads to a better result in terms of mean squared error.

To obtain a variance formula for \hat{I}_{hd} , we can apply a standard Taylor linearisation argument. Using the fact that $E(\bar{g}_{hd}) = G_{hd}$, this yields:

$$\operatorname{var}(\hat{I}_{hd}) \approx \operatorname{var}\left(\frac{\bar{y}_{hd} - I_{hd}\bar{g}_{hd}}{G_{hd}}\right) = \frac{1}{G_{hd}^2} \operatorname{var}(\bar{e}_{hd}),$$

with \bar{e}_{hd} the sample mean of residuals $E_{hdb} = Y_{hdb} - I_{hd}G_{hdb} = G_{hdb}(P_{hdb} - I_{hd})$. For the variance of \bar{e}_{hd} , approximate expressions similar to (3) and (4) could be derived.

The take-all stratum and the total SPPI

As noted above, the population may contain units with large turnovers $X_{hh} \ge X_h/n_h$. These have to be placed in a separate 'take-all' stratum and selected with probability $\pi_{hb}=1.$ We denote these units by $b=N_h+1,N_h+2,...,N_h^*$ in the population and by $i=n_h+1, n_h+2, \ldots, n_h^*$ in the sample, with $n_h^*=n_h+N_h^*-N_h$.

The sample size n_h^st indirectly determines the boundary of the take-all stratum, and hence N_h , n_h and X_h . The take-all stratum in sector h consists of (at least) the q_h largest units in the population, where q_h can be determined as follows: find the smallest possible $q_h \in \{0,1,\dots,n_h^*-1\}$ so that the inequality $X_{hb} \geq X_h/(n_h^*-q_h)$ holds for at most q_h units in the population. Here, X_h denotes the total turnover apart from the q_h units with the largest turnover values. (We will assume that such a value of q_h exists. Otherwise, an alternative option might be to take $q_h = n_h^*$. This would yield a cut-off sample of the n_h^* largest units, not a PPS sample.)

The definition of the price indices (1) and (5) has to be extended to incorporate the take-all stratum. For the sector-level index, this yields:

$$I_h^* = \frac{\sum_{b=1}^{N_h^*} X_{hb} P_{hb}}{\sum_{h=1}^{N_h^*} X_{hb}} = \frac{X_h}{X_h^*} I_h + \frac{1}{X_h^*} \sum_{b=N_h+1}^{N_h^*} X_{hb} P_{hb},$$

with $X_h^*=\sum_{b=1}^{N_h^*}X_{hb}=X_h+\sum_{b=N_h+1}^{N_h^*}X_{hb}$ and I_h given by (1). Since the take-all stratum is completely observed and I_h can be estimated by \hat{I}_h from (2), we can estimate I_h^* by

$$\hat{I}_h^* = \frac{X_h}{X_h^*} \hat{I}_h + \frac{1}{X_h^*} \sum_{i=n_h+1}^{n_h^*} x_{hi} p_{hi}.$$
 (8)

By using Expression (2), \hat{I}_h^* can be written as a weighted sample mean of price mutations:

$$\hat{I}_{h}^{*} = \frac{\sum_{i=1}^{n_{h}^{*}} \omega_{hi} p_{hi}}{\sum_{i=1}^{n_{h}^{*}} \omega_{hi}}, \qquad \omega_{hi} \propto \begin{cases} X_{h}/n_{h} & \text{if } \pi_{hi} < 1, \\ x_{hi} & \text{if } \pi_{hi} = 1. \end{cases}$$
(9)

Furthermore, the second term in (8) does not contribute to the variance of \hat{I}_h^* , so that $\operatorname{var}(\hat{I}_h^*) = (X_h/X_h^*)^2 \operatorname{var}(\hat{I}_h)$, with $\operatorname{var}(\hat{I}_h)$ given by Expression (4).

For the domain-level index I_{hd} and its estimator \hat{I}_{hd} , similar extensions I_{hd}^* and \hat{I}_{hd}^* can be developed. In particular:

$$\hat{I}_{hd}^* = \frac{\sum_{i=1}^{n_h^*} \omega_{hdi} p_{hdi}}{\sum_{i=1}^{n_h^*} \omega_{hdi}}, \qquad \omega_{hdi} \propto \begin{cases} (X_{hd}/n_h) \times (g_{hdi}/\bar{g}_{hd}) & \text{if } \pi_{hi} < 1, \\ \chi_{hdi} & \text{if } \pi_{hi} = 1. \end{cases}$$
(10)

In practice, X_{hd} is unknown but it can be replaced by its proxy $G_{hd}X_h$.

Finally, we mentioned in the introduction that SN also publishes a total SPPI, aggregated across economic sectors $h=1,\ldots,H$. This total SPPI is defined and estimated as

$$I_{tot}^* = \sum_{h=1}^H W_h^* I_h^*, \qquad \hat{I}_{tot}^* = \sum_{h=1}^H W_h^* \hat{I}_h^*, \tag{11}$$

with W_h^* the share of sector h in the total turnover of all sectors $h=1,\ldots,H$. In practice, these weights are based on macro-integrated data from the National Accounts. Regarding the variance of \hat{I}_{tot}^* , we observe that

$$\operatorname{var}(\hat{I}_{tot}^*) = \sum_{h=1}^{H} (W_h^*)^2 \operatorname{var}(\hat{I}_h^*) \approx \sum_{h=1}^{H} \left(W_h^* \frac{X_h}{X_h^*} \right)^2 \sigma_{ph}^2 \left\{ \frac{1}{n_h} - \frac{1}{N_h} (1 + CV_{wh}^2) \right\}, \quad (12)$$

since an independent PPS sample is drawn from each sector h.

3. Sample allocation

3.1 Allocation formulas

In this section, we will consider the choice of sample size n_h^st in each sector. The total sample size that is available for all SPPIs is fixed at, say, n^* units. We want to allocate this total sample size across the sectors h = 1, ..., H, taking into account that these sectors differ in terms of economic importance and heterogeneity of individual price indices.

A natural starting point is to try to find the allocation $(n_1^*,...,n_H^*)$ with $\sum_{h=1}^H n_h^* = n^*$ that minimises the variance of the total SPPI, $var(\hat{I}_{tot}^*)$, given by Formula (12). This is actually a difficult, non-standard optimisation problem, because several quantities that occur in the target function $(n_h, N_h, X_h, \sigma_{ph}^2)$ and CV_{wh}^2 may depend indirectly on n_h^st through the delineation of the take-all stratum. In principle, we could attempt to solve this problem numerically. However, given that the variance of \hat{I}_{tot}^* is not the only concern here (see below) and that, in practice, σ_{ph}^2 will be estimated from a small sample, it seems reasonable to simplify the problem.

To obtain a simpler problem, we note that, by the definition of the take-all stratum:

$$X_h^* = \sum_{h=1}^{N_h^*} X_{hb} \ge X_h + \frac{n_h^* - n_h}{n_h} X_h = \frac{n_h^*}{n_h} X_h.$$

Hence, $X_h/X_h^* \le n_h/n_h^* \le \sqrt{n_h/n_h^*}$; the last inequality holds because $0 \le n_h \le n_h^*$. Substituting this inequality into Formula (12), we find that (approximately)

$$\operatorname{var}(\hat{I}_{tot}^*) \le \sum_{h=1}^{H} (W_h^*)^2 \frac{n_h}{n_h^*} \sigma_{ph}^2 \left\{ \frac{1}{n_h} - \frac{1}{N_h} (1 + CV_{wh}^2) \right\} \le \sum_{h=1}^{H} (W_h^*)^2 \frac{\sigma_{ph}^2}{n_h^*}. \tag{13}$$

Finally, we assume that σ_{ph}^2 does not depend on the delineation of the take-all stratum; this is in line with the assumption $\beta=0$ that was used in the derivation of (4).

Instead of minimising $\mathrm{var}(\hat{I}_{tot}^*)$ directly, we propose to minimise upper bound (13) for all $(n_1^*, ..., n_H^*)$ with $\sum_{h=1}^H n_h^* = n^*$. This problem has the same structure as the well-known Neyman allocation problem (see, e.g., Cochran, 1977, pp. 98-99). The optimal solution is therefore given by:

$$n_h^* = \frac{W_h^* \sigma_{ph}}{\sum_{q=1}^H W_q^* \sigma_{pq}} n^*, \quad h = 1, \dots, H.$$
 (14)

Thus, more sample units are allocated to sectors with larger contributions to the service economy (in terms of W_h^*) and/or more heterogeneous price indices (in terms of σ_{ph}).

Allocation (14) does not take into account that SN also wants to publish reliable domain-level SPPIs. That is to say, it may be advantageous to allocate more sample units to some sectors that consist of relatively many domains, to ensure that these domains are covered sufficiently by the sample. As SN has little information about the population at the domain level, we do not attempt to find an 'optimal' allocation strategy that directly takes the variances of the domain-level SPPIs into account. Instead, we propose an allocation strategy that incorporates information about the domains through a cost function.

Suppose that sector h consists of D_h domains and that the average number of domains in which an enterprise from this sector is active equals \bar{A}_h . As mentioned above, each of the n_h^* sampled units from sector h is asked to report price indices for all domains in which it is active. Thus, the costs of data collection per unit can be considered higher when units tend to be active across multiple domains (i.e., when \bar{A}_h/D_h is relatively large) and lower when units tend to work in isolated domains (i.e., when A_h/D_h is small).

By the above reasoning, let $C_h = \bar{A}_h/D_h \le 1$ be a measure of the relative costs of data collection per unit in sector h. The total relative costs associated with a particular sample allocation are then equal to $C_{tot} = \sum_{h=1}^{H} C_h n_h^*$. The allocation that minimises upper bound (13) for a given value of C_{tot} can be derived analogously to the Neyman allocation (Cochran, 1977, pp. 96-98):

$$n_h^* = \frac{W_h^* \sigma_{ph} / \sqrt{C_h}}{\sum_{g=1}^H W_g^* \sigma_{pg} / \sqrt{C_g}} n^* = \frac{W_h^* \sigma_{ph} \sqrt{D_h / \bar{A}_h}}{\sum_{g=1}^H W_g^* \sigma_{pg} \sqrt{D_g / \bar{A}_g}} n^*, \quad h = 1, \dots, H.$$
(15)

Here, we have implicitly chosen \mathcal{C}_{tot} in such a way that the total sample size still equals n^* . Allocation (15) has the desirable properties that – all other things being equal - more units will be sampled from a sector if it contains more domains (higher D_h) and/or if enterprises tend to be active in isolated domains (lower \bar{A}_h). This means that, in comparison to (14), we can expect a sample with better coverage at the domain level.

To apply Formula (15), we have to estimate σ_{ph} and \bar{A}_h from the existing SPPI panels. For the estimation of σ_{ph} , we cannot use the formulas in Knottnerus (2011b) because the existing panel is not a PPS sample. Recall from Section 1 that the existing panel in sector h is treated *de facto* as a simple random sample of size, say, m_h . An associated estimator for I_h is (cf. Knottnerus, 2011a):

$$\hat{I}_{h,SRS} = \frac{\sum_{i=1}^{m_h} x_{hi} p_{hi}}{\sum_{i=1}^{m_h} x_{hi}}.$$
 (16)

It can be shown (see Appendix A) that an asymptotically unbiased estimator for σ_{ph}^2 from a simple random sample is given by

$$\hat{\sigma}_{ph,SRS}^2 = \frac{\sum_{i=1}^{m_h} x_{hi} (p_{hi} - \hat{I}_{h,SRS})^2}{\sum_{i=1}^{m_h} x_{hi}} + \widehat{\text{var}}(\hat{I}_{h,SRS}).$$
(17)

Here, $\widehat{\mathrm{var}}(\widehat{I}_{h,SRS})$ denotes an asymptotically unbiased variance estimator for $\widehat{I}_{h,SRS}$, for instance (see, e.g., Knottnerus, 2011a):

$$\widehat{\text{var}}(\widehat{I}_{h,SRS}) = \frac{1}{\overline{X}_h^2} \left(1 - \frac{m_h}{N_h} \right) \frac{1}{m_h(m_h - 1)} \sum_{i=1}^{m_h} x_{hi}^2 \left(p_{hi} - \widehat{I}_{h,SRS} \right)^2.$$

In the application to be discussed in Section 3.2, we used $\hat{\sigma}_{ph,SRS}^2$ to estimate σ_{ph}^2 .

3.2 **Results**

We applied the theory from the previous subsection to obtain an allocation for the 28 economic sectors that are currently sampled for the Dutch SPPI. Each sector contains between one and six service domains. The total number of domains is 52. We used the GBR of the first quarter of 2013 as a population frame. The total population across all sectors contained about 235 000 units. The data for 2013 of the existing SPPI panels were used to estimate σ_{ph} and \bar{A}_h . To obtain a robust estimate for σ_{ph} , we computed $\hat{\sigma}^2_{ph,SRS}$ for each quarter of 2013 separately and then took the median value.

Two different methods are currently used to collect price information on services from the sampled units (OECD/Eurostat, 2014): based on actual transaction prices (ATP) or based on model prices for standard products (MP). Of the 28 sectors in the Dutch SPPI, 8 are observed using ATP and 20 are observed using MP. From our data, we found that larger values of $\hat{\sigma}_{ph,SRS}^2$ – i.e., more heterogeneous price indices – tend to occur for sectors that are observed using ATP. This was in line with the expectations of the SPPI production staff.

The total sample size n^* was 1500. We considered the following allocation scenarios:

- A. Use the allocation of the current panels, inflated to a total sample size of 1500.
- B. Use the allocation given by Formula (15).
- C. As B, with the additional restriction that $n_h^* \ge 15/C_h$ in each sector.
- D. As C, with the additional restriction that the standard error of \hat{I}_h^* is at most equal to some fixed upper bound \mathcal{U}_h .
- E. As D, with W_h^* replaced by $\sqrt{W_h^*}$ in Formula (15).
- F. As E, with the sample size for the sector "Advertising agencies" fixed a priori.

Scenario A was included to compare our results with the current allocation. Scenario B is the basic allocation strategy that was proposed in Section 3.1. Under this scenario, the accuracy of the sector-level and domain-level SPPIs is not explicitly taken into account. To address this, we introduced two additional constraints. The

restriction added in Scenario C ensures that the expected effective sample size in each domain equals at least 15. (This number was chosen by the SPPI production staff.) Under Scenario D, in addition an upper bound was placed on the standard error of the sector-level SPPI. In consultation with the production staff, we fixed this upper bound at 0.5 index points for sectors observed using MP and 1.0 index points for sectors observed using ATP. Scenario E maintained these restrictions but used the square root of the sector weights in Formula (15) to reduce the dependence of the allocation on economic importance. Finally, Scenario F was introduced to treat the sector "Advertising agencies" separately. This sector had by far the largest value of $\hat{\sigma}^2_{ph,SRS}$ and was therefore assigned about one-fifth of the total sample under Scenarios B–E. According to the production staff, the large value of $\hat{\sigma}^2_{ph,SRS}$ is caused by low data quality in this sector and should not be used as a basis for sample allocation. In consultation with the production staff, we fixed the sample size for this sector at $n_h^st=80$ under Scenario F. The additional restrictions for Scenarios C–F were implemented heuristically and iteratively, by fixing any n_h^st that failed a restriction to the nearest feasible value and re-calculating (15) for the remaining sectors.

Table 3.2.1 summarises the results of these allocation scenarios in terms of the standard errors (in index points) of the total SPPI and sector-level SPPIs. By comparing the results for Scenario A and the other scenarios, it is seen that a substantial improvement in the accuracy of the total SPPI could be achieved by reallocating the current sample. The largest improvement in accuracy of \hat{I}_{tot}^* occurred under Scenario B. However, with this scenario the gained accuracy at the total level would be offset by a large drop in the accuracy of sector-level SPPIs for sectors observed using MP. As expected, the refinements introduced by Scenarios C, D and E caused a decrease in the accuracy of the total-level SPPI – albeit a slight one – but also a substantial improvement at the sector level. On balance, we concluded that Scenario E gave the best results here. As explained above, Scenario F was introduced to deal with a particular problem with our data. The latter allocation scenario was eventually chosen.

3.2.1 Table: The effect of various allocation scenarios on the accuracy of the estimated SPPI (all S.E.s in index points).

| Scenario | S.E.(total) | mean of S.E. (sector) | | | |
|----------|-------------|-----------------------|------------------|-----------------|--|
| | | all sectors | ATP sectors only | MP sectors only | |
| Α | 0.238 | 0.538 | 0.939 | 0.377 | |
| В | 0.135 | 0.857 | 0.658 | 0.936 | |
| С | 0.137 | 0.511 | 0.709 | 0.432 | |
| D | 0.151 | 0.442 | 0.741 | 0.323 | |
| E | 0.154 | 0.420 | 0.685 | 0.313 | |
| F | 0.183 | 0.434 | 0.772 | 0.299 | |

4. Panel rotation

SPPI estimation for dynamic populations

In practice, the SPPI is calculated as a chain index rather than as a direct index, to take population dynamics into account. At the population level, the chain-index formulation of the SPPI of sector h for quarter q in year t is defined recursively by:

$$I_{h}^{*}(t,q;0) = I_{h}^{*}(t-1,u;0) \times I_{h}^{*}(t,q;t-1,u),$$

$$I_{h}^{*}(t,q;t-1,u) = \frac{\sum_{b} X_{hb}(t-1,u;ref)P_{hb}(t,q;t-1,u)}{\sum_{b} X_{hb}(t-1,u;ref)},$$

$$X_{hb}(t-1,u;ref) = X_{hb}(ref) \times P_{hb}(t-1,u;ref).$$
(18)

Here, $I_h^*(t-1,u;0)$ denotes an SPPI for the last quarter of year t-1 with respect to the base period, and $I_h^*(t,q;t-1,u)$ denotes a price index for the current quarter with respect to the last quarter of the previous year. Similarly, $P_{hb}(t-1,u;ref)$ and $P_{hb}(t,q;t-1,u)$ denote corresponding price indices for enterprise b. The yearly adaptation of turnover weights that occurs in the last line of (18) is known as price updating. See, e.g., van der Grient and de Haan (2011) for more details.

It can be shown that, if the populations of enterprises and services remain fixed between the base period and guarter q of year t, then the chain index (18) and the direct index (1) (or rather, its extension from Section 2.3 that incorporates the takeall stratum) yield identical values. In practice, these populations do change over time: new enterprises are born, others cease to exist, and some enterprises may change their activities (services) and therefore move to different domains or even sectors.

At SN, these changes in the population are handled at the yearly transitions, by letting each short-term index $I_h^*(t,q;t-1,u)$ refer only to the population of enterprises that are active within sector h in both periods. These short-term indices are then chained together using Expression (18). An additional complication is that the variable $X_{hh}(ref)$ is not defined for enterprises that were born after the original reference period. Moreover - and particularly relevant for PPS sampling -, enterprises can grow or shrink over time, which means that weights based on $X_{hh}(ref)$ may not reflect the importance of units in the population at later time points, particularly if long chains are used. We therefore replace $X_{hb}(t-1,u;ref)$ in Expression (18) by $X_{hb}(t-1,u;ref_t) = X_{hb}(ref_t) \times P_{hb}(t-1,u;ref_t)$, where ref_t denotes the most recent year for which turnover values are available in the GBR at the end of year t-1. In practice, $ref_t = t-2$.

To estimate $I_h^*(t,q;t-1,u)$, a panel of enterprises selected by PPS sampling can still be used, but now the panel needs to be updated at each yearly transition. Firstly, the panel should be made representative for the current population by removing units that are no longer active (in sector h) and by selecting new-born units and units that

have moved to sector h. Secondly, the inclusion probabilities for all units should be based on the most recent available turnover information. That is to say, for the sample of size $n_h^*(t)$ that is taken in year t, it should hold that

$$\pi_{hb}(t) = \begin{cases} n_h(t)X_{hb}(ref_t)/X_h(ref_t) & \text{if } X_{hb}(ref_t) < X_h(ref_t)/n(t), \\ 1 & \text{if } X_{hb}(ref_t) \ge X_h(ref_t)/n(t). \end{cases}$$
(19)

Here, $X_h(ref_t)$ denotes the sum of $X_{hb}(ref_t)$ for all active units outside the take-all stratum, and $n_h(t)$ denotes the sample size outside the take-all stratum for year t. In addition to these panel updates that are necessary to avoid selection bias, SN also wants to apply panel rotation to reduce the burden on responding enterprises. We will discuss methods for achieving both objectives in the next subsection. The use of panel rotation in addition to panel updating does not affect the estimation procedure.

Based on a PPS sample of active enterprises in sector h with inclusion probabilities $\pi_{hb}(t)$ given by (19), the short-term sector-level index $I_h^*(t,q;t-1,u)$ can be estimated analogously to Expression (9):

$$\hat{I}_{h}^{*}(t,q;t-1,u) = \frac{\sum_{i=1}^{n_{h}^{*}(t)} \omega_{hi}(t-1,u;ref_{t})p_{hi}(t,q;t-1,u)}{\sum_{i=1}^{n_{h}^{*}(t)} \omega_{hi}(t-1,u;ref_{t})},$$

$$\omega_{hi}(t-1,u;ref_{t}) = \omega_{hi}(ref_{t}) \times p_{hi}(t-1,u;ref_{t}),$$

$$\omega_{hi}(ref_{t}) \propto \begin{cases} X_{h}(ref_{t})/n_{h}(t) & \text{if } \pi_{hi}(t) < 1, \\ x_{hi}(ref_{t}) & \text{if } \pi_{hi}(t) = 1. \end{cases} \tag{20}$$

For the domain-level SPPIs and total SPPI, similar definitions as chain indices can be given and the associated PPS-based estimators follow analogously. It should be noted that no straightforward expressions exist for the variances of these estimated chain indices. In Section 3, we therefore used the variances of the associated direct indices as an approximation to derive a sample allocation.

4.2 Methods for panel rotation

To reduce the response burden, SN wants to rotate some of the enterprises out of the sample at each yearly transition and replace them with new units. In the context of a PPS sample, it is obvious that panel rotation cannot be applied to the take-all stratum. For units outside the take-all stratum, panel rotation is applied to each sector independently. For simplicity, we will consider one sector and suppress the index h in the notation for the remainder of this section. Denote the panel for years t-1 and t without the take-all stratum by S(t-1) and S(t).

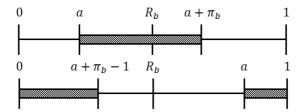
To update the PPS panel at the yearly transition between t-1 and t, we begin by determining the new take-all stratum for year t, using the procedure described in Section 2.3. We then remove any units from S(t-1) that are no longer active (within this sector). Next, a PPS sample is drawn from the subpopulation of new-born units and added to the current panel. The inclusion probability of new-born unit b for this step is given by $n(t-1)X_b(ref_t)/X(ref_{t-1})$; i.e., we mimic $\pi_b(t-1)$ from Expression (19) but use $X_b(ref_t)$, as $X_b(ref_{t-1})$ is undefined for new-born units. This step ensures that new-born units and continuing units have the same point of departure for panel rotation, which is important to avoid selection bias in the long run. Denote the updated panel without the take-all stratum by $\check{S}(t-1)$. Next, the updated panel is rotated. The amount of panel rotation is controlled by the so-called rotation fraction. We define a rotation fraction of λ (0 $\leq \lambda \leq 1$) to mean that $\lambda \times 100\%$ of the units in $\check{S}(t-1)$ are not part of S(t). Ideally, the rotation procedure should yield a new panel of size n(t), in which each unit is given its exact inclusion probability $\pi_b(t)$, while also achieving the desired rotation fraction.

Various methods for rotating PPS samples have been proposed. We will compare four of them here. The first three are variations of so-called Permanent Random Number (PRN) methods. PRNs are often used for the general problem of sample coordination (Ohlsson, 1995; Lindblom, 2014), of which panel rotation is a special case. The PRN principle entails that each unit in the population is assigned an independent random number R_b from the uniform distribution on [0,1) which does not change over time. New units are assigned a PRN at birth. Samples are drawn by selecting units based on their PRNs.

1. Poisson sampling with PRNs

Probably the simplest PRN method that is suitable for PPS sampling is obtained by considering the PPS sample as a Poisson sample. In this case, the selection mechanism is independent across units. To select an initial Poisson sample in year 0 from a population of units with PRNs $R_1, ..., R_N$ and inclusion probabilities $\pi_1(0), \dots, \pi_N(0)$, one chooses a starting point $\alpha(0) \in [0,1)$ and selects all units with $a(0) < R_b \le a(0) + \pi_b(0)$ (Ohlsson, 1995). It is easy to see that this indeed yields a sample with the desired first-order inclusion probabilities. When $a(0) + \pi_b(0) > 1$, the selection interval is truncated at 1 and carried on from 0. This is illustrated in Figure 4.2.1.

4.2.1 Figure: An illustration of Poisson sampling using PRNs. Unit b is selected in the sample if, and only if, its PRN R_h lies in the shaded part of the interval [0, 1].



To apply panel rotation to a Poisson sample that is drawn this way, one simply moves the starting point a(t-1) to a new position $a(t) \ge a(t-1)$. This causes some units to be removed from the sample, because their PRNs do not belong to their new selection intervals, while other units may enter the sample. For a given population

and sample, it is straightforward to work out the minimal adjustment to a(t-1)that is required to obtain a desired rotation fraction λ . This approach also works for method 2 and 3 below.

An advantage of Poisson sampling for panel rotation is that it produces PPS samples that exactly respect the nominal first-order inclusion probabilities of all units in the population at all times. Of the four methods that will be considered in this paper, only Poisson sampling has this property. On the other hand, because this method selects units independently, the sample size is random. Thus, the panel size is not known beforehand and may vary considerably from year to year, which is an important practical drawback. [In addition, the randomness of the sample size might increase the variance, but this can be alleviated by using a ratio estimator such as (10).] Alternative, fixed-size sampling methods are therefore of interest. Here, we consider two simple PRN-based methods. Recently, Grafström and Matei (2015) have suggested an alternative coordination method based on conditional Poisson sampling (i.e., conditional on the sample size); we did not include this more complicated approach in the present study.

2. Sequential Poisson sampling with PRNs

We rewrite the PRN of unit b for year t as

$$r_b(t) = (R_b - a(t)) \mod 1 = \begin{cases} R_b - a(t) & \text{if } a(t) \le R_b \\ R_b - a(t) + 1 & \text{if } R_b < a(t) \end{cases}$$
(21)

and define $\rho_b(t) = r_b(t)/\pi_b(t)$. For Poisson sampling, it can be shown that unit b is selected in the PPS panel for year t precisely when $\rho_b(t) \le 1$. Ohlsson (1995, 1998) proposed to obtain an approximate PPS sample of fixed size n(t) by sorting the population in ascending order of $\rho_h(t)$ and selecting the first n(t) units. This method is known as sequential Poisson sampling. The sample size is now fixed, but the actual inclusion probability of unit b may not be exactly equal to $\pi_h(t)$. Some bias may therefore be incurred in the HT estimator with the nominal inclusion probabilities.

3. Pareto sampling with PRNs

Pareto sampling, due to Rosén (1997), works similarly to sequential Poisson sampling but uses the following PRN transformation instead of $\rho_b(t)$:

$$\tilde{\rho}_b(t) = \frac{r_b(t)/[1 - r_b(t)]}{\pi_b(t)/[1 - \pi_b(t)]}.$$
(22)

Rosén (2000) argued that this transformation should improve the approximation to the nominal inclusion probabilities in practice, in comparison to sequential Poisson sampling. In a simulation study, Ohlsson (2000) verified this for small sample sizes $(n \le 4)$. Aires and Rosén (2005) showed by simulation that, for populations of sizes $N \leq 200$, the Pareto sampling method works well under a variety of conditions. Nonetheless, the Dutch SPPI involves populations that are much larger and the turnover distribution in these populations is more skewed than was considered by Aires and Rosén (2005).

Aires (1999) derived a recursive method for computing the realised inclusion probabilities under Pareto sampling. Using this method, it is – in theory – possible to make adjustments to obtain the exact nominal inclusion probabilities with Pareto sampling. However, this approach is too computationally demanding for our application. (The median population size of the SPPI sectors is about 3300; the median sample size is about 40.)

4. Circular systematic PPS sampling

The final method that we consider is not a PRN method, but a relatively simple extension of the systematic PPS sampling method that is already used at SN for fixed panels and cross-sectional PPS samples. This method was proposed by Knottnerus and Enthoven (2012). The idea is to draw systematic PPS samples by cycling through a randomly ordered list; when the bottom of the list is reached, we start again at the top. Panel rotation is applied in two steps. Denote the size of the current panel (after panel updating but before rotation) by $\check{n}(t)$. First, a simple random subsample of $\max\{\lambda \check{n}(t), \check{n}(t) - n(t)\}$ units is removed from the panel. Then, an additional PPS sample is drawn from the units that are currently not in the panel, so that the total panel size becomes n(t). The first step uses the fact that a simple random subsample of a PPS sample is again a PPS sample. Like methods 2 and 3, this method yields fixedsize samples with inclusion probabilities that may differ from the nominal ones.

With any sampling method where the panel is being updated and/or rotated, there might be some units that alternate from year to year between being (just) inside the take-all stratum and outside it with an inclusion probability close to 1.0. These units might then be rotated in and out of the sample quite often, which is undesirable in practice. To alleviate this problem, Knottnerus and Enthoven (2012) suggested to define a slightly larger take-all stratum by including all units with

$$\max\{\pi_h(t), \pi_h(t-1), \pi_h(t-2)\} \ge 1$$
 and $\pi_h(t) \ge \gamma$

for some value $0 < \gamma < 1$. We followed this suggestion with $\gamma = 0.8$.

4.3 **Simulation study**

To compare the usefulness of the above panel rotation methods for the Dutch SPPI, we conducted a simulation study. To this end, we created a synthetic population with price indices for ten years, based on real SPPI panel data and real sampling frames for the sector "Road transport of goods". This sector consists of five domains. Population dynamics and turnover distributions were modelled on the sampling frames for 2013 and 2014. The quarterly price mutations for the synthetic population were modelled on the real SPPI panel data of 2013 and 2014. We refer to Appendix B for more details on the construction of this synthetic population. In particular, price mutations were imputed in such a way that the sector-level and domain-level SPPIs for the synthetic population approximated the published quarterly indices for "Road transport of goods" in the period 2005–2014. In what follows, we therefore describe

the results as if the study refers to this ten-year period. The population size varied from 9,815 units in "2005" to 10,908 units in "2014".

Panel surveys were simulated using each of the four rotation methods from Section 4.2, by drawing a PPS sample from the synthetic population of "2005" and then performing yearly panel updates and rotation until "2014". Different PRNs were assigned in each simulation round, but within each round the same PRNs were used for the three PRN-based methods. For each simulated panel, the sector and domain SPPIs were also estimated using the theory of Section 4.1.

The panel rotation methods were evaluated based on the difference between the actual and nominal inclusion probabilities and the accuracy of the estimated indices. After R simulation rounds, the actual inclusion probability of unit b in year t can be estimated by $\hat{\pi}_{bR}(t) = R^{-1} \sum_{r=1}^{R} \iota\{b \in S_r^*(t)\}$, with $S_r^*(t)$ the panel for year t in round r and $\iota\{.\}$ an indicator function. Under the hypothesis that a rotation method produces PPS samples with the nominal inclusion probabilities $\pi_h(t)$, $\iota\{b \in S_r^*(t)\}$ is distributed as a Bernoulli variable with mean $\pi_b(t)$ and variance $\pi_b(t)[1-\pi_b(t)]$. Hence, assuming that $R\pi_h(t)\gg 1$ and $R[1-\pi_h(t)]\gg 1$, the residual

$$T_{bR}(t) = \sqrt{R} \frac{\hat{\pi}_{bR}(t) - \pi_b(t)}{\sqrt{\pi_b(t)[1 - \pi_b(t)]}}$$
(23)

should follow a distribution that is approximately standard normal. Any large deviations from the standard normal distribution would indicate that some units are selected much more or less often than they should be according to their nominal inclusion probabilities. To evaluate the effect of such deviations on estimates, we also computed the empirical bias and root mean squared error (RMSE) of the estimated sector and domain SPPIs, again based on R simulation rounds.

The simulations were repeated with different parameter settings. Here, we present the results for R = 20,000 samples of size $n^*(t) = 71$ in all years and a rotation fraction $\lambda = 0.1$; this corresponds to the actual situation in production for the sector "Road transport of goods". The results for other settings led to similar conclusions.

Figure 4.3.1 displays the histograms of $T_{bR}(2005)$ and $T_{bR}(2006)$ (the first and second year of the simulation). Given the assumptions stated above Formula (23), we only included units with $0.01 \le \pi_h(t) \le 0.99$ here. For comparison, the standard normal density is plotted as a red dashed line. It can be seen that all four methods produced reasonable PPS samples for 2005, the starting year. For 2006, when panel rotation was applied, some differences occurred. As expected, the Poisson sampling method still achieved the nominal inclusion probabilities; in fact, for this method the distribution of $T_{bR}(t)$ was similar for all years (not shown here). Of the other methods, Pareto sampling approximated the nominal inclusion probabilities very well. Sequential Poisson sampling performed reasonably well, but not as well as Pareto sampling. With the circular systematic PPS method, some very large residuals occurred. The results for the other years (2007–2014) were similar.

4.3.1 Figure: Histograms of the residuals $T_{bR}(t)$ for each rotation method, with $n^*(t) = 71$ and $\lambda = 0.1$, based on R = 20,000simulations: (a) t = 2005 (first year); (b) t = 2006.

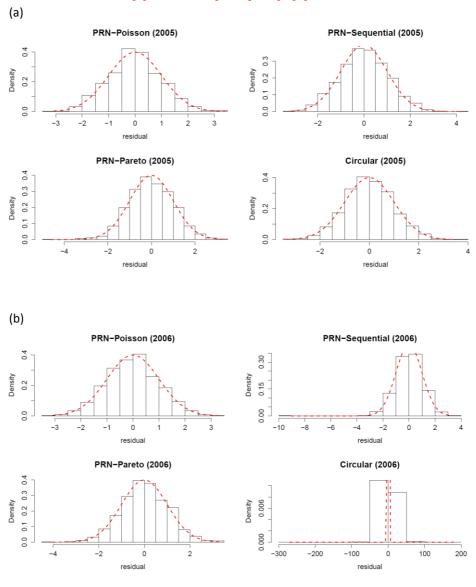
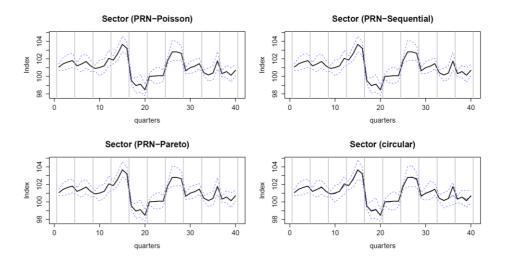


Figure 4.3.2 displays the estimated sector price indices in comparison to the true population index. We computed the empirical bias and RMSE of the estimated indices for each quarter in 2005–2014; Table 4.3.3 summarises the results across the 40 quarters in this period. Both the figure and the table show that the accuracy of the estimated indices was similar for all three PRN methods. In particular, the random sample size of the Poisson method did not increase the variance of the estimator. The circular systematic PPS method yielded estimates that were slightly less accurate. This was also the only method for which a significant bias occasionally occurred. The most striking example can be seen in Figure 4.3.2 for the last quarter of the penultimate year. For the domain-level SPPIs, similar results were found (not shown here).

4.3.2 Figure: Estimated sector SPPI for each rotation method, with $n^*(t) = 71$ and $\lambda = 0.1$, based on R = 20,000 simulations. Vertical grey lines indicate years. Each plot contains the true population index (black line), the average estimated index (solid blue line) and its empirical 5% and 95% quantiles (dashed blue lines).



4.3.3 Table: Bias and RMSE (in index points) of the estimated sector SPPI (distribution over 40 quarters), with $n^*(t) = 71$ and $\lambda = 0.1$, based on R = 20,000 simulations.

| | | PRN-Poisson | PRN-Sequent. | PRN-Pareto | Circular PPS | | | |
|------|--------|-------------|--------------|------------|--------------|--|--|--|
| bias | min. | -0.01 | -0.01 | -0.01 | -1.00 | | | |
| | median | 0.00 | 0.00 | -0.00 | -0.00 | | | |
| | max. | 0.07 | 0.03 | 0.06 | 0.17 | | | |
| | | | | | | | | |
| RMSE | min. | 0.20 | 0.20 | 0.20 | 0.22 | | | |
| | median | 0.46 | 0.46 | 0.46 | 0.49 | | | |
| | max. | 0.69 | 0.69 | 0.69 | 1.26 | | | |

An explanation for the relatively poor performance of the circular systematic PPS method may lie in the use of simple random sampling to remove units from the panel. As noted above, the resulting subsample of the panel for year t-1 is again a PPS sample, but importantly, it is a PPS sample with inclusion probabilities proportional to turnover in year ref_{t-1} , not ref_t . Therefore, this method will yield incorrect inclusion probabilities for units with large growths or declines in turnover. In fact, the large bias in the estimated index based on the circular systematic method that was seen in Figure 4.3.2 for the last quarter of the penultimate year could be traced back to a single unit with a sudden large increase in turnover combined with an usually large price mutation. For the three PRN methods, the selection of the panel for year tis based only on turnover values in year ref_t . These methods are therefore more robust to growing and shrinking units.

In principle, it should be possible to improve the circular systematic method by using a PPS subsample rather than a simple random subsample to remove units from the panel, to account for differences between $\pi_b(t-1)$ and $\pi_b(t)$. We did not investigate this further within the present study.

Based on the results of this simulation study, we propose to use Pareto sampling to obtain panel rotation for the Dutch SPPI. This method achieves the same accuracy and approximately unbiased estimation as Poisson sampling, but with fixed sample sizes.

Retrospective assignment of PRNs for Pareto sampling

In the discussion so far, it has been assumed that PRNs can be assigned to the population of enterprises before the initial PPS panel is drawn, to be used for panel updating and rotation in later years. While this is true for most sectors in the Dutch SPPI, there are also a few sectors for which a PPS panel has been set up previously using the (non-PRN-based) systematic sampling method mentioned in Section 2.1. For these sectors, we would like to be able to apply the same panel rotation method, based on Pareto sampling, using the current panel as a starting point. This is possible only if we can assign PRNs to the enterprises in these sectors in such a way that the initial Pareto sample coincides with the current panel. Obviously, we also require that panel rotation using these "pseudo-PRNs" should still yield samples for later periods with approximately correct inclusion probabilities and hence estimated price indices that are approximately unbiased.

This is a special case of a general problem that is known in the literature as the retrospective assignment of PRNs for sample coordination (Ernst, 2001; Fay, 2014). Exact solutions to this problem have been proposed for some specific PRN methods, but these rely on computational work that is feasible only for very small populations and/or samples. For the Dutch SPPI, we propose to use the following heuristic procedure which appears to give reasonable results in practice.

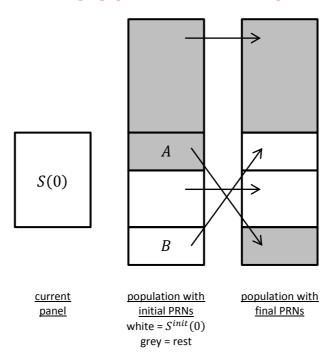
Let S(0) denote the current PPS sample of size n(0) which has been obtained by a non-PRN-based selection method. (We ignore the take-all stratum here as its selection does not depend on PRNs.) We carry out the following steps (see also Figure 4.4.1):

- 1. Randomly assign initial PRNs R_h^{init} to all units in the population and draw a new PPS sample $S^{init}(0)$ of size n(0) by Pareto sampling, for some arbitrary starting point a(0). That is to say, $S^{init}(0)$ consists of the n(0) units in the population with the smallest values of $\tilde{\rho}_{h}^{init}(0)$, computed from R_{h}^{init} through (21) and (22).
- 2. Compare the current panel S(0) to $S^{init}(0)$. Let A denote the subset of units in S(0) that are not selected in $S^{init}(0)$, and suppose that there are l such units. Conversely, there must then be l units in $S^{init}(0)$ that are not selected in S(0); denote this subset of units by B.

- 3. Re-arrange both sets of units A and B separately in order of their inclusion probabilities $\pi_h(0)$. Denote the transformed initial PRN of unit j in this order by $\tilde{\rho}_{Aj}^{init}(0)$ for set A and $\tilde{\rho}_{Bj}^{init}(0)$ for set B.
- 4. Assign the final PRNs R_h as follows:
 - a. Units in the population outside of $S(0) \cup S^{init}(0)$ and the n(0) lunits in the overlap $S(0) \cap S^{init}(0)$ all retain their initial PRNs: $R_b = R_b^{init}$.
 - b. Unit *j* in the ordered set *A* is assigned the transformed initial PRN of unit j in the ordered set B, and vice versa: $\tilde{\rho}_{Aj}(0) = \tilde{\rho}_{Bj}^{init}(0)$ and $ilde{
 ho}_{Bj}(0)= ilde{
 ho}_{Aj}^{init}(0).$ Finally, the associated PRNs R_{Aj} and R_{Bj} are obtained by applying the inverse transformations of (22) and (21).

In Step 4, the final PRNs are assigned in such a way that the associated Pareto sample of size n(0) with starting point a(0) is equal to S(0), as we intended. This means that, in principle, these PRNs can be used to rotate the current panel by Pareto sampling in future years $(t \ge 1)$.

4.4.1 Figure: Illustration of a procedure for the retrospective assignment of PRNs. The middle and right rectangles represent the target population at the end of Step 1 and Step 4, respectively.



The idea behind this procedure is that PRN-based methods for sample selection are valid when the assigned PRNs are drawn from the uniform distribution on [0,1). Therefore, heuristically, we may change some of the PRNs in a population (as in Step 4 above) provided that this does not affect the distribution of PRNs. In Step 3, the units are ordered such that transformed PRNs will be interchanged between units with similar inclusion probabilities. It can be shown that in this case the changed PRNs indeed still follow an approximate uniform distribution; see Appendix C.

To check whether the above heuristic procedure yields acceptable results in practice, we applied it to the same synthetic population that was used for the simulation study in Section 4.3. We simulated R = 20,000 samples of size $n^*(t) = 71$ for a period of forty quarters with a yearly rotation fraction $\lambda = 0.1$, as before, but this time the initial PRNs were adjusted using the above heuristic procedure before the first year of simulation.

4.4.2 Figure: Histograms of the residuals $T_{bR}(t)$ for Pareto sampling with retrospectively assigned PRNs for four simulated years (2005 = first year), with $n^*(t) = 71$ and $\lambda = 0.1$, based on R = 20,000 simulations: (a) units ordered by inclusion probability; (b) units ordered by transformed initial PRN.

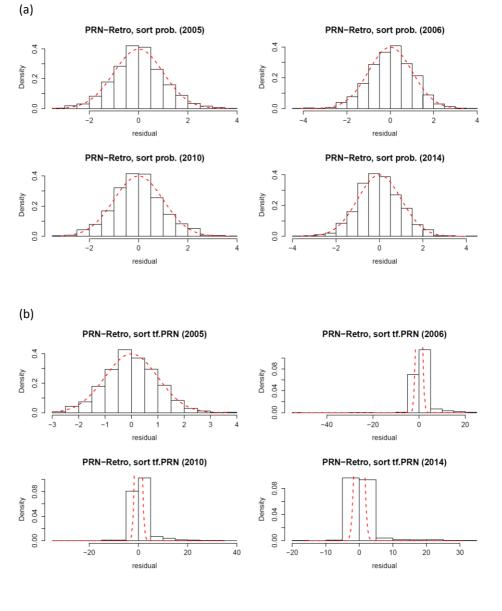


Figure 4.4.2(a) displays histograms of the residuals $T_{bR}(t)$ of the realised inclusion probabilities, computed by Formula (23) as before, for the first, second, fifth and tenth year of the simulated period. It is seen that the realised inclusion probabilities under the heuristic procedure are close to their nominal counterparts. By comparing these results to Figure 4.3.1, it appears that the retrospective assignment of PRNs by this procedure does not have an adverse effect on the Pareto sampling method, i.e., the quality of the approximation to the inclusion probabilities is similar to that for Pareto sampling with the initial PRNs.

To illustrate the importance of Step 3 in our procedure, we have also simulated an alternative approach in which the units in sets A and B are not ordered by their inclusion probabilities $\pi_b(0)$ but by their transformed initial PRNs $\tilde{
ho}_h^{init}(0)$, with all other steps left unchanged. The resulting residuals are displayed in Figure 4.4.2(b). In this case, it is seen that the procedure yields incorrect inclusion probabilities from the second year onwards, when the retrospectively assigned PRNs are used for panel rotation. This suggests that the uniform distribution of the initial PRNs is distorted by this alternative procedure.

In fact, it follows from the proof in Appendix C that the quality of the approximation to the uniform distribution under our heuristic procedure depends on the degree of alignment between inclusion probabilities of units in sets A and B. In Step 3 above, this alignment was obtained simply by arranging both sets of units in order of their inclusion probabilities. In general, a better alignment may be achieved by matching the units in both sets through some more advanced matching procedure, e.g., by minimising the total discrepancy between inclusion probabilities of matched units over all possible alignments. We did not investigate this further, as the above simple procedure appeared to yield results that were sufficiently accurate for our application.

5. Discussion

In this paper, we have described the recent redesign of the sampling and estimation strategy for the Dutch producer price indices on services. The new panels are based on a PPS sample design, stratified by NACE sector, with annual turnover as a size variable. The grouping of NACE codes into sectors was done in such a way that all published price indices are either sector-level indices or more detailed indices about domains that exactly fit within a sector. As there is no population frame of services available below the sector level, the PPS panels were drawn instead from a sampling frame of enterprises. As described in Section 2, all relevant price indices could be estimated from these panels. In particular, a ratio estimator was used to estimate price indices on service domains within the stratification by sector.

To distribute the total available sample size across different economic sectors, we have proposed a Neyman-like allocation. In this allocation, we aimed to optimise the accuracy of the total service price index (a weighted average of the sector indices), while also ensuring that the individual sector-level indices are sufficiently accurate. In addition, we added constraints on the expected sample sizes at the domain-level to ensure that each domain was sufficiently covered. In this study, we used a heuristic method to obtain an approximate solution to this restricted allocation problem. Others have developed exact algorithms for sample allocation under restrictions; see, e.g., Friedrich et al. (2015).

Finally, a panel updating and rotation strategy was introduced. Regular panel updates to add new-born units and remove units that are no longer active are necessary to ensure that a panel remains representative over time when the population is dynamic. In addition, regular panel rotation of continuing units can be useful to reduce the response burden of units that are selected into the panel and to reduce the risk of panel attrition due to non-response. As discussed in Section 4.1, the introduction of panel updating and rotation has several consequences for the definition and estimation of the target price index, notably the need to use a chain index and the price updating of turnover weights. For PPS sampling in particular it is also important to account for the fact that enterprises will grow or shrink over time. In our application, we have used the most recent available yearly turnover values as a basis for sample selection and weighting.

Based on the results of a simulation study, we decided to use Pareto sampling with Permanent Random Numbers as a panel rotation method. Although this approach does not achieve the exact nominal inclusion probabilities, we found the deviations to be negligible in our application. This result is in line with previous research (Ohlsson, 2000; Aires and Rosén, 2005).

Two advantages of Pareto sampling are that it yields fixed-size panels and that it is relatively easy to apply in practice, without a need for extensive computations. Several exact, fixed-size approaches to panel rotation - or, more generally, sample

coordination - have been developed for PPS sampling in the past, but typically these methods are complex and computationally feasible only for very small populations and/or samples. One exception may be the conditional Poisson sampling method that was recently proposed by Grafström and Matei (2015), which appears to be computationally feasible also for larger sample sizes, although it is still much more complex than the methods we considered here. In our application, Pareto sampling yielded acceptable results but there may be conditions under which the approximate nature of this method becomes problematic. For future research, it would therefore be interesting to compare Pareto sampling, conditional Poisson sampling and possibly other panel rotation methods in a larger simulation study that involves a wider variety of conditions.

In Section 4.4, we also proposed a simple heuristic procedure for the retrospective assignment of PRNs in cases where a previous PPS panel has been selected by a non-PRN-based method and one would still like to rotate this panel using Pareto sampling. This procedure is not exact, but it appears to provide a reasonable approximation in our application. Again, it would be interesting for future research to test the wider applicability of this heuristic procedure in a more extensive simulation study.

The results of this paper are currently being implemented in practice at Statistics Netherlands. New PPS panels of enterprises are being introduced for producer price indices on services, one sector at a time, to replace the old panels based on simple random sampling. The size of each new panel is derived from the allocation under Scenario F in Section 3.2. According to current plans, the yearly panel updating and rotation strategy will be applied to these new panels for the first time in the last guarter of 2017.

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Appendix A: Justification of Formula (17)

In this appendix, we will show that $\hat{\sigma}^2_{ph,SRS}$ from (17) is an asymptotically unbiased estimator for σ_{ph}^2 under a simple random sampling design. Firstly, we note that

$$\sum_{i=1}^{m_h} x_{hi} (p_{hi} - \hat{I}_{h,SRS})^2 = \sum_{i=1}^{m_h} x_{hi} (p_{hi} - I_h)^2 - (\hat{I}_{h,SRS} - I_h)^2 \sum_{i=1}^{m_h} x_{hi}.$$
 (24)

This may be derived from the following decomposition

$$\sum_{i=1}^{m_h} x_{hi} (p_{hi} - \hat{I}_{h,SRS})^2 = \sum_{i=1}^{m_h} x_{hi} \{ (p_{hi} - I_h) + (I_h - \hat{I}_{h,SRS}) \}^2,$$

by using (16) to establish that

$$2(I_h - \hat{I}_{h,SRS}) \sum_{i=1}^{m_h} x_{hi}(p_{hi} - I_h) = -2(\hat{I}_{h,SRS} - I_h)^2 \sum_{i=1}^{m_h} x_{hi}.$$

Next, we can use (24) to rewrite the expected value of $\hat{\sigma}_{ph,SRS}^2$ as follows:

$$E(\hat{\sigma}_{ph,SRS}^2) = E\left\{\frac{\sum_{i=1}^{m_h} x_{hi} (p_{hi} - I_h)^2}{\sum_{i=1}^{m_h} x_{hi}}\right\} - E(\hat{I}_{h,SRS} - I_h)^2 + E\{\widehat{\text{var}}(\hat{I}_{h,SRS})\}.$$
(25)

The right-most term in (25) is approximately equal to $var(\hat{I}_{h,SRS})$. The second term represents the mean squared error of $\hat{I}_{h.SRS}$. Since $\hat{I}_{h.SRS}$ is a ratio estimator, its bias under a simple random sampling design is known to be of the order $O(1/m_h)$; see, e.g., Cochran (1977, p. 160). Hence, $E(\hat{I}_{h.SRS} - I_h)^2 \approx \text{var}(\hat{I}_{h.SRS})$ and we may approximate Expression (25) by

$$E(\hat{\sigma}_{ph,SRS}^2) \approx E\left\{\frac{\sum_{i=1}^{m_h} x_{hi} (p_{hi} - I_h)^2}{\sum_{i=1}^{m_h} x_{hi}}\right\} = E\left\{\frac{\sum_{i=1}^{m_h} z_{hi}}{\sum_{i=1}^{m_h} x_{hi}}\right\},\,$$

with $z_{hi} = x_{hi}(p_{hi} - I_h)^2$. Again, we can use the fact that the expected ratio of two sample totals is approximately equal to the corresponding ratio of population totals, apart from an error term of the order $O(1/m_h)$. It follows that:

$$E(\hat{\sigma}_{ph,SRS}^2) \approx \frac{\sum_{b=1}^{N_h} Z_{hb}}{\sum_{b=1}^{N_h} X_{hb}} = \frac{\sum_{b=1}^{N_h} X_{hb} (P_{hb} - I_h)^2}{X_h} = \sum_{b=1}^{N_h} W_{hb} (P_{hb} - I_h)^2 = \sigma_{ph}^2$$

and the proof is completed.

Appendix B: Construction of a synthetic population

The synthetic population that was used in the simulation study of Section 4.3 was constructed in two phases. In the first phase, we formulated models for various features of the population, which were then estimated on real data of enterprises in the sector "Road transport of goods" with five domains. In the second phase, we applied these estimated models to generate new, synthetic data. Both phases consisted of several steps, which will be summarised below.

A. Modelling phase

- 1. (population dynamics) Using the sampling frames of 2013 and 2014, we computed the fractions of births and deaths in the sector between these years.
- 2. (turnover growth rate) Using all continuing units in the two sampling frames, we estimated a linear regression of the yearly turnover in 2014 on the yearly turnover in 2013.
- 3. (number of domains) Using the observed data of the current SPPI panel for the first quarter of 2014, we estimated a Poisson regression model for the number of domains (minus one) in which an enterprise is active.
- 4. (distribution of units across domains) Using the same data as in step A.3, we estimated a logistic regression model, for each domain separately, for the probability that an enterprise is active within that domain.
- 5. (price mutations) Using the observed data of the current SPPI panel, we estimated a regression model for each domain and each quarter of 2014 for the price mutation with respect to the last quarter of 2013 [i.e., the price mutation that occurs in a chain index such as (18)]. Since the event of no change in observed prices occurred unusually often, we introduced a separate modelling step to estimate the probability of observing a price mutation unequal to zero.

B. Generating phase

- 1. (set parameters) We chose a simulation period of ten years (i.e., forty quarters). In the construction that follows, we used the published quarterly price index for each domain in the sector "Road transport of goods" over the period 2005-2014 as a target population index.
- 2. (population dynamics) For the first year of simulation ("2005"), the population consisted of all units in the real sampling frame of 2013 with positive turnover values. For the remaining years, the estimated population dynamics from step A.1 were applied to remove some units from the population at random and add new units. For the purpose of this simulation study, all yearly births and deaths were supposed to occur between the last quarter of year t-1 and the first quarter of
- 3. (turnover growth rate) For the first year of simulation, we used the turnover values from the real sampling frame of 2013 (in line with step B.2). For continuing units, turnover values in later years were generated iteratively from the

- regression model that was estimated in step A.2. For each new born unit, the turnover value in its first year was drawn at random from the (previously generated) turnover values of continuing units for that year.
- 4. (number of domains) We assigned a fixed number of active domains to each unit in the synthetic population by drawing from the Poisson model that was estimated in step A.3.
- 5. (distribution of units across domains) We used the estimated logistic regression models from step A.4 to assign each unit in the synthetic population to one or more domains, taking into account its total number of active domains assigned in step B.4. By default, we distributed the total turnover of each unit evenly across its active domains.
- 6. (price mutations) We used the estimated models from step A.5 to construct a time series of (at most) forty quarterly price mutations for each unit for each of its active domains, by iterating across the four quarters for which these models were estimated.
- 7. (derive final population price indices) Based on the synthetic population data generated in steps B.2–B.6, we computed the realised chain indices for each domain. As some of the above steps involved random draws and the population is finite, these realised indices were not exactly equal to the target indices defined in step B.1. In the simulation study, the realised population price indices found in the present step were used as the target for estimation.

Appendix C: Justification of the procedure in Section 4.4

In Section 4.4, we introduced a heuristic procedure for the retrospective assignment of PRNs in the case of Pareto sampling. As discussed there, this procedure can be justified if it yields PRNs with a distribution that is approximately equal to the uniform distribution on [0,1). We will now prove that this is indeed the case.

Firstly, recall that R_b is uniformly distributed on [0,1) if, and only if, it holds that $P(R_b \le c) = c$ for all $c \in [0,1)$. It is not difficult to show that if R_b is uniformly distributed on [0,1), then, for a fixed starting point a(t), so is $r_h(t)$ from (21). In fact:

$$P(r_b(t) \le c) = P((R_b - a(t)) \mod 1 \le c)$$

$$= \{P(R_b \ge a(t)) \times P(R_b - a(t) \le c)\}$$

$$+ \{P(R_b < a(t)) \times P(R_b - a(t) + 1 \le c)\}$$

$$= (1 - a(t)) \times (c + a(t)) + a(t) \times (c + a(t) - 1)$$

$$= c + a(t) - a(t)$$

$$= c$$

for all $c \in [0,1)$. For the second equality we used (21) and for the third equality we used the fact that R_b is uniformly distributed. Conversely, if $r_b(t)$ is uniformly distributed, then so is R_b ; this can be shown by applying the same argument to $R_b = (r_b(t) + a(t)) \bmod 1.$

Next, we consider the final PRNs from the heuristic procedure in Section 4.4. We only have to consider the units in subsets A and B, since for all other units $R_b = R_b^{init}$. Let $r_{Aj}(0) = \left(R_{Aj} - a(0)\right) \mod 1$ for unit j in the ordered set A, and let $r_{Bj}^{init}(0) =$ $\left(R_{Bj}^{init}-a(0)\right) \mod 1$ for unit j in the ordered set B. We find that, for all $c\in [0,1)$,

$$P(r_{Aj}(0) \le c) = P\left(\tilde{\rho}_{Aj}(0) \le \frac{c/(1-c)}{\pi_{Aj}(0)/[1-\pi_{Aj}(0)]}\right)$$

$$\approx P\left(\tilde{\rho}_{Bj}^{init}(0) \le \frac{c/(1-c)}{\pi_{Bj}(0)/[1-\pi_{Bj}(0)]}\right)$$

$$= P(r_{Bj}^{init}(0) \le c)$$

$$= c.$$

In the first line, we used the fact that the transformation $r_b(t) \rightarrow \tilde{\rho}_b(t)$ defined in (22) is a monotone increasing function from [0,1) to $[0,\infty)$, and in the third line we used that this function is invertible. In the second line, we substituted $\hat{\rho}_{Bi}^{init}(0) =$ $\tilde{\rho}_{Ai}(0)$ and $\pi_{Bi}(0) \approx \pi_{Ai}(0)$. It follows that $r_{Ai}(0)$ is approximately uniformly distributed and, as noted above, this is equivalent to showing that R_{Ai} is approximately uniformly distributed. The proof that R_{Bj} is approximately uniformly distributed is analogous.

Explanation of symbols

Empty cell Figure not applicable

Figure is unknown, insufficiently reliable or confidential

Provisional figure

Revised provisional figure

2014-2015 2014 to 2015 inclusive

2014/2015 Average for 2014 to 2015 inclusive

2014/'15 Crop year, financial year, school year, etc., beginning in 2014 and ending in 2015

2012/'13-2014/'15 Crop year, financial year, etc., 2012/'13 to 2014/'15 inclusive

Due to rounding, some totals may not correspond to the sum of the separate figures.

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